Monetary Policy with Racial Inequality*

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Abstract

I develop a heterogeneous-agent New-Keynesian model featuring racial inequality in income and wealth, and studies heterogeneous effects of monetary policy. Black and Hispanic workers gain more from accommodative monetary policy than White workers, because of higher labor market risks and a higher fraction of hand-to-mouth, although wealthy White gain from asset price appreciation. I also find that replacing the overall unemployment rate with the Black unemployment rate as a policy target is equivalent to making monetary policy more accommodative with the overall unemployment rate, because the unemployment rate for all racial groups move in parallel over the business cycle.

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1 Introduction

As inequality in income and wealth grew over time, and racial inequality is gaining recognition, the question as to what the monetary authority can and should do about racial inequality has becomes more important. According to the conventional view, the monetary authority can only smooth business cycles mainly using policy rate adjustments, and the long-term structural issues such as racial disparities cannot be dealt with by monetary policy. However, the opinion that the monetary authority has to do something about the racial disparities is getting stronger. Then Democratic presidential nominee Biden proposed in a speech in July 2020 that Congress amend the Federal Reserve Act to *"add to that responsibility and aggressively target persistent racial gaps in job, wages, and wealth."* The Federal Reserve is considered by many to have taken a step towards more emphasis on inequality already. For example, Chair Powell of the Federal Reserve unveiled the new strategy which *"emphasizes that maximum employment is a broad-based and inclusive goal"* in August 2020. However, it is a fair assessment that there is no consensus yet as to what the monetary authority can and should do to. The fact that the Federal Reserve recently hosted a series of Racism and the Economy conferences indicates that the role of the monetary authority is still an open question.

Against such background, this paper builds a heterogeneous-agent New-Keynesian (HANK) model with racial inequality in labor market risks and wealth, and studies how monetary policy affects different racial groups. The goal of the paper is modest. It is not intended to answer why racial inequalities in income and wealth exist. Instead, I take the observed racial disparities, embed into a canonical HANK model in a tractable manner, so that the model can be used to study how monetary policy affects different racial groups differently. An important assumption is that workers of different racial groups have common preferences. Instead, racial heterogeneity in unemployment and hand-to-mouth shocks and rate-of-return heterogeneity are used to replicate the observed racial heterogeneity in income and wealth. I use the HANK model, since it is an extension of the New-Keynesian Dynamic Stochastic General Equilibrium (NK-DSGE) model, which is a workhorse model used by monetary authorities, and introducing heterogeneous agents is crucial to capture various dimensions of racial inequality. With these considerations in mind, I try to stay close to the standard NK-DSGE model as much as possible, but at the same time try to incorporate various dimensions of observed racial disparities. My hope is that the model presented in this paper can be a benchmark model to think about the interactions between monetary policy and racial inequality.

The HANK model developed in this paper incorporates two important dimensions of racial differences that I present in the next section. First, Hispanic and Black workers face a higher risk of unemployment, and the risk rises disproportionately during a recession. When the shock driving the business cycle is common for all racial groups, this means that accommodative monetary policy brings down the unemployment rate of Hispanic and Black workers, and benefit them disproportionately. Indeed, Bartscher et al. (2021) compute that a -25bp reduction in the policy rate lowers the unemployment rate of Black workers 0.34pp more than that of White workers. And the model is successfully calibrated to replicate this empirical finding. Second, more Hispanic and Black workers are either poor or wealthy hand-to-mouth, i.e., liquidity constrained. The combination of the two is crucial in thinking about the role of monetary policy for different racial groups. The heterogeneous labor income risks imply that accommodative monetary policy could alleviate unemployment risks of Hispanic and Black workers to a larger extent, while the racial heterogeneity in hand-to-mouth implies that, since many of them are liquidity constrained, their consumption responds more to a lower unemployment rate and a higher wage caused by accommodative monetary policy.

There are four main findings. First, the model can replicate the facts that the unemployment rate for Black workers is more volatile over the business cycle, and the unemployment rate for Black workers declines 0.34pp more than that for White workers in response to a -25bp monetary policy shock, because of the higher job separation rate among Black workers. A shorter expected duration of jobs among Black workers makes the number of vacancy posting for Black workers more sensitive to the business cycle and monetary policy. Second, the model is consistent with the empirical finding by Ganong et al. (2020) that consumption of Black workers is 50% more sensitive to income shocks than White workers. This is because the higher fraction of both poor and wealthy hand-to-mouth among Black workers, documented in the next section. Third, all racial groups gain from a cut in the policy rate, but Black and Hispanic workers gain more than White workers, by about 50%. This is due to the combination of the higher labor market risks that they are facing, and a higher fraction of hand-to-mouth. Fourth, while welfare effects of accommodative monetary policy are increasing in the degree of labor market risks, welfare effects of monetary accommodation are not monotonically decreasing in wealth. The gains due to hand-to-mouth decreases with wealth, but loss from asset price appreciation increases with wealth. Finally, I find that replacing the overall unemployment rate with the Black unemployment rate as a monetary policy target is equivalent to making monetary policy more accommodative with the overall unemployment rate. This is because the unemployment rate for all racial groups move in parallel over the business cycle.

This paper contributes to four strands of literature. First, to the literature studying monetary policy in the presence of racial inequality, the current paper contributes by developing the first HANK model featuring racial inequality. This emerging literature includes a recent paper by Bartscher et al. (2021), who empirically studies racial inequality in wealth. Another recent paper by Lee et al. (2021) emphasizes racial heterogeneity in consumption basket, which implies that inflation affects different racial groups differently over the business cycles. They build a stylized macro model with two (Black and White) agents to study policy implications. Aliprantis et al. (2019) ask what is behind the racial wealth gap, using a steady-state incomplete-market model. Cajner et al. (2017) document racial difference in labor market outcomes.

Second, the current paper extends the literature of HANK (heterogeneous-agent New-Keynesian) models by introducing racial inequality. Papers such as Kaplan et al. (2018), Gornemann et al. (2021), and Bayer et al. (2020) combine the incomplete-market heterogeneous-agent model with aggregate uncertainty (Krusell and Smith (1998), which is Bewley-Aiyagari-Huggett model with aggregate shocks) with New-Keynesian nominal frictions, to investigate interactions between heterogeneity and monetary policy. The current paper is the first to introduce race as one dimension of heterogeneity into the otherwise standard HANK model. An important property of the HANK model is that if more consumers are either poor or wealthy hand-to-mouth, the model generates a stronger response of consumption when income increases. What the current paper emphasizes is that racial minorities are more likely to be hand-to-mouth, and thus their consumption is more strongly affected by monetary policy. As a result, the HANK model exhibits strong amplification of shocks.

Third, the current paper extends the literature of developing the macro model with search frictions in the labor market by introducing racial heterogeneity in labor market risks. Andolfatto (1996) and Merz (1995) first introduce search frictions in the labor market into a canonical real business cycle (RBC) model. Nakajima (2012a) and Krusell et al. (2010) introduce labor market search into the incomplete-market heterogeneous-agent model with aggregate uncertainty. Gornemann et al. (2021) add New-Keynesian friction into such model so that monetary policy can affect unemployment risks. The current paper is a natural extension of Gornemann et al. (2021) in the sense that monetary policy can affect different racial groups differently partly because they face different labor market risks.

Finally, the paper makes a small contribution to literature making computation of HANK models easier and more accessible. Solving a HANK model is not easy because of the large state space (distribution of heterogeneous agents), and solving the optimal decision problem for heterogeneous agents. The first paper that solves a heterogeneous-agent macro model (Krusell and Smith (1998)) employs global approximation, which is slow, especially with rich heterogeneity. Therefore, various methods relying on local approximation (perturbation) have been developed. Reiter (2009) proposes the first popular local-approximation method to solve the HANK model. This method can be understood as an extension of the local-approximation method developed for solving a representative-agent macro model by Schmitt-Grohé and Uribe (2009). More recently, efficient continuous-time version of the local-approximation method is developed to solve the model in Kaplan et al. (2018), and a more efficient local-approximation method is developed to solve the model in Bayer et al. (2019).[1] The modest contribution of the current paper to this literature is to develop a toolkit called `jhank` toolkit, which is available in Fortran90, Julia, and Matlab, to implement the simple local-approximation method by Reiter (2009) a little more easily. The current paper is an example of how to use the `jhank` toolkit.[2]

The remainder of the paper is organized as follows. Section 2 presents racial differences in terms of labor market characteristics and wealth holding. These are the facts that motivate the model constructed in Section 3. Section 4 explains calibration. Section 5 investigates wealth inequality in the model. The next five sections present the main results. Section 6 studies marginal propensity to consume (MPC) in the model. Section 7 shows how workers of different racial groups are affected by a monetary policy shock. Section 8 is about monetary transmission of the model. Section 9 studies how different monetary policy rules can affect different racial groups differently over the business cycle. In Section 10, I assume that the model economy goes through a severe recession mimicking the Great Recession, and investigate how different monetary policy rules could mitigate the negative consequences facing different racial groups differently. Section 11 concludes. An Appendix follows, containing additional details about racial inequality (Appendix A), hand-to-mouth (Appendix B), and equations characterizing the equilibrium (Appendix C), and steady-state values of variables (Appendix D).

[1] Codes to implement these solution methods can be found at Moll's homepage (`https://benjaminmoll.com`) and Luetticke's homepage (`https://www.ralphluetticke.com/`).

[2] `jhank` toolkit will be available at `https://makotonakajima.github.io/jhank/`.

2 Racial Inequality in Income and Wealth

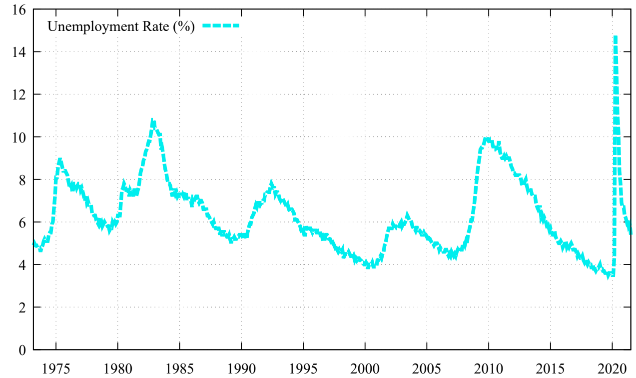
This section documents racial differences in the U.S. in terms of labor market (Section 2.1) and wealth (Section 2.2) characteristics. I focus on four major racial groups in the U.S., namely, White, Asian, Hispanic, and Black.[3] In Section 2.3, I look at workers with different education attainment within each racial group. The key takeaway from this section is that racial groups which face high labor market risks are also the groups which exhibit low liquid wealth, which makes it difficult to smooth consumption expenditures. The stylized facts presented here motivate how I embed permanent differences in the model that I build in Section 3.

2.1 Labor Market Characteristics

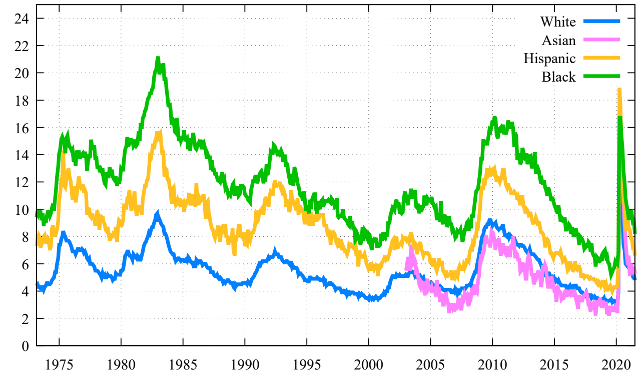
This section presents differences in labor market characteristics across four racial groups. Figure 1 shows the overall unemployment rate (Panel (a)), the unemployment rate for four racial groups (Panel (b)), the unemployment rate normalized by the average unemployment rate for respective groups (Panel (c)), the unemployment rate gaps (Panel (d)), the labor force participation rate for four racial groups (Panel (e)), and median usual weekly earnings, normalized by the overall median usual weekly earnings each year, for four racial groups (Panel (f)). All data are monthly frequency covering from 1973 to 2021, except for the normalized unemployment rate in Panel (c) and the weekly earnings presented in Panel (f). The normalized unemployment rate ends in January 2020 since the significant increase during the COVID-19 pandemic makes it difficult to see the trend, while the weekly earnings is annual and covers from 1979 to 2020. Sources for Figure 1 as well as Table 1 are Current Population Survey (CPS).

As is well known, the overall unemployment rate is countercyclical, sharply rising during recessions and gradually going down during expansions (Panel (a)). Regarding the unemployment rate for each racial group (Panel (b)), three characteristics can be pointed out. First, there are permanent differences in levels. The unemployment rate for Black workers is consistently the highest among the four racial groups. The Hispanic unemployment rate is the second. The unemployment rate for White and Asian workers are similar and lower than that of Black and Hispanic workers. As shown in Table 1, the average unemployment rate is 11.8% for Blacks, 8.8% for Hispanics, 4.9% for Asians, 4.9%, and 5.5% for Whites. Second, although the levels are different, the unemployment rate for all racial groups move in parallel. As shown in Table 1, the correlation coefficients between the overall unemployment rate and the unemployment rate for four racial groups are all above 0.9. Third, the unemployment rate for Hispanic and Black workers are more volatile, but the volatility is approximately proportional to the level. As shown in Table 1, the standard deviation of the overall unemployment rate is 1.70, but it is 1.56 for White workers, and 2.50 for Hispanics and 3.12 for Blacks. But the coefficient of variation (ratio of standard deviation to mean) is similar across all racial groups; it is 0.27 for the overall

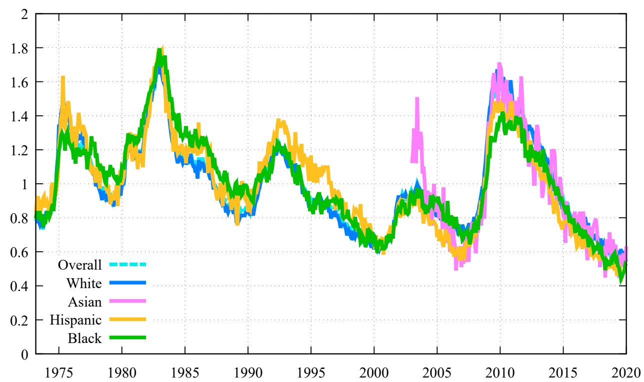
[3] I follow the definitions of races used by the Census Bureau in terms of the three racial groups used in the empirical analysis — White, Black (same as African), and Asian. I exclude American Indians, Alaskan Natives, Native Hawaiians and Other Pacific Islanders, and Two or More Races from the analysis. These groups made up 2.1% of the labor force on average between 2003 and 2018. Hispanic is an identity and a Hispanic person can be of any race. When I compute numbers for Hispanics, I include all individuals who identify as Hispanic, regardless of the race of the individuals. When I compute numbers for Whites, Blacks, and Asians, I exclude those who identify as Hispanic.



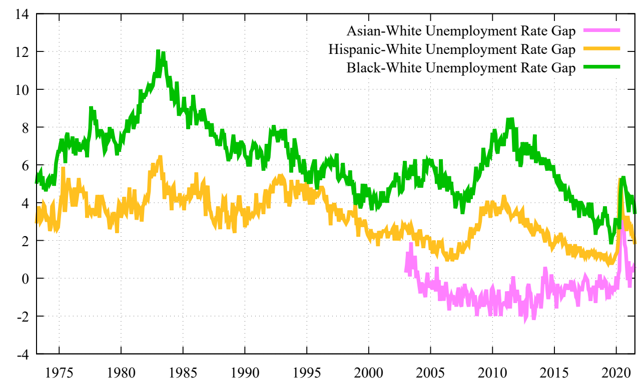
(a) Overall Unemployment Rate



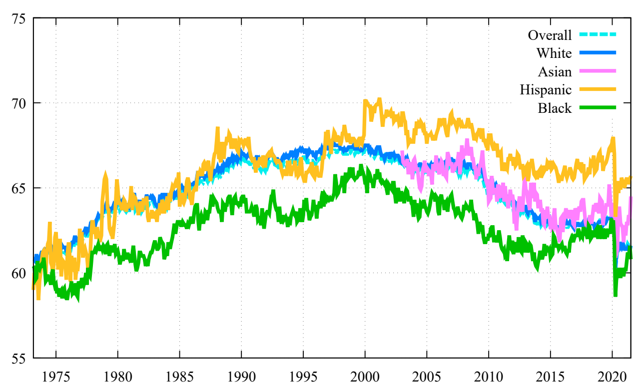
(b) Unemployment Rate for Four Racial Groups



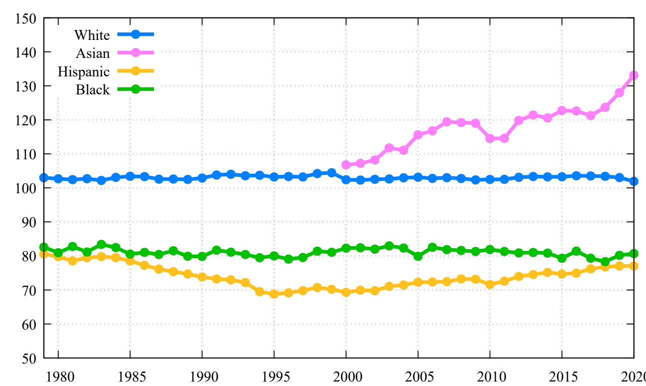
(c) Normalized Unemployment Rate



(d) Unemployment Rate Gap



(e) Labor Force participation rate



(f) Median Earnings Relative to Overall Median

Figure 1: Racial Differences in Labor Market Characteristics

unemployment rate, 0.28 for Whites, 0.29 for Hispanics, and 0.26 for Blacks. Asians' coefficient of variation (0.42) is higher because their data start from 2003, and the short period contains two large recessions. In Panel (c), the unemployment rate for the overall labor force as well as for four racial groups are shown, after normalized by the respective mean unemployment rate. For example, a value of 1 for the Black workers in Panel (c) means that the unemployment rate

Table 1: Labor Market Statistics of Four Racial Groups

	Overall	White	Asian	Hispanic	Black
Unemployment Rate (UR)					
Average	6.29	5.53	4.90	8.75	11.80
Standard Deviation	1.70	1.56	2.05	2.50	3.12
Coefficient of Variation	0.27	0.28	0.42	0.29	0.26
Correlation with Overall UR	1.000	0.997	0.909	0.937	0.919
Unemployment Rate Gap					
Average	–	–	–0.57	3.22	6.27
Correlation with Overall UR	–	–	–0.019	0.655	0.702
Labor Force Participation Rate (LFPR)					
Average	64.7	64.9	64.7	66.0	62.6
Correlation with Overall LFPR	1.000	0.998	0.906	0.764	0.913
Real Median Usual Weekly Earnings					
Dollar Values	748	770	887	551	605
Relative to Overall	100.0	103.3	118.7	73.7	80.9

Note: Source for the unemployment rate and the labor force participation rate is the CPS, from March 1973 to July 2021, except for Asian, whose data are available from January 2003. The source for real median usual weekly wage is also the CPS, from 2003 to 2018.

is equal to the Black unemployment rate (11.8%). What is remarkable in Panel (c) is that the five lines are generally on top of each other, meaning that all the unemployment rates move in sync, and the volatility is close to proportional to the average unemployment rate. The exception is the Asian unemployment rate in the early 2000s; it is either due to some data issue at the beginning of covering Asians, or due to the declining trend of the Asian unemployment rate.

Panel (d) of Figure 1 contains what is called the unemployment rate gap. For example, Black-White unemployment rate gap is the difference between the Black unemployment rate and the White unemployment rate. As shown in Table 1, the average unemployment rate gap is 3.2% for Hispanic and White, 6.3% for Black and White, while it is –0.6% between Asian and White. Since the unemployment rate for Blacks and Hispanics is consistently more volatile but moves in parallel with that of Whites, the unemployment rate gaps are naturally countercyclical, going up in recessions and going down in expansions. Besides, not surprisingly, the gaps for Hispanic and Black workers are positively correlated with the overall unemployment rate. As shown in Table 1, the correlation coefficient with the overall unemployment rate is 0.70 for Black-White unemployment rate gap and 0.65 for Hispanic-White unemployment rate gap. For Asian-White gap, correlation is close to zero (–0.02) but it is not important as the unemployment rate for White and Asian workers are almost on top of each other.

Although there is no labor force participation decision in the model, let's look at the labor force participation rate in the U.S. data (Panel (e)). The overall labor force participation rate gradually went up, from 60.8% in 1973 to 67.3% in 2000, and gradually went down since. The participation rate is 63.3% in December 2019, before the COVID-19 pandemic started. As with the unemployment rate, the labor force participation rate for all racial groups exhibits a similar

Table 2: Monthly Transition Rates

	Male			Female		
	White	Hispanic	Black	White	Hispanic	Black
EU rate	1.2	2.2	2.3	1.0	1.7	1.7
UE rate	25.6	28.4	18.2	25.0	20.4	17.2
EN rate	2.1	2.5	3.4	3.1	4.8	3.7
NE rate	4.6	7.5	4.9	3.8	5.1	4.5
UN rate	18.5	19.5	24.8	24.5	31.7	28.7
NU rate	2.4	5.1	5.1	1.7	3.3	4.0

Note: U = unemployment, E = employment, and N = out-of-labor-force. Monthly transition rates in percent. Source is Cajner et al. (2017). Based on the CPS microdata, longitudinally matched, from 1994 and 2016.

trend as the overall participation rate. The correlation coefficient with the overall participation rate is 0.998 for Whites, 0.906 for Asians, 0.764 for Hispanics, and 0.913 for Blacks (Table 1). The correlation for the Hispanic labor force participation rate is lower because of the sudden upward shift in 2000. The Hispanic participation rate tracks closely the White participation rate until 2000, but is consistently higher than the White participation rate by 2.5 percentage points on average after 2000. This might be due to some changes not related to actual changes of labor force participation decision among Hispanic workers. Similar to the unemployment rate, there are permanent differences in the level of the participation rate across racial groups, although the differences are smaller compared with those of the unemployment rate. The Black participation rate is consistently lower than others, while Asian participation rate is close to that of Whites. The average Black labor force participation rate is 62.6%, while the rate is 64.9% for Whites and 64.7% for Asians. The average Hispanic participation rate is 66.0%, which is higher than the participation rate of White workers because of the jump in 2000.

Panel (f) of Figure 1 shows the median usual weekly earnings for four racial groups, normalized by the overall median usual weekly earnings for each year. The median weekly earnings of White workers is consistently higher than the overall median by 3%, as confirmed in Table 1. The median weekly earnings of Blacks is consistently lower by about 20% than the overall median. The median Hispanic weekly earnings went down from 1978 to late 1990s, but went up since then, but consistently below Black median earnings. The average for Hispanic workers is 73.7% of the overall median. The median earnings for Asian workers shows significant growth since 2000. This is because the proportion with college degree among Asian workers is above 50%, unlike other racial groups, which implies that the median earnings of Asian workers is significantly affected by rising college premium. On average, the median weekly earnings of Asians is about 20% higher than the overall median earnings. This rise in earnings among Asian workers might be related to the declining unemployment rate, which can be seen in Panel (c).

I showed that the unemployment rate is consistently higher among Hispanic and Black workers, but it is because their job-finding rate is lower, or is it because their separation rate is higher? Table 2 answers this question. The numbers in the table are taken from Cajner et al. (2017), who use CPS microdata from 1994 to 2016 to compute monthly transition rates among three labor market status (employed, unemployed, and out-of-labor-force), separately for three racial groups (White, Hispanic, and Black) and two genders. In terms of the transition rates between employment and unemployment, which is the focus of the current paper, there are three takeaways from the first block of Table 2. First, both Hispanic and Black workers exhibit a higher separation (EU) rate than White workers. This makes the average job tenure of Black and Hispanic workers shorter, and pushes up their unemployment rate. Among males, the EU transition probability is 2.2% per month for Hispanics, and 2.3% for Blacks, compared with 1.2% for White workers. There is a similar tendency among female workers. Second, Black workers have a lower job-finding rate (UE transition rate) than White workers, making their unemployment rate even lower. Among Black males, the UE transition rate is 18.2% per month, which is about 70% of the White males' UE transition rate (25.6%). The job-finding rate for Black and White females is similar to their male counterparts. Third, however, Hispanic males have a higher UE transition rate (28.4%) than White males. Probably this is due to the type of jobs and industries that many Hispanic workers work in. This is why the unemployment rate among Hispanic workers is lower than the Black unemployment rate even thought both racial groups exhibit similarly high separation rate. Among Hispanic females, the job-finding rate is slightly lower than White females, but still higher than Black females.

In terms of the flow rates going into and getting out of labor force, what stands out is that all transition rates are higher among Black and Hispanic workers than White workers. In other words, labor market status is less stable among Black and Hispanic workers. Specifically, Black and Hispanic workers exhibit a higher EN rate, NE rate, UN rate, and NU rate, compared with White workers. Black and Hispanic workers exhibit a higher probability of getting out of labor force (E+U into N) and a higher probability of coming back to labor force (N to E+U) than White workers.

2.2 Hand-to-Mouth and Wealth Holding

In this section, I use the Survey of Consumer Finances (SCF), which is a cross-sectional household survey of wealth conducted by the Board of Governors of the Federal Reserve System every three years, from 1989 to 2016, and document wealth inequality across racial groups. Regarding sample selection, I follow Kaplan et al. (2014) and include households whose head is between 22 and 79 years old, and report strictly positive non-financial income.[4] Since the SCF over-samples wealthier households, I use the sample weights provided by the SCF in computing all statistics. Table 3 summarizes the results, and Appendix A provides additional results.[5] Table 3 has five columns. The first column includes all racial groups. The second column includes only households whose head is White. The third column is labeled as Asian, but indeed includes households whose head is "other" racial groups, i.e, neither White, Hispanic, or Black. I label this column as Asian, because majority of the households of "other" races are Asian. This is a limitation of using the Extract Public Dataset of the SCF. The fourth and the fifth columns are associated with households whose head is Hispanic and Black, respectively.

[4] Non-financial income is the sum of wage income from work, and various transfers from the government, such as unemployment insurance, and social security.

[5] Figure A.1 in Appendix A shows that the fraction of hand-to-mouth households remained stable. Table A.1 in Appendix A contains the fraction of hand-to-mouth and statistics of wealth, according alternative measures of wealth.

Table 3: U.S. Wealth Distribution for Four Racial Groups

	Overall	White	Asian	Hispanic	Black
Measures of Hand-to-Mouth Households					
Total Hand-to-Mouth	30.3	25.2	27.9	48.7	47.0
Poor Hand-to-Mouth	11.3	7.2	12.1	27.9	22.3
Wealthy Hand-to-Mouth	19.0	18.0	15.8	20.8	24.7
Measures of Wealth					
Mean Total Wealth	365,994	451,376	368,207	104,132	93,566
Relative to White	81.1	100.0	81.6	23.1	20.7
Median Total Wealth	89,564	127,725	82,123	9,553	16,706
Relative to White	70.1	100.0	64.3	7.5	13.1
Mean Illiquid Wealth	264,006	318,743	291,230	93,147	83,733
Median Illiquid Wealth	78,998	111,262	69,449	7,219	14,781
Mean Liquid Wealth	101,987	132,633	76,977	10,984	9,833
Median Liquid Wealth	2,777	5,268	4,750	207	293

Note: The source is the the Survey of Consumer Finances (SCF), Extract Public dataset. Averages of 1989 to 2016 waves (10 waves, since the SCF is available every three years) are shown. I follow Kaplan et al. (2014) with respect to the definitions of hand-to-mouth and wealth, as well as sample selection (households whose head is between 22 and 79 years old, and their non-financial income is strictly positive, are included). The sample household weights provided by the SCF is used. With the Extract Public dataset, Asians are bunched together with all the other racial groups. Dollar amounts are shown in 2010 dollars.

The upper block of Table 3 summarizes the fraction of households which are classified as hand-to-mouth according to the definition of Kaplan et al. (2014).[6] Simply put, households whose liquid wealth holding is less than half of the non-financial income per pay period (2 weeks) is classified as hand-to-mouth. Moreover, if a household which is classified as hand-to-mouth has zero or negative illiquid wealth, the household is classified as poor hand-to-mouth, while a hand-to-mouth household with strictly positive illiquid wealth is called wealthy hand-to-mouth. Total wealth is the sum of liquid and illiquid wealth. Averaged between 1989 and 2016, 30.3% of all households are hand-to-mouth. Among them, about 1/3 (11.3%) are poor hand-to-mouth, while the remaining 2/3 (19.0%) are wealthy-hand-to-mouth. These numbers are very close to the numbers that Kaplan et al. (2014) report using pooled data from 1989 to 2010.[7]

As can be seen from the second to fifth columns of the top block, there is significant heterogeneity in terms of the proportions of hand-to-mouth across racial groups. Among White households, there are less hand-to-mouth households. The total fraction of hand-to-mouth

[6] Following Kaplan et al. (2014), liquid wealth is the sum of checking, saving, money market, and call accounts, directly held pooled investment funds, directly held individual stocks and bonds, net of credit card balance. Illiquid wealth is the sum of certificate of deposits, saving bonds, cash value of life insurance, all kinds of retirement accounts, value of primary and other residences, net equity in non-residential real estate, net of mortgages and other types of home equity loans.. See Appendix B for more details about how Kaplan et al. (2014) define hand-to-mouth.

[7] According to them, the fraction of hand-to-mouth is 31.2%, 39% of which (12.1% of total households) is poor hand-to-mouth, while the remaining 62% (19.2% of total) are wealthy hand-to-mouth.

households among White is 25.2%, among which 7.2% are poor hand-to-mouth and 18.0% are wealthy hand-to-mouth. All numbers are below the overall ratios. The fractions of hand-to-mouth among Asian households are close to the overall fractions; 12.1% are poor hand-to-mouth, and 15.8% are wealthy hand-to-mouth, and thus 27.9% are total hand-to-mouth households. On the other hand, there are significantly more hand-to-mouth households among Hispanics and Blacks. Among Hispanic and Black households, almost half (48.7% for Hispanics, 47.0% for Blacks) are hand-to-mouth. Among Hispanic hand-to-mouth households, the fraction of poor hand-to-mouth is particularly high, at 27.9%. This number is close to three times the overall proportion, and more than four times the proportion of poor hand-to-mouth among White households. The fraction of wealthy hand-to-mouth is only slightly higher than the overall fraction, at 20.8%. Among Black households, 22.3% are poor hand-to-mouth, which is about twice as high as the overall fraction, and three times as high as White households. Among Black hand-to-mouth households, 24.7% are wealthy hand-to-mouth, compared with 19.0% among overall households.

The lower block of Table 3 shows mean and median total wealth, liquid wealth and illiquid wealth, for all households as well as four racial groups. Across all households, mean total wealth is 366,000 dollars in 2010 dollars, while median total wealth is 90,000 dollars. For both mean and median total wealth, all minority groups hold less wealth than White households. The difference is especially large among Hispanic and Black households. Mean wealth of Asian, Hispanic, and Black households are 81.6%, 23.1%, and 20.7% of mean wealth of White households, respectively. Median total wealth for Asian, Hispanic, and Black households are 64.3%, 7.5%, and 13.1%, of median total wealth of White households, respectively. For all households, liquid wealth makes up a smaller portion of total wealth, but the fraction is significantly smaller for Hispanic and Black households. Median liquid asset holding for Hispanic households is only 207 dollars. For Black households, it is only 293 dollars. This small liquid asset holding is consistent with the large fraction of hand-to-mouth households for these two racial groups.

2.3 Race and Education

In the next section, I build a model in which workers of different racial groups are different in terms of labor market risks, liquidity, and the rate of return of savings. This is a parsimonious way to capture the differences across racial groups in the data presented in this section while assuming that the preferences are the same, and could be justified for short-run analysis like the current paper. However, the approach is admittedly ad-hoc in the sense that deeper causes which create racial differences in labor market outcomes and wealth holding are not explicitly modeled. The differences across racial groups could be manifestation of heterogeneity in education, skills, sector or type of jobs, etc. In this section, I investigate how much of the racial differences presented in this section can be attributed to differences in education attainment. Specifically, Table 4 shows the unemployment rate and the proportion of hand-to-mouth across different racial groups and different education attainment.

The first block of Table 4 shows the composition of education attainment in the labor force, for different racial groups. Asian workers tend to have higher education attainment than White workers. The fraction with at least Bachelor's degree among White workers is 36.2%, while more than half (55.3%) of Asian workers have at least Bachelor's degree. On the other hand,

Table 4: Race and Education

	Overall	White	Asian	Hispanic	Black
Labor Force Composition (%)					
Less than high school	10.9	6.7	7.7	31.3	10.9
High school diploma	28.2	27.5	17.8	30.6	33.8
Some college	28.6	29.7	19.2	23.7	32.7
Bachelor's degree or more	32.2	36.2	55.3	14.3	22.6
Total	100.0	100.0	100.0	100.0	100.0
Unemployment Rate (%)					
Less than high school	13.9	14.4	7.8	11.0	24.8
High school diploma	8.1	7.1	5.9	8.4	13.5
Some college	5.9	5.1	5.9	6.6	9.6
Bachelor's degree or more	3.2	2.8	3.8	4.1	4.9
Total	6.5	5.4	4.9	8.2	11.5
Hypothetical	–	6.3	5.3	6.5	10.0
% accounted for	–	13.6	70.3	62.0	25.3
Proportion of Hand-to-Mouth: Less than Bachelor's Degree (%)					
Total hand-to-mouth	37.3	31.4	41.8	52.2	51.6
Poor hand-to-mouth	15.4	10.1	19.7	30.9	26.2
Wealthy hand-to-mouth	21.9	21.3	22.1	21.2	25.5
Proportion of Hand-to-Mouth: Bachelor's Degree or More (%)					
Total hand-to-mouth	17.1	15.3	14.9	30.5	30.1
Poor hand-to-mouth	3.5	2.6	4.7	11.6	8.5
Wealthy hand-to-mouth	13.6	12.8	10.2	18.8	21.6
Proportion of Hand-to-Mouth (%)					
Total Hand-to-Mouth	30.3	25.2	27.9	48.7	47.0
Hypothetical	–	27.7	32.1	44.3	43.8
% accounted for	–	11.7	-150.1	18.8	14.7

Note: Source for the labor force composition and the unemployment rate is the CPS, Annual Social and Economic Supplement. Averages from 2003 to 2018 are shown. Source for the fraction of hand-to-mouth is the SCF, averaged across 10 waves between 1989 and 2016. For non-White, the *hypothetical* unemployment rate is computed by using the non-White unemployment rate for each education level and the education composition among the White labor force. For White, the Black educational composition and the White unemployment rate for each education level are used to compute the hypothetical unemployment rate. The fraction of the difference in the unemployment rate between non-White and White that can be accounted for by the difference in the education composition is shown in *Fraction accounted for*. The definition of hand-to-mouth follows Kaplan et al. (2014). The *hypothetical* fractions of hand-to-mouth are computed similarly as the *hypothetical* unemployment rate.

Hispanic workers exhibit the opposite tendency. The fraction of workers without high school diploma is 6.7% among Whites but 31.3% among Hispanics. Meanwhile, the proportion with at least Bachelor's degree among Hispanic workers is only 14.3%, significantly lower than Whites. Education attainment of Black workers is somewhere between the Whites' and Hispanics'. The proportion without high school diploma among Black workers is 10.9%, higher than Whites', while the proportion of Black workers with Bachelor's degree (22.6%) is lower than Whites'.

One might think that the difference in the composition of education attainment could explain racial differences, for example, in the unemployment rate. However, even the unemployment rate with the same education attainment is different across racial groups, as shown in the second block of Table 4. Among White workers without high school diploma, the unemployment rate is 14.4%. Asian (7.8%) and Hispanic (11.0%) workers without high school diploma have a lower unemployment rate. For Hispanic workers, it is probably because of the type of jobs or sectors they search for a job. On the other hand, the unemployment rate is 24.8% among Black workers without high school diploma. Among workers with Bachelor's degree, Asian (3.8%), Hispanic (4.1%) and Black (4.9%) workers exhibit a higher unemployment rate than Whites (2.8%). There is a similar tendency among workers with some college.

Since the unemployment rates associated with the same educational attainment are different across racial groups, it is not likely that controlling the differences in the composition of education attainment is enough to account for all the racial differences. But let's try. The results are shown in the bottom two lines of the second block. For a minority racial group, the "Hypothetical" unemployment rate is computed by combining the composition of education attainment among Whites and the unemployment rate associated with different education attainment of the minority racial group. The row labeled "% accounted for" represents the fraction of the difference in the unemployment rate between White and non-White which can be accounted for by controlling the composition of educational attainment. The hypothetical Black unemployment rate is 10.0%, which is only slightly lower than the actual Black unemployment rate (11.5%). Therefore, only about a quarter of the difference in the unemployment rate between Blacks and Whites can be accounted for by the difference in educational attainment. If I do the opposite hypothetical, combining the education composition of Black workers and the unemployment rate associated with different education attainment among White workers, the hypothetical unemployment rate is 6.3%, slightly higher than the actual unemployment rate among White workers (5.4%) but far lower than that of Black worker (11.5%). In the end, only 14% of the Black-White difference in the unemployment rate can be accounted for by the difference in educational attainment. In short, the difference in the educational attainment between White and Black workers can explain only up to 1/4 of the difference in the overall unemployment rate. However, a larger fraction of the difference in the unemployment rate can be accounted for by the difference in educational attainment for Asian and Hispanic workers. The hypothetical unemployment rate for Hispanics is 6.5%, which is sizably lower than the actual Hispanic unemployment rate (8.2%). This implies that the difference in the education attainment between Whites and Hispanics can account for 62% of the difference in their unemployment rate. For Asians, the Asian unemployment rate is lower than the White one, but if the educational composition of White workers is used, the hypothetical Asian unemployment goes up, to 5.3%, which is close to the White unemployment rate. Indeed, the difference in the educational attainment can account for 70% of why the Asian unemployment rate is lower than that of White workers. In sum, for Hispanic and Asian workers, the difference in the composition of educational attainment can account for a large part of the difference in the unemployment between them and the White workers, although the difference is smaller from the beginning. These results echo the findings of Cajner et al. (2017), who also use the CPS and control for differences in age, marital status, and state they live in, on top of educational attainment and reach the same conclusion; they find that the Black-White difference in the unemployment cannot be explained by differences in observable characteristics, while the Hispanic-White differences in the unemployment rate can be explained largely by differences in educational attainment.

The third block of Table 4 shows the proportion of hand-to-mouth households for different racial groups without Bachelor's degree. There are three main takeaways. First, even among households with the same education attainment (without Bachelor's degree), the fraction of hand-to-mouth is higher among Asians (41.8%), Hispanics (52.2%) and Blacks (51.6%) than Whites (31.4%). Second, interestingly, the difference is mainly due to a higher proportion of poor hand-to-mouth households among Asians (19.7%), Hispanics (30.9%) and Blacks (26.2%) compared with White households (10.1%). Third, on the other hand, the proportion of wealthy hand-to-mouth households is similar across White (21.3%), Asian (22.1%), Hispanic (21.2%), and Black (25.5%) households. The fourth block shows the proportion of hand-to-mouth for each racial groups with Bachelor's degree. Again, even if we restrict our attention to households with Bachelor's degree, there are large racial differences in the proportion of hand-to-mouth, although the proportions are lower for all groups compared with those without Bachelor's degree. The proportion of total hand-to-mouth is higher among Hispanics (30.5%) and Blacks (30.1%) compared with Whites (15.3%) and Asians (14.9%). This is true for both poor hand-to-mouth and wealthy hand-to-mouth.

The last block of Table 4 contains the actual and *hypothetical* fractions of total hand-to-mouth and the percentage of the difference in hand-to-mouth that can be accounted for by the differences in education composition, for each racial group. The *hypothetical* fraction of hand-to-mouth is constructed in a similar way as the *hypothetical* unemployment rate shown in the second block of the table. Specifically, the hypothetical fraction of hand-to-mouth for Blacks is computed by combining the proportion of hand-to-mouth for two educational groups among Black households and the composition of education attainment among Whites. The hypothetical fraction of hand-to-mouth among Black households is 43.8%, which is close the actual fraction among Black households (47.0%). In other words, accounting for the Black-White difference in educational composition cannot explain much of the Black-White difference in the fraction of hand-to-mouth. The last row states that only 15% of the difference can be accounted for by the difference in educational attainment. If the hypothetical fraction of hand-to-mouth is constructed in the opposite way, by combining the education composition of Blacks and the proportion of hand-to-mouth for two education groups among Whites, the hypothetical fraction of hand-to-mouth is 27.7%, instead of 25.2% among White households, implying that the difference in the educational attainment can only account for 12% of the Black-White difference in the fraction of hand-to-mouth. The proportion of the White-Hispanic difference in hand-to-mouth that can be accounted for by educational attainment is slightly larger at 19%. For Asians, the actual fraction of hand-to-mouth is already close to that of White households, and if the educational composition among White households is applied for Asian households, the fraction of hand-to-mouth among Asians becomes even higher than the actual proportion of hand-to-mouth.

3 Model

Time is discrete and infinite, starting from $t = 0$. The economy is populated by workers, investment firms, capital firms, labor firms, intermediate good firms, final good firms, mutual funds, the government, and the monetary authority. The model is intended to stay close to the canonical one-asset heterogeneous-agent model with the standard New Keynesian nominal frictions, with the main innovation being having multiple types capturing different races.

3.1 Worker

There are mass of infinitely-lived workers. Let me first discuss dimensions of heterogeneity of workers. A worker is characterized by permanent type s, discount factor β_t, persistent idiosyncratic productivity shock p_t, unemployment shock $e_t \in \{1, 2\}$, wealthy hand-to-mouth shock $h_t \in \{1, 2\}$, and asset holding a_t. The permanent type s is used to capture racial differences in labor market risks, average level of earnings, liquidity risks, and the rate of return. The basic idea is to assume the same preferences across racial groups, but use s to capture heterogeneity in the economic environment that different racial groups face in the current U.S. economy. The discount factor shock and the persistent productivity shock are found to help the model replicate the observed inequality in wealth (Krusell and Smith (1998), De Nardi and Fella (2017)). The unemployment shock is endogenous and heterogeneous across racial groups and is thus an important channel through which monetary policy has diverse effects to different racial groups. Asset holding is the only endogenously chosen variable, and is crucial in generating the poor hand-to-mouth workers in the model. The problem of a worker is the following:

$$\max_{\{c_t, a_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{\tilde{t}=0}^{t} \beta_{\tilde{t}} \right) u(c_t) \tag{1}$$

subject to:

$$c_t + p_t^a a_{t+1} = (1 + r_t + \iota_s)\, p_t^a a_t + \begin{cases} (1 - \tau_t) w_t p_t \eta_s & \text{if } e_t = 1 \\ \min(\phi_0 w_t p_t \eta_s, \phi_1 \overline{wp\eta}_s) & \text{if } e_t = 2 \end{cases} \tag{2}$$

$$a_{t+1} \geq \begin{cases} 0 & \text{if } h_t = 1 \\ (1 - \delta_h) a_t & \text{if } h_t = 2 \end{cases} \tag{3}$$

Both β_t and p_t follow a first-order Markov process, with transition probabilities $\pi_{\beta_{t+1}|\beta_t}$ and $\pi_{p_{t+1}|p_t, e_t, e_{t+1}}$, respectively. The transition probabilities for p_t depend also on the employment status in period t and $t + 1$, since the productivity shock p_t remains the same during an unemployment spell. As for the unemployment shock, $e_t = 1$ and 2 denote being employed, and unemployed, respectively. $\pi_{e_{t+1}|s, e_t}$ denote the transition probabilities of e for a type-s worker. Specifically, when a worker is employed, the worker loses its job and becomes unemployed in the next period at a type-s specific but exogenous separation rate λ_s. If the worker is unemployed, the worker finds a job and becomes employed at a job-finding rate $f_{s,t}$, which is endogenously determined. The wealthy hand-to-mouth shock h_t is i.i.d. and $h_t = 1$ with probability $1 - \pi_s^h$ and $h_t = 2$ with probability π_s^h.[8] Notice that the probability of wealthy hand-to-mouth shock is also type-specific.

[8] Although it is reasonable to think that the wealthy hand-to-mouth shock might be persistent and state-dependent, I assume it is i.i.d. for two reasons. First, it helps computationally to have a smaller state space

In the maximand (1), $u(c)$ is the period utility function with the functional form of $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$, where σ is the coefficient of relative risk aversion. c_t is consumption. $\mathbb{E}$ is an expectation operator. (2) is the budget constraint, and (3) represents the liquidity constraint, which depends on the realization of h_t. In equation (2), p_t^a is the price of assets, r_t represents the average rate of return of assets. ι_s is a type-specific factor for the rate of return of assets. Basically, ι_s makes the return from holding assets permanently different for different racial groups. The last term of (2) is non-financial income, which depends on the current employment status. For the employed ($e_t = 1$), τ_t is the UI tax rate, w_t is the wage per efficiency unit, and η_s is the productivity for type-s worker. An unemployed ($e_t = 2$) receives UI benefits. The amount of UI benefits is the replacement rate ϕ_0 times the would-be (pre-tax) labor income, with the upperbound of $\phi_1 \overline{w}\overline{p}\overline{\eta}_s$, where $\overline{w}$ is the steady-state wage, $\overline{p}$ is the average labor productivity shock, $\overline{\eta}_s$ is the average η_s across all types, and ϕ_1 controls the level of the upperbound of UI-benefits, relative to average wage.

The liquidity constraint (3) has two cases depending on h_t. If $h_t = 1$, the worker is not subject to hand-to-mouth shock, and the liquidity constraint basically states that the worker cannot borrow, but can use all the assets for current consumption. If $h_t = 2$, the worker is liquidity constrained in a way that captures wealthy hand-to-mouth. Specifically, a_{t+1} is constrained to be above $(1 - \delta_h)a_t$. In other words, the worker which is subject to the wealthy hand-to-mouth shock can use only a fraction δ_h of the current asset holding a_t for consumption smoothing, while the fraction $(1-\delta_h)a_t$ remains illiquid and cannot be used for current consumption. This is a simplified way to introduce wealthy hand-to-mouth in a model which abstracts from two-asset setup like the one developed by Kaplan et al. (2018). In a related set up, Bayer et al. (2020) assume that workers can adjust illiquid asset holding with an i.i.d. probability. The wealthy hand-to-mouth shock here can be considered as a one-asset version of their setup.

3.2 Investment Firm

Competitive investment firms purchase final goods from final good firms and convert into investment goods and sell to capital firms at price p_t^i, subject to a quadratic investment adjustment cost, and a marginal efficiency of investment (MEI) shock z_t^{MEI} (Justiniano et al. (2010)). The problem of investment firms is as follows:

$$\max_{\{i_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{\tilde{t}=0}^{t} Q_{\tilde{t}} \right) \left[i_t z_t^{MEI} \left(p_t^i - \frac{\psi_i}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right) - i_t \right] \tag{4}$$

The term $\frac{\psi_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$ represents the quadratic investment adjustment cost. The MEI shock follows the AR(1) process below:

$$\log z_{t+1}^{MEI} = \rho_{MEI} \log z_t^{MEI} + \epsilon_{t+1}^{MEI} \tag{5}$$

for storing the type distribution of workers. By assuming that the wealthy hand-to-mouth shock is i.i.d., the size of the state space halves. Second, the data related to hand-to-mouth is obtained using the Survey of Consumer Finances (SCF). Since the SCF is a cross-sectional data and available every three years, I cannot compute the persistence or state-dependence of the wealthy hand-to-mouth shock using the SCF.

where $\epsilon_{t+1}^{MEI} \sim N(0, \sigma_{MEI}^2)$. Future profits are discounted by a discount factor Q_t. Current profits of investment firms can be defined as:

$$d_t^{INV} = i_t z_t^{MEI} \left(p_t^i - \frac{\psi_i}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right) - i_t \tag{6}$$

3.3 Capital Firm

The problem of competitive capital firms can be characterized as follows:

$$\max_{\{k_t, n_t\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \left(\prod_{\tilde{t}=0}^{t} Q_{\tilde{t}} \right) \left[r_t^k n_t k_t - p_t^i i_t z_t^{MEI} \right] \tag{7}$$

subject to:

$$k_{t+1} = (1 - \delta_0 n_t^{\delta_1}) k_t + i_t z_t^{MEI} \tag{8}$$

where r_t^k is the real rate of return of capital. Following Greenwood et al. (1988), $\delta_0 n_t^{\delta_1}$ is the depreciation rate which depends on the level of utilization n_t. Current profits of the capital firms are:

$$d_t^{CAP} = r_t^k n_t k_t - p_t^i i_t z_t^{MEI} \tag{9}$$

3.4 Labor Firm

Labor firm can be unmatched or matched with a worker. An unmatched labor firm can post a vacancy in a type-s market by paying a vacancy posting cost κ_s. Whether an unmatched firm posting a vacancy is matched with a worker or not is determined by a matching function. If matched, the firm and the worker produces and sells labor services to intermediate good firms, and the revenue is shared between the worker (wages) and the firm (profits). The bargaining is simplified by assuming the following simple surplus sharing rule.[9]

$$w_t = \omega_0 \overline{x} + \omega_1 (\log x_t - \log \overline{x}) + \omega_2 (\log \pi_t - \log \overline{\pi}) \tag{10}$$

ω_0 captures the worker's share out of total surplus in the steady state, with $\overline{x}$ being the steady-state rental rate of labor services or labor productivity. ω_1 captures the elasticity of wage with respect to labor productivity. ω_2 is intended to capture nominal wage rigidity. With $\omega_2 \in (-1, 0)$, a higher inflation rate implies a lower real wage, but not one-to-one with the inflation rate.

[9] A common choice for the surplus sharing rule is the generalized Nash bargaining. Nakajima (2012a) uses the generalized Nash bargaining in a heterogeneous-agent model with the same labor market frictions. However, the generalized Nash bargaining would make it significantly hard to solve the model, as the bargaining outcome would depend on all the characteristics of the worker in the bargaining, including savings. Since savings are endogenous, the workers internalize the effect to bargaining outcome when making the saving decision. In the end, Nakajima (2012a) quantitatively shows that the bargaining outcome is not sensitive to savings. Gornemann et al. (2021) also use a simple surplus sharing rule like the one used here.

The value function of a labor firm matched with type-(s,p) worker can be recursively defined as follows:

$$J_t(s,p) = (x_t - w_t)p\eta_s + \mathbb{E}_t Q_{t+1}(1-\lambda_s)\sum_{p'} \pi_{p'|p,1,1} J_{t+1}(s,p') \tag{11}$$

where $x_t - w_t$ is the profits per efficiency unit. $p\eta_s$ represents the individual productivity of the worker. Each period, after production, the match is destroyed at the separation rate λ_s. The type-s of the worker that the firm is matched to does not change, but the individual labor productivity p changes according to $\pi_{p'|p,e=1,e'=1}$, where $e = e' = 1$ means the worker remains employed.

Unmatched firms keep entering the markets until the expected profits of entering the type-s market are equal to the vacancy posting cost, as follows:

$$\kappa_s = \frac{\mu v_{s,t}^{\alpha} u_{s,t}^{1-\alpha}}{v_{s,t}} \sum_p \pi_{p|s,e=2} J_t(s,p) \tag{12}$$

$m(v_{s,t}, u_{s,t}) = \mu v_{s,t}^{\alpha} u_{s,t}^{1-\alpha}$ is the matching function, where μ is the matching efficiency, and α is the elasticity of matches with respect to the number of vacancies posted. $v_{s,t}$ and $u_{s,t}$ are the number of vacancies and the number of unemployed (and thus job-searching) workers in type-s market, respectively. Labor market search and matching occurs before production in period t. $\pi_{p|s,e=2}$ denotes the distribution of individual productivity p among the currently unemployed ($e=2$) workers of type-s. This expected zero profit condition virtually determines the equilibrium $v_{s,t}$ for each type-s. Once $v_{s,t}$ is determined, the job-finding rate for a type-s worker, $f_{s,t}$, can be characterized as:

$$f_{s,t} = \frac{\mu v_{s,t}^{\alpha} u_{s,t}^{1-\alpha}}{u_{s,t}} \tag{13}$$

Current total profits of labor firms in period t, d_t^{LAB}, can be characterized as follows:

$$d_t^{LAB} = \int \mathbb{1}_{e=1}(x_t - w_t)p\eta_s \, d\, m_{t+1} - \sum_s \kappa_s v_{s,t} = (x_t - w_t)\ell_t - \sum_s \kappa_s v_{s,t} \tag{14}$$

3.5 Final Good Firm

Following the standard New Keynesian model, there are continuum of intermediate good firms indexed by $j \in [0,1]$ producing differentiated output $y_t(j)$ at nominal prices $P_t(j)$. These intermediate goods are bundled into a final output y_t with the following production function, with elasticity of substitution among intermediate goods $\epsilon_p > 1$.

$$y_t = \left(\int_0^1 y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj\right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{15}$$

Profit maximizing problem of a representative final good firm is:

$$\max_{\{y_t(j)\}} P_t \left(\int_0^1 y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} - \int_0^1 P_t(j) y_t(j) dj \tag{16}$$

First order condition with respect to an intermediate goods output $y_t(j)$ is:

$$y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_p} y_t \tag{17}$$

This is virtually a demand function by the final good firm, taken as given by intermediate good firms. The aggregate price index P_t in the equation above is characterized by:

$$P_t^{1-\epsilon_p} = \int_0^1 P_t(j)^{1-\epsilon_p} dj \tag{18}$$

Final good firms make zero profits in equilibrium.

3.6 Intermediate Good Firm

Intermediate good firms produce differentiated goods j, using the following production technology:

$$y_t(j) = z_t^{TFP} (k_t(j) n_t(j))^{\theta} \ell_t(j)^{1-\theta} \tag{19}$$

where $k_t(j)$ is capital, $n_t(j)$ is the level of utilization, and $\ell_t(j)$ is labor used by the intermediate firm j. z_t^{TFP} is total factor productivity shock and follows the AR(1) process below:

$$\log z_{t+1}^{TFP} = \rho_{TFP} \log z_t^{TFP} + \epsilon_{t+1}^{TFP} \tag{20}$$

where $\epsilon_{t+1}^{TFP} \sim N(0, \sigma_{TFP}^2)$. The nominal profit of the intermediate good firm j, $D_t(j)$, is as follows:

$$D_t(j) = P_t(j) y_t(j) - R_t^k (k_t(j) n_t(j)) - X_t \ell_t(j) - \frac{\psi_1}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\pi} \right)^2 y_t P_t - \psi_0 P_t \tag{21}$$

$P_{t-1}(j)$ is the nominal price of intermediate good j in the previous period. P_t is the aggregate price of intermediate goods in the current period, taken as given by an intermediate good firm j. R_t^k and X_t are the nominal rental rate of capital and labor, respectively. Following the standard New-Keynesian setup (Rotemberg (1982)), I assume a quadratic nominal price adjustment cost, with ψ_1 determining the degree of nominal price rigidity. $\overline{\pi}$ is the steady-state inflation rate. ψ_0 is a fixed cost, which ensures that profits (and dividends) are zero in the steady-state.

Dividing by the nominal price of intermediate goods P_t, real profit of an intermediate good firm j is:

$$\frac{D_t(j)}{P_t} = \frac{P_t(j)}{P_t} y_t(j) - \frac{R_t^k}{P_t} (k_t(j) n_t(j)) - \frac{X_t}{P_t} \ell_t(j) - \frac{\psi_1}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \overline{\pi} \right)^2 y_t - \psi_0 \tag{22}$$

And the optimization problem of an intermediate good firm j is as follows:

$$\max_{\{P_t(j),(k_t(j)n_t(j)),\ell_t(j)\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \prod_{\tilde{t}=0}^{t} Q_{\tilde{t}} \frac{D_t(j)}{P_t} \tag{23}$$

subject to:

$$y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} y_t \tag{24}$$

$$y_t(j) = z_t^{TFP}(k_t(j)n_t(j))^{\theta}\ell_t(j)^{1-\theta} \tag{25}$$

Total real profits of intermediate good firms, d_t^{INT}, can be defined as follows:

$$d_t^{INT} = \int_0^1 \frac{D_t(j)}{P_t} dj \tag{26}$$

3.7 Government

The government runs the unemployment insurance (UI) program. The budget of the UI program is balanced each period, by adjusting the UI tax rate τ_t each period. The government budget constraint is as follows:

$$\tau_t \int \mathbb{1}_{e=1} w_t p \eta_s \, d\, m_{t+1} = \int \mathbb{1}_{e=2} \min(\phi_0 w_t p \eta_s, \phi_1 \overline{wp\eta}_s) \, d\, m_{t+1} \tag{27}$$

where m_{t+1} is the type distribution of workers in period t after labor market transitions.

3.8 Monetary Authority

Monetary policy is characterized by the following Taylor rule with interest rate smoothing:

$$\frac{R_t}{\overline{R}} = \left(\frac{R_{t-1}}{\overline{R}}\right)^{\rho_R} \left(\frac{\pi_t}{\overline{\pi}}\right)^{(1-\rho_R)\phi_\pi} \left(\frac{y_t}{\overline{y}}\right)^{(1-\rho_R)\phi_y} z_t^{MP} \tag{28}$$

where the first term represents the interest rate smoothing, and ρ_R is the smoothing parameter. z_t^{MP} is a monetary policy shock, which follows the AR(1) process below.

$$\log z_{t+1}^{MP} = \rho_{MP} \log z_t^{MP} + \epsilon_{t+1}^{MP} \tag{29}$$

where $\epsilon_{t+1}^{MP} \sim N(0, \sigma_{MP}^2)$. The assumed timing is that R_t is applied to nominal asset saved in period t (and return is paid in period $t+1$). ϕ_π and ϕ_y represent the response of monetary policy against inflation and output, respectively. Taking log of both sides yields:

$$\begin{aligned} \log R_t = (1-\rho_R)\log \overline{R} + \rho_R \log R_{t-1} \\ + (1-\rho_R)[\phi_\pi(\log \pi_t - \log \overline{\pi}) + \phi_y(\log y_t - \log \overline{y})] + \log z_t^{MP} \end{aligned} \tag{30}$$

3.9 Mutual Funds

Workers own all the firms and other assets through mutual funds. This is a way to avoid portfolio choice problem at the individual level, and force all workers to have the same representative portfolio. Without this assumption, workers could choose to have different portfolio composition. The price of a share of mutual funds is p_t^a and the dividend of mutual funds is d_t. d_t is defined as follows:

$$d_t = d_t^{INV} + d_t^{CAP} + d_t^{LAB} + d_t^{INT} \tag{31}$$

where d_t^{INV},d_t^{CAP}, d_t^{LAB} and d_t^{INT} denote real period-t profits from investment firms, capital firms, labor firms, and intermediate good firms, respectively. Total number of mutual fund shares is fixed at $\overline{a}$. The discount factor that is used for intertemporal decision of firms can be defined as:

$$Q_{t+1} = \frac{p_t^a}{p_{t+1}^a + d_{t+1}} \tag{32}$$

Remember workers of different type-s face different rate of returns of assets ι_s. In aggregate, the following budget constraint holds for mutual funds:

$$r_t + \int \iota_s a_t \, m_{t+1} = \frac{d_t}{p_t^a} \tag{33}$$

Given the type distribution m_{t+1}, dividends d_t, and the price of assets p_t^a, this equation gives the average rate of return for workers, r_t.

3.10 Characterizing the Equilibrium

In this section, I will derive some equations that characterize the equilibrium. The complete list of equations characterizing the equilibrium is in Appendix C. Worker's optimal saving decision is characterized by the following equations:

$$u'(c_t) = \beta_t \mathbb{E}_t \frac{(1 + r_{t+1} + \iota_s)p_{t+1}^a}{p_t^a} u'(c_{t+1}) \tag{34}$$

$$a_{t+1} = \begin{cases} \max\{((1 + r_t + \iota_s)p_t^a a_t + (1 - \tau_t)w_t p_t \eta_s - c_t)/p_t^a, \underline{a}_t\} & \text{if } e_t = 1 \\ \max\{((1 + r_t + \iota_s)p_t^a a_t + \min(\phi_0 w_t p_t \eta_s, \phi_1 \overline{wp\eta}_s) - c_t)/p_t^a, \underline{a}_t\} & \text{if } e_t = 2 \end{cases} \tag{35}$$

where

$$\underline{a}_t = \begin{cases} 0 & \text{if } h_t = 1 \\ (1 - \delta_h)a_t & \text{if } h_t = 2 \end{cases} \tag{36}$$

As for investment firms, taking first order condition yields the following equilibrium condition:

$$z_t^{MEI} p_t^i = 1 + \frac{z_t^{MEI}\psi_i}{2}\left[3\frac{i_t^2}{i_{t-1}^2} - 4\frac{i_t}{i_{t-1}} + 1\right] - \mathbb{E}_t Q_{t+1} z_{t+1}^{MEI} \psi_i \frac{i_{t+1}^2}{i_t^2}\left[\frac{i_{t+1}}{i_t} - 1\right] \tag{37}$$

Notice that, if we impose steady-state conditions ($i_{t-1} = i_t = i_{t+1}$ and $z_t^{MEI} = z_{t+1}^{MEI} = 1$), the above equation becomes $p_t^i = 1$.

Capital firms decide the utilization rate and the capital accumulation. Taking the first order condition with respect to utilization rate h_t in the problem of capital firms yields:

$$r_t^k = p_t^i \delta_0 \delta_1 n_t^{\delta_1 - 1} \tag{38}$$

The first order condition with respect to k_{t+1} yields:

$$p_t^i = \mathbb{E}_t Q_{t+1}\left[r_{t+1}^k n_{t+1} + (1 - \delta_0 n_{t+1}^{\delta_1}) p_{t+1}^i\right] \tag{39}$$

As for the intermediate good firms and final good firms, I focus on a symmetric equilibrium in which all intermediate firms choose the same price in period t. Therefore $P_t = P_t(j)$ and $y_t = y_t(j)$ for all j in equilibrium. The Lagrangian for an intermediate good firm j is as follows:

$$\begin{aligned}\mathbb{E}_0 \sum_{t=0}^{\infty}\left(\prod_{\tilde{t}=0}^{t} Q_{\tilde{t}}\right)\Bigg[\frac{P_t(j)}{P_t} y_t(j) - \frac{R_t^k}{P_t}(k_t(j) n_t(j)) - \frac{X_t}{P_t}\ell_t(j) - \frac{\psi_1}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\pi}\right)^2 y_t \\ -\psi_0 + \lambda_t^f\left\{y_t(j) - \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} y_t\right\} + mc_t\left\{z_t^{TFP}(k_t(j)n_t(j))^{\theta}\ell_t(j)^{1-\theta} - y_t(j)\right\}\Bigg]\end{aligned} \tag{40}$$

where mc_t is the Lagrange multiplier for production technology, and can be interpreted as the marginal cost of producing one unit of intermediate goods. First order conditions are:

$$\frac{P_t(j)}{P_t} + \lambda_t^f - mc_t = 0 \tag{41}$$

$$\frac{R_t^k}{P_t} - mc_t z_t^{TFP}\theta(k_t(j)n_t(j))^{\theta-1}\ell_t(j)^{1-\theta} = 0 \tag{42}$$

$$\frac{X_t}{P_t} - mc_t z_t^{TFP}(1-\theta)(k_t(j)n_t(j))^{\theta}\ell_t(j)^{-\theta} = 0 \tag{43}$$

$$\begin{aligned}\left[\frac{1}{P_t}y_t(j) - \psi_1\left(\frac{P_t(j)}{P_{t-1}(j)} - \bar{\pi}\right) y_t \frac{1}{P_{t-1}(j)} + \lambda_t^f \epsilon_p P_t(j)^{-\epsilon_p - 1} P_t^{\epsilon_p} y_t\right] \\ + \mathbb{E}_t Q_{t+1}(-\psi_1)\left(\frac{P_{t+1}(j)}{P_t(j)} - \bar{\pi}\right) y_{t+1} P_{t+1}(j)(-P_t(j)^{-2}) = 0\end{aligned} \tag{44}$$

substituting in $r_t^k = R_t^k/P_t$, $x_t = X_t/P_t$, $\pi_t = P_t/P_{t-1}$, $P_t(j) = P_t$, $y_t(j) = y_t$, $k_t(j)n_t(j) = k_t n_t$, and $\ell_t(j) = \ell_t$, the above conditions become:

$$1 + \lambda_t^f - mc_t = 0 \tag{45}$$

$$r_t^k = mc_t z_t^{TFP}\theta(k_t n_t)^{\theta-1}\ell_t^{1-\theta} \tag{46}$$

$$x_t = mc_t z_t^{TFP}(1-\theta)(k_t n_t)^{\theta}\ell_t^{-\theta} \tag{47}$$

$$\left[y_t - \psi_1(\pi_t - \bar{\pi}) y_t \pi_t + \lambda_t^f \epsilon_p y_t\right] + \mathbb{E}_t Q_{t+1}\left[\psi_1(\pi_{t+1} - \bar{\pi}) y_{t+1}\pi_{t+1}\right] = 0 \tag{48}$$

By substituting out λ_t^f:

$$r_t^k = mc_t z_t^{TFP} \theta (k_t n_t)^{\theta-1} \ell_t^{1-\theta} \tag{49}$$

$$x_t = mc_t z_t^{TFP} (1-\theta)(k_t n_t)^{\theta} \ell_t^{-\theta} \tag{50}$$

$$[y_t - \psi_1 (\pi_t - \overline{\pi}) y_t \pi_t + (mc_t - 1)\epsilon_p y_t] + \mathbb{E}_t Q_{t+1} [\psi_1 (\pi_{t+1} - \overline{\pi}) y_{t+1} \pi_{t+1}] = 0 \tag{51}$$

Notice that $P_{t-1}(j)$ is eliminated from the system of equations characterizing the optimal decision of intermediate good firms, which means that we do not need to keep track of past price levels. The amount of dividends from intermediate good firms, d_t^{INT}, can be computed as follows:

$$d_t^{INT} = y_t - r_t^k n_t k_t - x_t \ell_t - \frac{\psi_1}{2} (\pi_t - \overline{\pi})^2 y_t - \psi_0 \tag{52}$$

Total number of mutual fund shares and aggregate labor supply are obtained by aggregating up individual worker's share holding and labor supply:

$$\overline{a} = \int a \, d\, m_t \tag{53}$$

$$\ell_t = \int \mathbb{1}_{e=1} p \eta_s \, d\, m_t \tag{54}$$

Aggregating up the budget constraint of individual households yields the following:

$$c_t = d_t + w_t \ell_t \tag{55}$$

Substituting the dividends yields the following aggregate resource constraint:

$$y_t = c_t + i_t + z_t^{MEI} i_t \frac{\psi_i}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 + \sum_s \kappa_s v_{s,t} + \frac{\psi_1}{2} (\pi_t - \overline{\pi})^2 y_t + \psi_0 \tag{56}$$

The left-hand-side is total output. The right-hand-side consists of aggregate consumption expenditures, investment, investment adjustment cost, vacancy posting cost, nominal price adjustment cost, and fixed cost of production.

Although aggregate bond supply is assumed to be zero in equilibrium by assumption, no arbitrage condition between mutual fund shares and nominal bonds has to hold, which is characterized as follows:

$$R_t = \mathbb{E}_t \frac{1}{Q_{t+1}} \pi_{t+1} \tag{57}$$

Notice that R_t is the return of nominal bond from period t to $t+1$, so is determined in period t, while the discount factor Q_{t+1} and the inflation rate π_{t+1} are realized in period $t+1$.

4 Calibration

One period is a quarter. Section 4.1 discusses common parameters and Section 4.2 covers type-specific parameters. Tables 5 and Table 6 summarize calibration of the common parameters and the type-specific parameters, respectively. By imposing steady state conditions ($\mathbf{x}_t = \mathbf{x}_{t+1} = \overline{\mathbf{x}}$ and $\mathbf{y}_t = \mathbf{y}_{t+1} = \overline{\mathbf{y}}$), I can obtain equations characterizing steady-state values of aggregate variables. Steady-state values of the aggregate variables, together with how they are obtained, are summarized in Table A.2 in Appendix D.

Table 5: Summary of Calibration: Main Parameters

	Value	Description	Target
σ	2.0000	Relative risk aversion	Standard in literature.
β_1	0.9885	Discount factor (high)	See text.
β_2	0.8904	Discount factor (low)	See text.
ρ_β	0.9957	Persistence of discount factor shock.	Avg duration of β = 58 years.
ρ_p	0.9160	Persistence of individual prod. shock	Storesletten et al. (2001).
σ_p	0.3085	S.D. of individual prod. shock	Storesletten et al. (2001).
δ_h	0.0182	Liquidity for wealthy hand-to-mouth	Ratio of credit limit to wealth.
$\overline{a}$	1.0000	Total supply of mutual fund shares	Normalization.
μ	0.7962	Matching efficiency	Avg job-finding rate is 62.6%.
α	0.7761	Matching elasticity	Bartscher et al. (2021).
ω_0	0.9700	Steady-state wage share	Nakajima (2012b).
ω_1	0.4490	Sensitivity of wage to productivity.	Nakajima (2012b).
ω_2	–0.1326	Nominal wage stickiness.	Gertler et al. (2008).
ψ_i	0	Investment adjustment cost.	Volatility of PCE.
δ_0	0.0150	Avg depreciation rate	From NIPA.
δ_1	1.5833	Curvature of depreciation cost	Utilization rate $n = 1$ in steady state.
ϵ_p	20.000	Elasticity of substitution	Price mark-up of 5%.
θ	0.3000	Capital share of production	Labor share = 2/3.
ψ_0	0.1751	Fixed cost of production	Steady-state profit = 0.
ψ_1	38.080	Price adjustment cost	Avg price duration = 5 quarters.
ϕ_0	0.4610	UI replacement rate	Avg across U.S. states.
ϕ_1	0.5120	Upperbound of UI benefits	Avg across U.S. states.
ρ_R	0.8000	Interest rate smoother	Standard in literature.
ϕ_π	1.5000	Taylor response to inflation	Standard Taylor rule.
ϕ_y	0.1250	Taylor response to output	Standard Taylor rule.
$\overline{\pi}$	1.0050	Avg inflation rate	Annual inflation rate of 2%.
$\overline{R}$	1.0138	Avg nominal interest rate	Endogenously obtained.
ρ_{MP}	0.6100	Persistence of monetary policy shock	Kaplan et al. (2018).
σ_{MP}	0.0025	S.D. of monetary policy shock	Standard in literature.
ρ_{TFP}	0.9500	Persistence of TFP shock	Standard in literature.
σ_{TFP}	0.0029	S.D. of TFP shock	Justiniano et al. (2010).
ρ_{MEI}	0.8100	Persistence of MEI shock	Justiniano et al. (2010).
σ_{MEI}	0.1050	S.D. of MEI shock	Justiniano et al. (2010).

Note: Quarterly frequency.

4.1 Common Parameters

The coefficient of relative risk aversion is set at σ=2.0, a commonly-used value. I assume two values of β. Calibration of β_1 and β_2 is discussed in the next section, since the βs are calibrated together with type-dependent parameters. As for the transition probabilities of β, I assume symmetry, which implies that the transition matrix is characterized by one parameter, ρ_β, which is the probability that β stay the same from one period to the next. ρ_β is set at 0.9957, which is consistent with the average duration of β being 58 years (232 quarters). I choose 58 years since the statistics associated with hand-to-mouth are computed using households of ages 22-79 (Section 2.2). The transition probabilities of individual labor productivity shock p depend on the employment status before and after the employment transition in period t. If a worker is employed and remains employed (e_t=e_{t+1}=1), the transition probabilities are discrete version of an AR(1) shock with the persistence ρ_p and the standard deviation σ_p. ρ_p = 0.9160 and σ_p = 0.3085 are taken from Storesletten et al. (2001).[10] Since they estimate the parameters using annual data, I assume that, with probability of 0.75 (three quarters out of four), the individual productivity remains the same, while, with probability of 0.25 (one quarter out of four), individual productivity changes according to the discretized Markov process using the annual productivity shock. In case a worker remains unemployed (e_t=e_{t+1}=2) or a worker was unemployed and finds a job (e_t=2, e_{t+1}=1), I assume the individual labor productivity shock remains the same. The parameter controlling the amount of liquidity available when a worker is hit by a wealthy hand-to-mouth shock, δ_h is set at 0.0182. This is obtained by computing the ratio of median credit card limit across all households and median total wealth. Following Kaplan et al. (2014), the credit card limit is defined as equivalent to one-month earnings. The definition of total wealth is described in Section 2.2. The supply of mutual fund shares is set at $\overline{a} = 1$.

In order to pin down the matching efficiency parameter μ, first I assume that the average vacancy posting cost ($\overline{\kappa}$) is 1.5 month equivalent of average wage (= $0.5\overline{wp\eta}$). Using the zero-profit condition for labor firms posting a vacancy to hire an average worker, and the average job-finding rate of 63.4% per quarter, I can back up the matching efficiency parameter μ=0.7962. Once μ is fixed, I can compute type-specific κ_s that is consistent with the type-specific job-finding rate f_s. As for the elasticity of the matching function α, although α is typically estimated to be around 0.5 (Petrongolo and Pissarides (2001)), I calibrate α to be 0.7761, such that the Black-White unemployment gap shrinks by 0.34pp in response to a 25bp accommodative monetary policy shock (estimate by Bartscher et al. (2021)). I will discuss the effects of a monetary policy shock in Section 7.2. The simple wage function is extended from the one used in Nakajima (2012b). In particular, ω_0 = 0.97 reflects that profits for firms out of production is 3% of the total surplus. The sensitivity parameter of wage to changes in labor productivity is ω_1 = 0.449, which is computed by Hagedorn and Manovskii (2008). The sensitivity parameter of wage to inflation (price changes) is set at ω_2 = $-$0.1326, using the information provided by Gertler et al. (2008). In their estimated model with staggered nominal wage bargaining, the fraction 0.283 of firms re-optimize the nominal wage without restrictions, which means that the inflation rate does not affect the bargained wage in real terms. The nominal wage of the remaining (the fraction of 0.717) firms are not optimally adjusted. In particular, the elasticity of the nominal wage in terms of the inflation rate is estimated to be 0.815. In other words, the elasticity of real wage in terms of the inflation rate is $-$0.185. I do not model explicitly staggered nominal wage bargaining, but using these two pieces of information implies that the average elasticity of real wage to inflation is ω_2 = $-$0.185 x 0.717 = $-$0.1326.

As for the production sector, the parameter controlling the investment adjustment cost is set at

[10] σ_p is the average between its values in expansions and recessions, estimated by Storesletten et al. (2001).

ψ_i=0. The intention is to use this parameter to match the volatility of aggregate consumption, but it turns out that the volatility of aggregate consumption generated by the baseline model is already higher than the data with ψ_i=0. The average quarterly depreciation rate is set at δ_0 = 1.5%, following NIPA. The curvature parameter of the depreciation cost function is set at δ_1 = 1.583, which guarantees that the steady-state utilization rate is one. ϵ_p is set at 20, implying price mark-up of 5% (Bayer et al. (2019)). Capital share parameter of the production function θ is set at 0.30, which implies that the resulting labor share (after taking into account the price mark-up and the profits of labor firms) is about 2/3. The fixed cost of production is set at ψ_0=0.1751, making sure that the steady-state profit (and dividends) is zero. The price adjustment cost parameter ψ_1 is set at 38.08, such that, when converted into Calvo framework, the parameter value implies the nominal price is adjusted every five quarters on average (Gornemann et al. (2021)). This is a common value in the New-Keynesian literature.

In terms of the fiscal and monetary policy, the UI replacement rate and the upperbound of the UI benefits are average numbers across U.S. states. Specifically, the UI replacement rate is ϕ_0 = 0.461 and the upperbound of the UI benefits is ϕ_1 = 0.512 of average earnings. As for the monetary authority, interest rate smoothing parameter is set at ρ_R=0.80, which is standard in literature. The response parameters to inflation gap and to output gap are set at ϕ_π=1.5 and ϕ_y=0.125, respectively, again following the standard specification of Taylor rule. Average inflation rate is set at $\overline{\pi}$=1.005, implying 2% annual inflation target. Average nominal interest rate is $\overline{R}$=1.0138, which is obtained from the average inflation rate and the steady-state real rate of return, which is endogenously determined. Standard deviation of monetary policy shock is set at σ_{MP}=0.0025. The persistence of the monetary policy shock is set at ρ_{MP}=0.61, following Kaplan et al. (2018).

There are three aggregate shocks in the model, namely total factor productivity (TFP) shock z_t^{TFP}, marginal efficiency to investment (MEI) shock z_t^{MEI}, and monetary policy shock z_t^{MP}. The persistence and the standard deviation of the monetary policy shock are already discussed above. The persistence of the TFP shock is set at ρ_{TFP} = 0.95, which is standard in the literature. The persistence of the MEI shock is set at ρ_{MEI} = 0.81, following Justiniano et al. (2010). In calibrating the remaining two parameters — standard deviation of the TFP shock and the MEI shock, I use the standard deviation of output and the variance decomposition of output volatility in Justiniano et al. (2010). The standard deviation of detrended GDP in the U.S. data from 1980 to 2019 is 1.23%. According to the estimated model of Justiniano et al. (2010), 0.25 and 0.60 of the output variations in business cycle frequencies (between 6 and 32 quarters) are accounted by the TFP shock and the MEI shock, respectively.[11] Since I have only three aggregate shocks, and the parameters associated with the monetary policy shock are already calibrated using independent evidence, I calibrate the standard deviation of the TFP shock and that of the MEI shock such that (1) output volatility of the model is 1.23%, and (2) the ratio of fractions of the output variance accounted for by the two shock is 0.25/0.60. σ_{TFP} = 0.0029 and σ_{MEI} = 0.1050 satisfy the two targets simultaneously. The standard deviation of output in the model is 1.23%, and the model implies that 0.28, 0.05 and 0.66 of output fluctuations can be

[11] Contribution from other shocks are as follows: monetary policy shock = 0.04, government expenditure shock = 0.02, investment specific productivity shock = 0.00, price mark-up shock = 0.02, wage mark-up shock = 0.01, intertemporal preference shock = 0.05.

Table 6: Calibration: Different Types

s (Type)	π_s	η_s	f_s	λ_s	u_s/π_s	v_s	κ_s	π_s^h	ι_s
1 (White)	68.25	1.030	66.0	3.80	5.44	0.0293	1.431	19.4	0.002
2 (Asian)	5.08	1.184	69.7	3.59	4.90	0.0022	1.677	18.0	–0.009
3 (Hispanic)	15.28	0.735	69.1	6.20	8.23	0.0108	0.661	28.9	–0.078
4 (Black)	11.39	0.810	49.5	6.44	11.52	0.0069	0.820	31.8	–0.075

Note: π_s is the fraction of each type, in percent. η_s is productivity of each type, normalized such that the overall average is one. Both are obtained from the CPS. f_s is the quarterly job-finding rate, and λ_s is the quarterly separation rate, both in percent. They are obtained from Cajner et al. (2017) and converted from monthly to quarterly numbers. u_s is the number of unemployed workers of type-s. u_s/π_s is the unemployment rate, which is implied by the job-finding rate and the separation rate, in the steady state. v_s is the number of vacancy postings. κ_s is the vacancy posting cost. They are obtained from the steady-state conditions of the model. π_s^h is the probability of the wealthy hand-to-mouth shock, obtained from the proportions of wealthy hand-to-mouth in Table 3. ι_s is type-specific savings interest rate premium, calibrated to match the fraction of poor hand-to-mouth for each type, except for White, whose ι_s is pinned down to satisfy zero average premium.

accounted for by the TFP shock, the monetary policy shock, and the MEI shock, respectively.

4.2 Permanent Types

This section describes calibration of type-specific parameters. Table 6 summarizes the results. The basic principle is to assume common preferences across all racial types, but assume heterogeneity in labor market and financial conditions, to replicate racial differences documented in Section 2. Specifically, I assume 4 types, with $s = 1, 2, 3, 4$ representing White, Asian, Hispanic, and Black workers, respectively. The type-dependent parameters are associated with job-finding rate (κ_s), separation rate (λ_s), average earning levels (η_s), the probability of the wealthy hand-to-mouth shock (π_s^h), and the rate of return of savings (ι_s).

In Table 6, π_s in the first column shows the proportion of each type, computed as share within labor force, from the Current Population Survey (CPS), Annual Social and Economic Supplement. I take the average between 2003 and 2018. η_s is the average labor productivity for each type, obtained from the median usual weekly earnings for each racial group, reported by the Bureau of Labor Statistics (BLS). Values of η_s are normalized by the overall median usual weekly earnings. f_s and λ_s are quarterly job-finding rate and the quarterly separation rate, respectively. These numbers are based on the monthly transition rates reported by Cajner et al. (2017). Specifically, I convert the monthly transition rates into quarterly rates, and adjust both rates by the same proportion so that the implied steady-state unemployment rate for each racial group matches the unemployment rate in the CPS, shown in the next column as u_s/π_s. Once parameters of the matching function are set (discussed in the previous subsection), the number of vacancy postings for each type-s, v_s can be backed up using the formula for the job-finding rate. Then, the vacancy-posting cost for each type, κ_s can be backed up using the free-entry condition for each type-s. Basically, κ_s for each racial group guarantees that the job-finding rate for each racial group is realized in the steady state. The next two columns show the obtained values of v_s and κ_s. The next column of Table 6 shows the i.i.d. probability of the wealthy hand-to-mouth shock, π_s^h. These are obtained using the fraction of the wealthy-hand-to-mouth for each type-s, reported in Table 3.[12] The last column shows the type-specific premium to the rate of return to savings ι_s.

Let me discuss how ι_s is pinned down, together with other parameters. First of all, I fix the steady-state capital-output ratio to be 12 (annualized value of 3), which implies that the real interest rate is 0.875%.[13] Conditional on ι_s, this pins down the return of shares of mutual funds for households. β_1 (the higher discount factor) is used to guarantee that the demand for shares of the mutual funds is equal to its supply ($\overline{a} = 1$), together with other parameters discussed below. Now, I have five parameters — β_2, and ι_s to match the fraction of poor hand-to-mouth workers for four racial groups. In order to match the number of targets and that of parameters, I impose one condition: $\int \iota_s a \; dm = 0$, where m is the steady-state type distribution. This condition states that the average race-specific premium is zero. This leaves ι_s for three racial groups for the fraction of poor hand-to-mouth for three racial groups. I use ι_s for Asian, Hispanic, and Black workers to match the fraction of poor hand-to-mouth for these three racial minorities. Finally, β_2 (the lower discount factor) is used to make sure that the fraction of poor hand-to-mouth among White workers in the model matches the data counterpart.

Notice that the resulting race-specific premium ι_s varies greatly across racial groups. For White workers, the premium is positive and relatively small at 0.2%, meaning that their type-specific saving rate is $r + \iota_s$ = 0.875% + 0.2% = 1.075% per quarter. For Asian workers, ι_s is also relatively small but negative at -0.9%. It has to be lower than ι_s for White workers since the fraction of poor hand-to-mouth among Asians (12.1%) is higher than that of White workers (7.2%). Compared with White and Asian workers, ι_s for Hispanic (-7.8%) and Black (-7.5%) turn out to be quite large. These numbers imply that the quarterly rate of return of savings for Hispanic and Black workers is -7.0% and -6.6%, respectively. The rate of return of savings for Hispanic and Black workers turns out to be negative and large since ι_s is the only difference that workers of different racial groups face in the financial market. In other words, the differences in ι_s, by construction, capture variety of racial differences in the financial market conditions such as difficulty in purchasing a house or obtaining mortgage and thus enjoying high returns from owning a home, or portfolio allocation into stocks, which are associated with a higher average return. Under the assumption that preferences are common across racial groups, the large dispersion of ι_s could be interpreted as Hispanic and Black workers facing significant disadvantages in the financial market. In the literature trying to account for the Black-White wealth gap, Aliprantis et al. (2019) downplay that the Black-White differences in the rate of return in explaining the wealth gap, while Bartscher et al. (2021) and Derenoncourt et al. (2022) emphasize the differences in portfolio composition and differences in returns of assets in widening the wealth gap in the recent decades.

[12] Since workers with zero savings are poor hand-to-mouth and cannot be wealthy hand-to-mouth even if they are hit by the shock, π_s^h can be obtained by $\pi_s^{h2m\text{-}w}/(1 - \pi_s^{h2m\text{-}p})$, where $\pi_s^{h2m\text{-}p}$ and $\pi_s^{h2m\text{-}w}$ are the fraction of poor and wealthy hand-to-mouth for type-s, respectively.

[13] In the steady state, the real interest rate is equal to the rate of return of capital, which is characterized by Equation (49).

Table 7: Wealth Distribution for Four Racial Groups

	Overall	White	Asian	Hispanic	Black
Data					
Total Hand-to-Mouth	31.4	25.2	27.9	48.7	47.0
Poor Hand-to-Mouth	12.3	7.2	12.1	27.9	22.3
Wealthy Hand-to-Mouth	19.1	18.0	15.8	20.8	24.7
Mean total wealth	100.0	123.3	100.6	28.5	25.6
Median total wealth	24.5	34.9	22.4	2.6	4.6
Model					
Total Hand-to-Mouth	31.4	25.2	27.9	48.7	47.0
Poor Hand-to-Mouth	12.3	7.2	12.1	27.9	22.3
Wealthy Hand-to-Mouth	19.1	18.0	15.8	20.8	24.7
Mean total wealth	100.0	138.2	72.8	6.5	9.3
Median total wealth	24.3	62.4	24.3	0.9	2.0

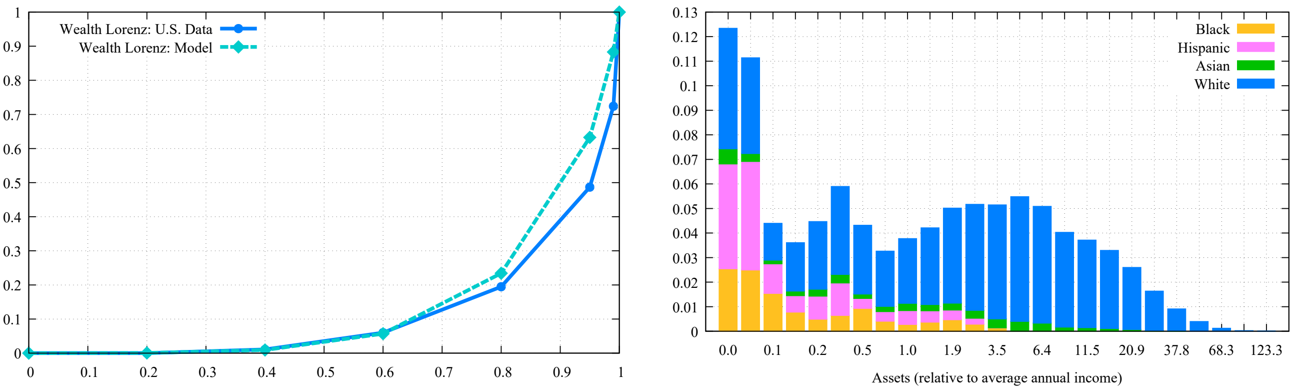
Note: Data are computed using the Survey of Consumer Finances. See the note for Table 3 for description of the data. Model statistics are computed using the steady state of the calibrated baseline model. Mean and median wealth are normalized such that the overall mean is 100.

5 Racial Inequality in the Model

The model exhibits racial inequality in terms of both income and wealth. Racial inequality of income is mostly exogenously set, capturing observed differences in average income and unemployment risks, as described in Section 4. On the other hand, inequality in wealth is endogenous. In this section, I compare the racial inequality in terms of wealth between the data and the model. Table 7 compares the proportion of hand-to-mouth households as well as mean and median wealth, in the U.S. data (top panel) and in the model (bottom panel). The data are mostly the same as those in Table 3, but wealth are normalized such that the overall mean wealth is 100.0 in this table, for easier comparison. The numbers for the model are taken from the steady state of the baseline model.[14]

Let's start with the proportion of hand-to-mouth (top three rows of each panel). The main takeaway is that the model matches the fraction of hand-to-mouth perfectly, by construction. As I discussed in the previous section, β_2 and ι_s are calibrated to match the proportion of poor hand-to-mouth, and p_s^h is set to match the proportion of wealthy hand-to-mouth. On the other hand, since the fraction of hand-to-mouth for different racial groups is matched in a somewhat crude way, using the heterogeneous rate of return of wealth, the model implies smaller wealth holding for minority groups with a high fraction of hand-to-mouth. This is shown in the last two rows of each panel. Specifically, median wealth (relative to overall mean wealth) is about 0.9 for Hispanics and 2.0 for Blacks in the model, while the median wealth is 2.6 for Hispanics and 4.6 for Blacks in the data. Mean wealth for Blacks and Hispanics is 9.3 and 6.5 in the model, compared with 25.6 and 28.5 in the data, respectively. For Asians, median wealth is close be-

[14] The proportion of hand-to-mouth including all racial groups is slightly different because I use the proportion of each racial group in the labor force in this table, while the racial composition is taken from the SCF in Table 3.



(a) Wealth Lorenz: Data and Model (b) Histogram of Wealth Distribution: Model

Figure 2: Wealth Distribution

tween the model (24.3) and the data (22.4), while mean wealth in the model (72.8) is slightly smaller than in the data (100.6). Among Whites, the model (138.2) replicates mean wealth in the data (123.3) well, while the model (62.4) overshoots median wealth in the data (34.9). The model tends to undershoot wealth holding for racial minority since I calibrate the model to match the fraction of poor hand-to-mouth using a parsimonious way, adjusting the rate of return of savings. On the other hand, the model overshoots wealth holding for White workers since their higher discount factor is calibrated to match the aggregate amount of savings.

How about the overall wealth distribution? Panel (a) of Figure 2 compares the wealth Lorenz curves of the data and the model, including all racial groups. Since the model is calibrated to match the fraction of poor hand-to-mouth, the Lorenz curve of the model is close to the empirical Lorenz curve for the bottom 80% of wealth distribution. Among the top 20 percent, the wealth is more unequally distributed in the data compared with the model, which is a common issue of the model without additional features such as entrepreneurs or high return of wealth for the wealthiest.[15] Gini index for wealth for all racial groups is 0.785 in the data, while it is 0.654 in the model. Panel (b) of Figure 2 shows a histogram of wealth for four racial groups in the model. There is large mass of White, Asian, Hispanic, and Black workers at the lower part of histogram, which represent a large proportion of poor hand-to-mouth, while the top end of wealth distribution is mostly represented by Whites.

6 Racial Differences in Marginal Propensity to Consume (MPC)

This section explores the heterogeneity in the marginal propensity to consume (MPC), which is measured as the consumption response to an unexpected one-time income transfer. Borrowing from Kaplan and Violante (2021), *since the MPC is a central concept in modern macroeconomics, the usefulness of these models is closely tied to their ability to reproduce the evidence on MPCs.* I focus on the heterogeneity in MPCs across racial groups, which tells how differently different racial groups could be affected by monetary policy. Table 8 shows how the MPC is dif-

[15] See Quadrini and Ríos-Rull (1997) and De Nardi and Fella (2017) for review of the literature.

Table 8: Racial Differences in Marginal Propensity to Consume

MPC (%)	All	Poor H2M	Wealthy H2M	No H2M
Overall	11.9	29.0	25.8	4.9
White	10.1	27.5	28.3	4.0
Asian	11.2	30.6	25.8	4.8
Hispanic	17.1	29.5	20.5	9.0
Black	15.9	30.8	21.0	7.3

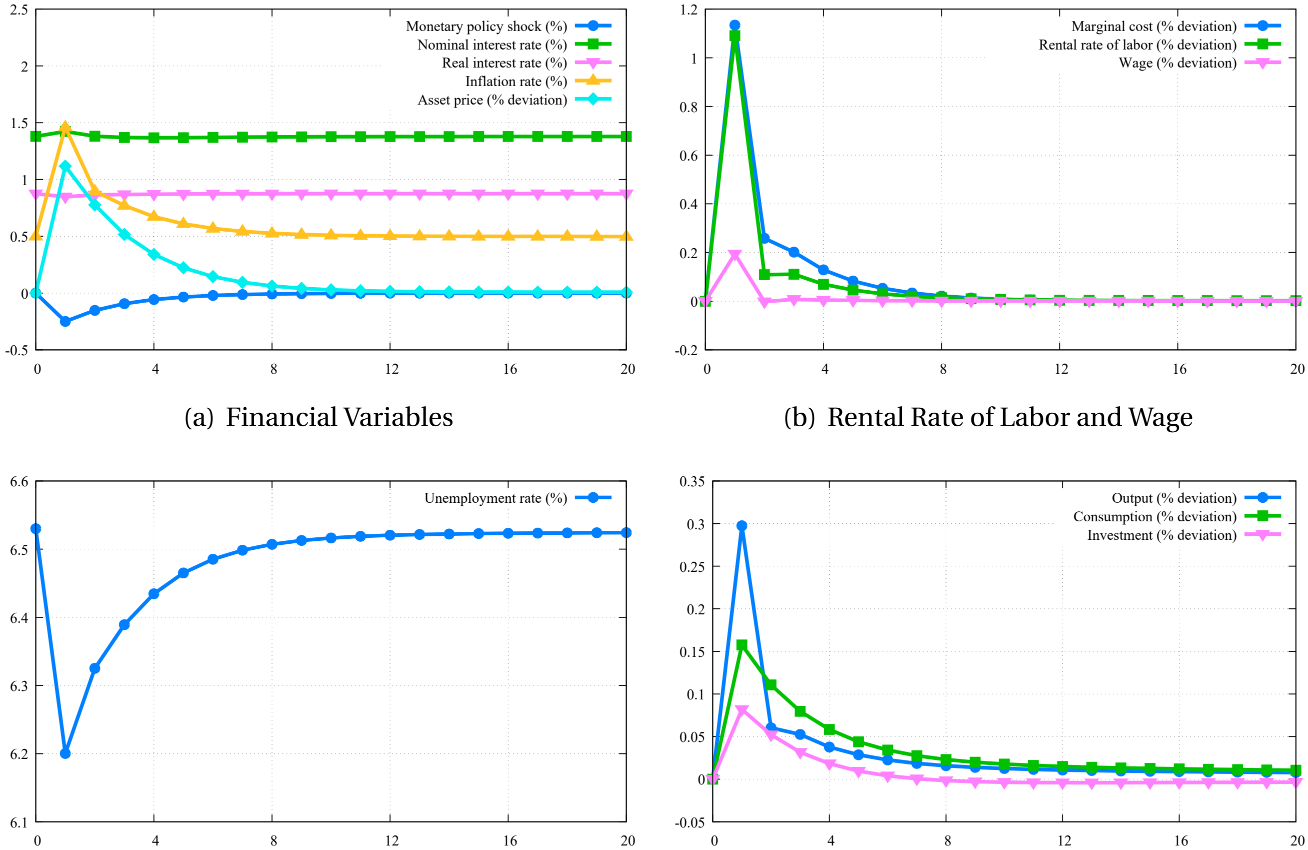
Note: MPC is defined as the quarterly percentage change in consumption in response to an unexpected one-time transfer of $500, divided by $500. No general equilibrium effects are considered.

ferent across different racial groups (rows) as well as how the MPC is different across workers who are either poor or wealthy hand-to-mouth or not hand-to-mouth (columns). Following the standard, the MPC in the baseline is defined as the immediate one-quarter response of consumption expenditures to an unexpected one-time transfer of $500 (converted into the model unit), divided by $500.[16]

The overall MPC in the calibrated model is 11.9%. This is slightly below the lower bound of the range of empirical estimates summarizes by Kaplan and Violante (2021), which is between 15% and 25%. The estimate of the MPC obtained by Ganong et al. (2020) is 23%. Interestingly, as can be seen in the first column, there is a noticeable racial heterogeneity. The MPC of White workers is 10.1%, and that of the Asian workers in 11.2%. On the other hand, the MPC of Hispanic and Black workers are 17.1% and 15.9%, respectively. The MPC of Black and Hispanic workers are 60-70% higher than White workers'. The result that the MPC of Black workers is 57% higher than the MPC of White workers is broadly consistent with Ganong et al. (2020), who find that Black households cut their consumption 50% more than White households when faced with a similarly-sized income shock. They find that the MPC of Hispanic households is only 20% higher than that of White households, but in my model, the MPC of Blacks and Hispanics have to be similar because they have similarly high fraction of hand-to-mouth, which is the crucial determinant of the MPC.

The second to last columns of Table 8 show that a high MPC is due to the presence of poor (second column) and (to a lesser extent) wealthy (third column) hand-to-mouth workers. On the other hand, workers who are neither poor hand-to-mouth nor wealth hand-to-mouth (fourth column) exhibit a low MPC which is typical in the representative-agent model. Overall, the MPC among workers who are not hand-to-mouth is 4.9%. The MPC among poor hand-to-mouth workers is 29.0%, while the MPC among wealthy hand-to-mouth workers is 25.8%. The main reason why Hispanic and Black workers exhibit a higher MPC is a composition effect; a larger proportion of them are either poor hand-to-mouth or wealthy hand-to-mouth.

[16] Kaplan and Violante (2021) study the MPC in heterogeneous-agent macro models. Carroll et al. (2017) argue that the MPC in heterogeneous-agent models critically depends on the wealth distribution in the model. Jappelli and Pistaferri (2010) review different approaches of measuring the MPC.



(a) Financial Variables

(b) Rental Rate of Labor and Wage

(c) Unemployment Rate

(d) Output and Components

Figure 3: Effects of Monetary Policy Shock: Macro Aggregates

7 Heterogeneous Effects of Monetary Policy with Racial Inequality

This section studies how an accommodative monetary shock affects different racial groups differently. Section 7.1 looks at the response of macroeconomic aggregates, before Section 7.2 investigates how an accommodative monetary policy shock affects different racial groups differently. Section 7.3 investigates welfare implications.

7.1 Macroeconomic Effects

Figure 3 summarizes macroeconomic effects in response to a quarterly -25bp (-100bp annually) accommodative monetary policy shock, up to 20 quarters. The effects are standard in the New-Keynesian DSGE model. Panel (a) contains financial variables. After the initial shock, the monetary policy shock gradually goes back to its steady-state level of zero (dark blue line). Because of the nominal rigidity, real interest rate also declines (pink line), while the nominal interest rate goes up slightly (green line), because the inflation rate (yellow line) and output (blue line in Panel (d)) pick up, and the nominal interest rate rises in response to both, following the Taylor rule. Asset price rises (cyan line), reflecting the stimulated economic activities.

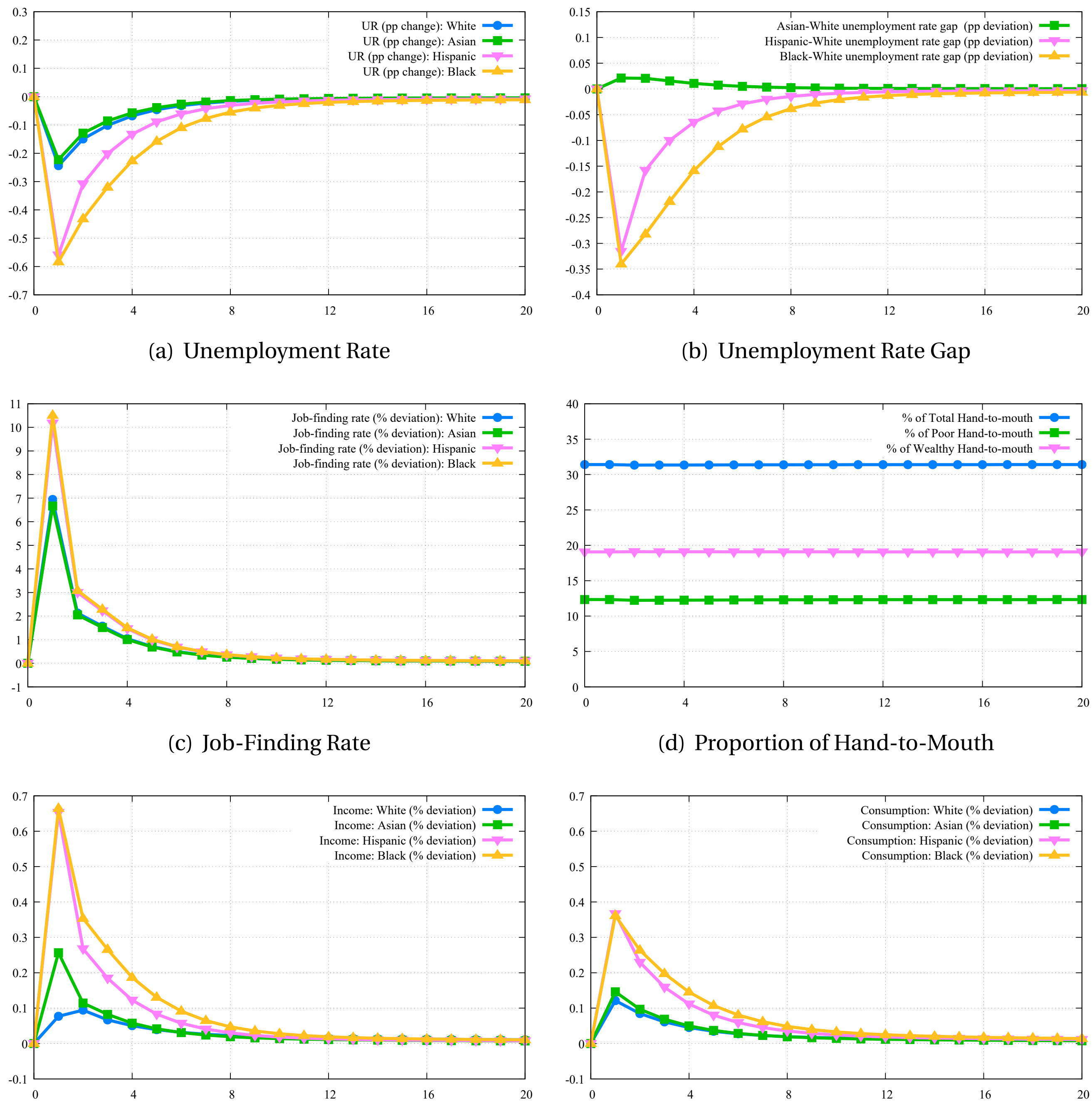
The response of the asset price in the model is qualitatively and quantitatively consistent with the empirical finding by Bernanke and Kuttner (2005). In the baseline model, the asset price gains by 1.1% on impact, while Bernanke and Kuttner (2005) find that an unanticipated -25bp cut in the policy rate is associated with about a 1% increase in broad stock indexes. On the other hand, Bartscher et al. (2021) find that stock prices rise by about 5% and house prices gain by about 2% to the same monetary policy shock. Besides, they find that the the asset prices exhibit in a persistent hump-shaped response, while the response in the baseline model is not persistent.

Panel (b) shows the response of the marginal cost (mc_t, blue line), the rental rate of labor (x_t, green line) and wage rate (w_t, pink line). Since the aggregate demand increases in response to a lower real interest rate, demand for labor increases, which pushes up the marginal cost and the rental rate of labor. Since wage is assumed to respond less elastically to changes in the rental rate of labor, wage increases, but less than the rental rate. As emphasized by Shimer (2005), when wage does not increase as much as the rental rate of labor, profits of labor firms increase more than the rental rate, and thus the number of vacancy postings responds strongly. Consequently, the unemployment rate declines sizably, by 0.33pp (Panel (c)).

Panel (d) shows output and its components. Output increases (blue line) as aggregate demand is stimulated, and both labor input (the unemployment rate goes down) and capital stock increase. Consumption increases (green line) for two reasons. First, there is a standard intertemporal substitution effect; a lower real interest rate discourages savings and brings forward consumption. Second, when a worker is hand-to-mouth, either because the worker has zero assets and thus is poor hand-to-mouth, or because the worker is hit by the wealthy hand-to-mouth shock and cannot use the whole savings for consumption even if the worker wants, a lower unemployment rate, a higher wage, and a higher asset price boost consumption of the constrained worker. Investment also increases (pink line), increasing capital stock.

7.2 Heterogeneous Effects of Monetary Policy to Different Racial Groups

Figure 4 shows how the accommodative monetary shock affects workers of different racial groups. Panel (a) shows that the unemployment rate declines for all four racial groups, but the unemployment rate for Black and Hispanic workers declines more than that for White and Asian workers. Panel (b) shows the same differently; Panel (b) shows the unemployment rate gaps. For example, Black-White unemployment rate gap (yellow line) is the difference between the unemployment rate of Black workers and that of White workers, normalized such that the steady-state gap is zero. Since the Black unemployment rate declines more than the White unemployment rate, the Black-White unemployment rate gap declines, by 0.34pp on impact. As I discussed in the calibration section, Bartscher et al. (2021) estimate that, in the U.S., the Black-White unemployment rate gap shrinks by up to 0.34pp in response to a -25bp monetary policy shock. I calibrate the elasticity of the matching function, α, to replicate this empirical response. Panel (b) also show that the Hispanic-White unemployment rate gap responds to a similar magnitude as the Black-White unemployment rate gap, while Asian-White unemployment rate gap remains close to zero in response to the monetary policy shock. The latter is consistent with the fact that the responses of the unemployment rate are similar between White and Asian workers.



(a) Unemployment Rate

(b) Unemployment Rate Gap

(c) Job-Finding Rate

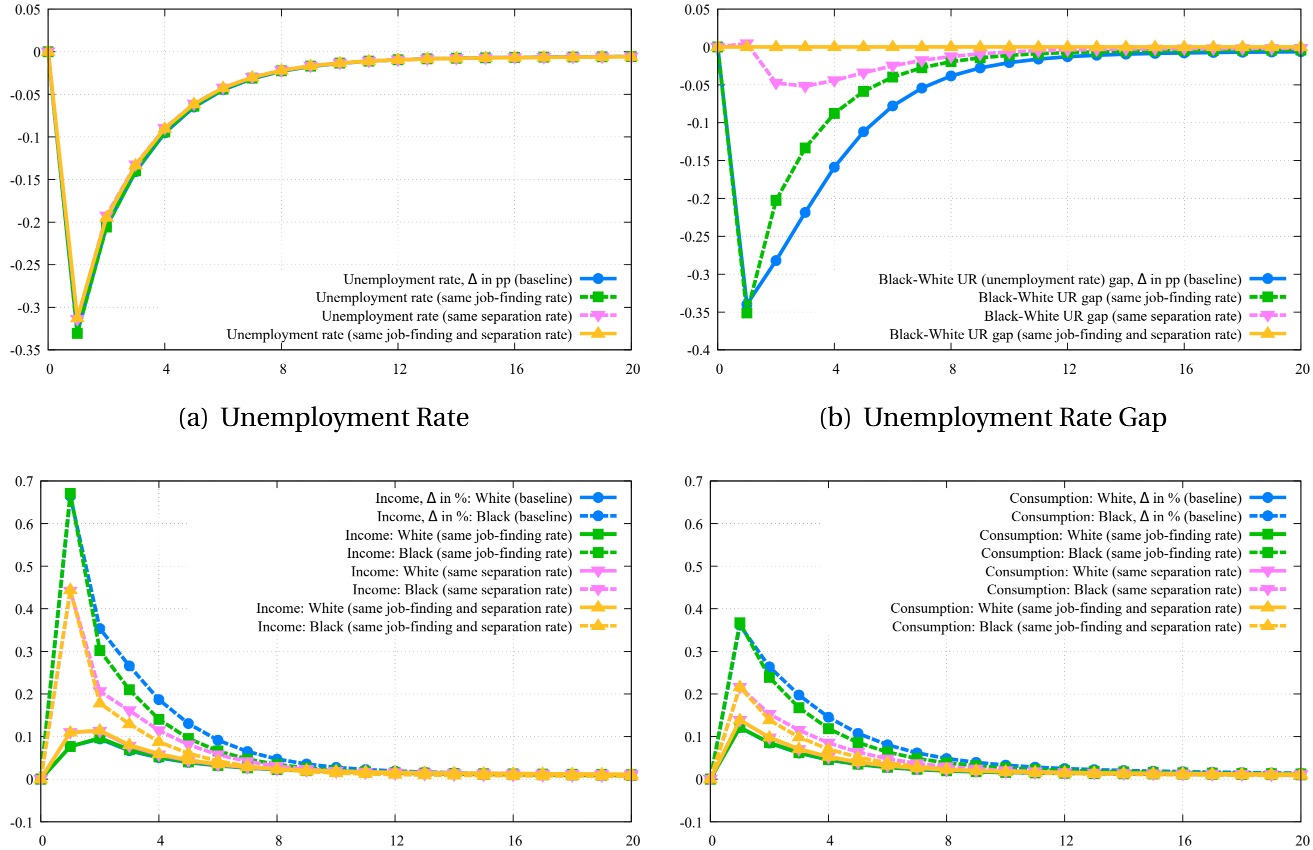
(d) Proportion of Hand-to-Mouth

(e) Income

(f) Consumption

Figure 4: Effects of Monetary Policy Shock: Racial Inequality

Why does the unemployment rate respond differently for difference racial groups? And why does the Black-White unemployment rate gap shrink in response to an accommodative monetary policy shock? Since the separation rate is type-specific but exogenously fixed, different responses of the unemployment rate for different racial groups are due to different responses of the job-finding rate (Figure 4(c)). In response to a −25bp monetary policy shock, the job-



(a) Unemployment Rate

(b) Unemployment Rate Gap

(c) Average Income

(d) Average Consumption

Figure 5: Heterogeneity in Labor Market Risks and Monetary Transmission

finding rate for Black (yellow line) and Hispanic (pink line) workers increases more than 10%, while the job-finding rate for White (blue line) and Asian (green line) workers increases by less than 7%. Mechanically, these differences in the response of the job-finding rate yield the different responses of the unemployment rate. Why does the job-finding rate for Black and Hispanic workers rise more than that of White and Asian workers? In order to answer this question, I study three alternative models. The results of the three alternative models as well as the baseline model (blue lines) are shown in Figure 5. In the first alternative model (green lines), I assume that the steady-state job-finding rates for all racial groups are set the same at the overall job-finding rate in the baseline model, while the separation rates are left unchanged.[17] In the second alternative model (pink lines), I do the opposite; I assume that the separation rates for all races are set at the same overall separation rate in the baseline model, while the job-finding rates are left at the levels as in the baseline model. In the third alternative model (yellow lines), I fix both the job-finding rates and the separation rates for four racial groups as equal to their respective overall average in the baseline model.

The impulse responses of the overall unemployment rate to the -25bp accommodative mon-

[17] More precisely, I adjust κ_s for each s so that the same job-finding rate is achieved in the steady state.

etary policy shock are shown in Panel (a), and the impulse responses of the Black-White unemployment gap are shown in Panel (b). When the steady-state job-finding rates for all racial groups are set the same, both the overall unemployment rate and the Black-White unemployment rate gap respond to the monetary policy shock similarly as the baseline model. In other words, the strong response of the Black-White unemployment rate gap in the baseline model is not due to the racial differences in the job-finding rate. On the other hand, when the separation rates for racial groups are set the same, the response of the Black-White unemployment rate gap almost disappears. Instead of declining up to 0.34pp in the baseline model, the Black-White unemployment rate gap declines up to 0.05pp in the model with the common separation rate. The overall unemployment rate declines slightly less than in the baseline model because the Black unemployment rate declines less. If both the job-finding and the separation rates are set the same, the unemployment rate for all racial groups move in parallel, and the Black-White unemployment rate gap doesn't respond to an accommodative monetary shock.

The intuition is the following. For Black workers, for whom the separation rate is high, a temporary increase in the rental rate of labor induced by the accommodative monetary policy shock affects the value of the labor firm matched with a Black worker more strongly, since labor productivity is higher for a larger fraction of expected duration of the match. Therefore, the increase in the number of vacancies posted in the labor market for Black workers is larger than that for White workers, which narrows the Black-White unemployment rate gap. This intuition of the baseline model is also valid in the alternative model with the same job-finding rate but the baseline race-specific separation rate. The only difference in this alternative model is that the unemployment rate gap reverts back faster once the monetary stimulus dissipates. This is because the job-finding rate generally is higher for Black workers in this alternative model, and thus once the effect of the stimulus to the job-finding rate weakens, the tapering of the job-finding rate is larger in level, which brings down the unemployment rate faster. The same can be seen by comparing the Black-White and the Hispanic-White unemployment rate gap in the baseline model (yellow and pink lines in Figure 4(b)). Both gaps decline by about the same magnitude because the separation rate is equally high for Hispanic and Black workers, but the Hispanic-White unemployment rate gap reverts back faster because the job-finding rate is higher among Hispanics. On the other hand, when the separation rate is assumed the same for all racial groups, the unemployment rate gap does not shrink on impact, because the value of a match with a Black worker and that with a White worker are affected by a temporary increase in the rental rate of labor by the same proportion, and thus their job-finding rate is affected by the same proportion. Since the job-finding rate is higher for White workers than for Black workers, the job-finding rate increases more for White workers in level, but the unemployment rate is lower among White workers. In the end, the change in the unemployment rate among White workers turns out to be similar to that of Black workers. The unemployment rate gap shrinks from the second quarter on after the monetary policy shock, because the job-finding rate for White workers is higher and thus their unemployment rate reverts back faster.

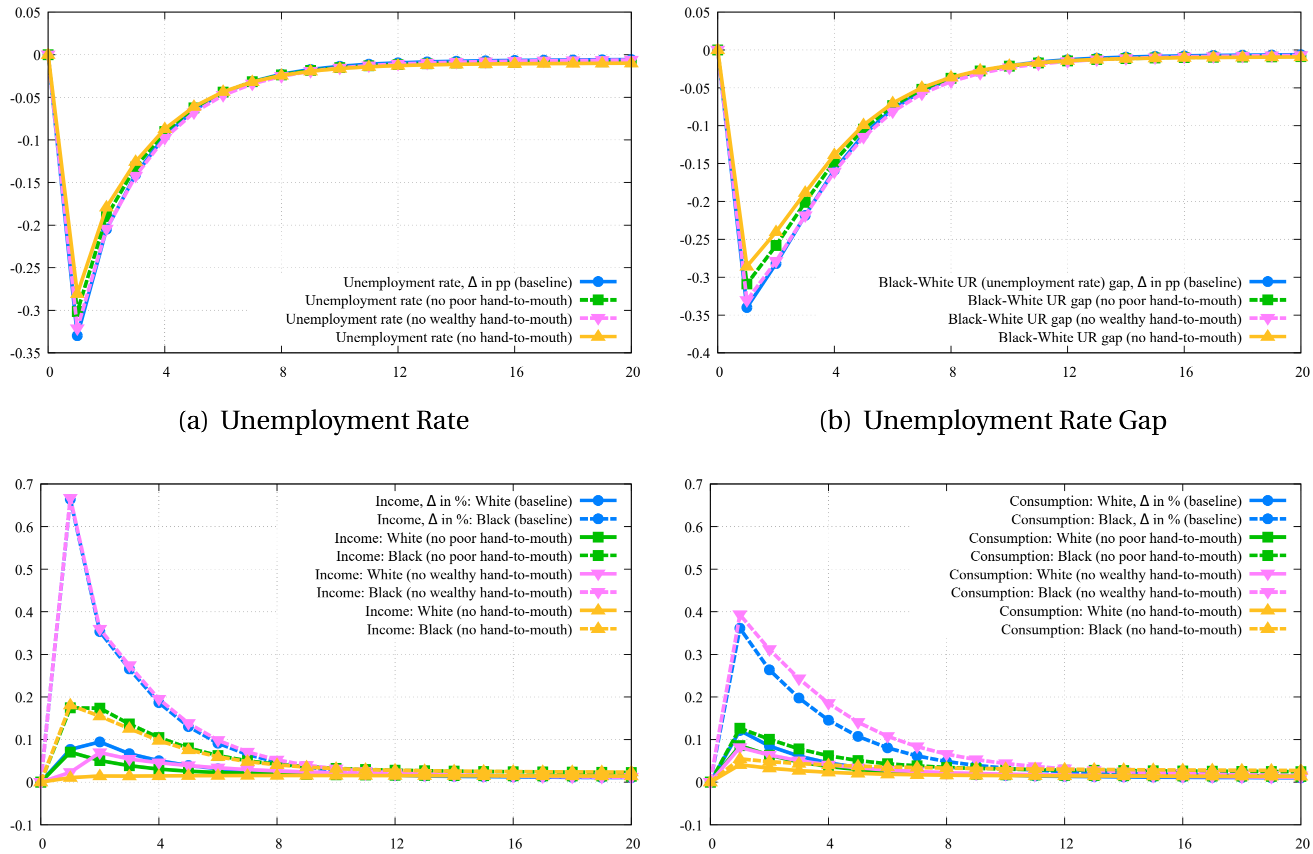
Figure 4(d) shows that the fraction of total, poor, and wealthy hand-to-mouth workers do not respond to the accommodative monetary shock in a sizable manner. Panel (e) shows how average income for four racial groups change in response to the monetary policy shock.[18] It shows

[18] Income does not include capital gain, i.e., change in p_t^a.

that Black and Hispanic average income increase more than Asian average income, and Asian income increases more than that of White. Income composition matters in understanding this heterogeneity. Since the Black and Hispanic unemployment rate decline the most, and employed workers earn more than what unemployed workers receive (UI benefits), their average income increases the most among racial groups. Specifically, Black and Hispanic average income increase by 0.66% and 0.65% on impact, respectively, while the White average income increases only by 0.08%. Black average income increases about 8.6 times more than White's. Meanwhile, although the unemployment rate of White and Asian workers decline similarly in response to the monetary accommodation, average income of White workers increases less than that of Asians (0.26%), because White workers hold significantly more assets on average, and thus financial income makes up a significantly bigger portion for their income. When the real interest rate declines in response to the monetary policy shock, financial income declines, and thus average total income of White workers does not increase as much as Asian's.

Panel (f) shows the response of average consumption for four racial groups. Average consumption responds more strongly for racial groups with a greater average income response. For example, Black (0.36%) and Hispanic (0.37%) average consumption increase more than that of Whites (0.12%) and Asians (0.15%). The ordering in response size is preserved also because the fraction of hand-to-mouth is higher among Hispanic and Black workers. On the other hand, the differences in the consumption response across racial groups are smaller than those of income. This is due to three effects. First is intertemporal consumption smoothing. Black, Hispanic, and Asian workers spread the additional income gains over time. Second is an intertemporal substitution effect; White workers bring forward more consumption than the increase in income because of the lower real interest rate induced by monetary accommodation. Third is the role of capital gains. Capital gains from the asset price increase give White workers additional resources for increasing consumption expenditures.

Panels (c) and (d) of Figure 5 show how average income and consumption for Black and White workers respond differently to the accommodative monetary policy shock, in the baseline model and three alternative models discussed above. Since the unemployment rate for Black and White workers in the alternative model with the same job-finding rate respond to the monetary policy shock in a similar way to the baseline model, income and consumption of White and Black workers in this alternative model (green lines) are close to those of the baseline model (blue lines). In the alternative model with the same separation rate, the unemployment rate for Black workers declines more than that of White workers, but not as much as in the baseline model (pink line in Panel (b)). Therefore, income and consumption of Black workers still increase more than those of White workers, but the differences in income and consumption between Black and White workers are smaller than in the baseline model. Finally, in the alternative model in which both the job-finding rate and the separation rate are the same across all racial groups (yellow lines), although the labor market risks that Black and White workers face are the same, the average income and consumption of Black workers still increase more than those of White worker, mainly due to income composition; as White workers earn significantly larger financial income, which declines in response to the accommodative monetary policy shock, the response of the total income of White workers are muted compared with that of Black workers, who do not earn much financial income. In terms of the consumption response, the average Black consumption responds more strongly to the monetary policy shock



(a) Unemployment Rate

(b) Unemployment Rate Gap

(c) Average Income

(d) Average Consumption

Figure 6: Hand-to-Mouth Heterogeneity and Monetary Transmission

than the White's because of the larger fraction of hand-to-mouth among Black workers.

In order to see the role of hand-to-mouth in shaping the heterogeneous effects of monetary policy, Figure 6 compares the impulse responses to the -25bp monetary policy shock in the baseline model (blue lines) and three alternative models in terms of hand-to-mouth heterogeneity. In the first alternative model (green lines), the model elements that generate poor hand-to-mouth workers — the discount factor shock and the heterogeneity in the interest rate premium — are turned off. Specifically, $\iota_s = 0, \forall s$ and the β is calibrated to clear the market of mutual fund shares as in the baseline model. As a result, the fraction of poor hand-to-mouth is less than 0.25% for all racial groups. In the second alternative model (pink lines), the wealthy hand-to-mouth shock is turned off. In the third alternative model (yellow lines), both poor and wealthy hand-to-mouth are turned off. Panels (a) and (b) compare the impulse response of the overall unemployment rate and the Black-White unemployment rate gap, respectively. Panel (c) compares the average income of Black and White workers in four models. Panel (d) compares the Black and White average consumption.

Panels (a) and (b) show that both the overall unemployment rate and the Black-White unemployment rate gap respond more strongly to the accommodative monetary policy shock in the presence of hand-to-mouth workers, especially poor hand-to-mouth workers. The baseline model with both poor and wealthy hand-to-mouth exhibits the strongest impulse responses, while, the alternative model in which both types of hand-to-mouth are shut off, the responses are the weakest among the four models. Between the two types of hand-to-mouth, the presence of poor hand-to-mouth matters more for the strength of the impulse responses. Overall, Figure 6(a) and (b) indicate that the presence of hand-to-mouth matters for monetary transmission. I come back to this issue in Section 8.

In terms of the average income of Black and White workers, the alternative models without either poor or wealthy hand-to-mouth indicate that the White average income responds less strongly to the monetary policy shock. As shown in Panel (a), these models exhibit a weaker stimulus effect, and thus a weaker response according to the Taylor rule. Therefore, the real interest rate in these models declines more than in the baseline model. Moreover, the increase in the White labor income is smaller. The combination of the two makes the response of the White average income weaker in these alternative models. In the alternative model without hand-to-mouth, the response of the White average income becomes negligible. In terms of the Black average income, the alternative models without poor hand-to-mouth exhibit a significantly weaker impulse response compared with the models with poor hand-to-mouth, including the baseline model. This is due to the difference in income composition. When there is no poor hand-to-mouth, Black and Hispanic workers hold significantly larger amount of assets in the model. Since financial income declines in response to the accommodative monetary policy shock, the decline in financial income strongly counteracts the increase in labor income, making the responses of total average income of racial minority significantly weaker.

Panel (d) shows that the response of the White average consumption to the accommodative monetary policy shock is weaker if either poor or wealthy hand-to-mouth is turned off. This result reflects the differences in the strength of the response of the White average income. As in the baseline model, the response of the White average consumption is stronger than that of income because of the asset price appreciation. As for the response of the Black average consumption, the alternative model without wealthy hand-to-mouth exhibits a stronger response than the baseline model, because Black workers hold more assets, and thus asset price appreciation has a stronger effect to consumption of Black workers. On the other hand, if poor hand-to-mouth is turned off, racial differences in the consumption response become close to non-existent.

7.3 Heterogeneous Welfare Effects of Monetary Policy

This section investigates welfare consequences of the −25bp monetary policy shock for different racial groups. Table 9 summarizes the results. Specifically, the table compares welfare effects between the baseline model and six alternative models which are already analyzed above. In the first three alternative models, either the heterogeneity in the job-finding rate or the heterogeneity in the separation rate is turned off. In the next three alternative models, either poor hand-to-mouth or wealthy hand-to-mouth is turned off. The top panel of Table 9 shows the maximum consumption response in percent to the monetary policy shock, which happens on impact in all models. The second panel shows the welfare effects of the accommodative monetary shock, measured in consumption equivalence variations (CEV).

Table 9: Heterogeneous Welfare Effects of Accommodative Monetary Shock

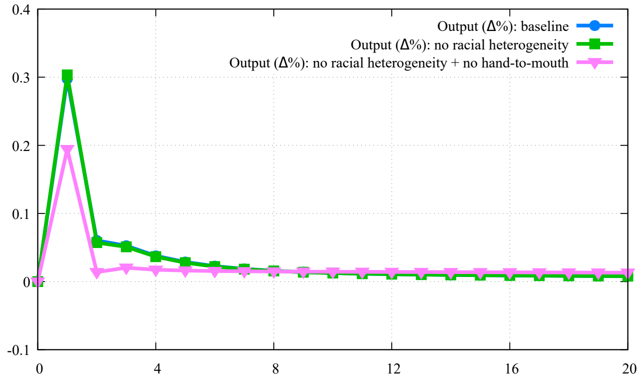
	Overall	White	Asian	Hispanic	Black
% Change in Consumption					
Baseline Model	0.160	0.121	0.146	0.367	0.361
Model with Same f_s	0.161	0.122	0.147	0.365	0.367
Model with Same λ_s	0.154	0.139	0.184	0.223	0.216
Model with Same f_s and λ_s	0.154	0.139	0.185	0.222	0.217
Model without Poor Hand-to-Mouth	0.093	0.086	0.084	0.117	0.127
Model without Wealthy Hand-to-Mouth	0.138	0.082	0.128	0.408	0.393
Model without Hand-to-Mouth	0.043	0.041	0.040	0.048	0.055
% Change in Welfare (in CEV)					
Baseline	0.025	0.020	0.018	0.030	0.033
Model with Same f_s	0.025	0.020	0.018	0.031	0.030
Model with Same λ_s	0.022	0.021	0.020	0.022	0.023
Model with Same f_s and λ_s	0.021	0.021	0.021	0.022	0.021
Model without Poor Hand-to-Mouth	0.024	0.023	0.022	0.027	0.030
Model without Wealthy Hand-to-Mouth	0.023	0.019	0.018	0.026	0.031
Model without Hand-to-Mouth	0.026	0.023	0.022	0.029	0.035

Note: In response to -25bp monetary policy shock. % Change in Consumption denotes the largest response of average consumption, which happens on impact. % Change in welfare is measured by consumption equivalence variations (CEV).

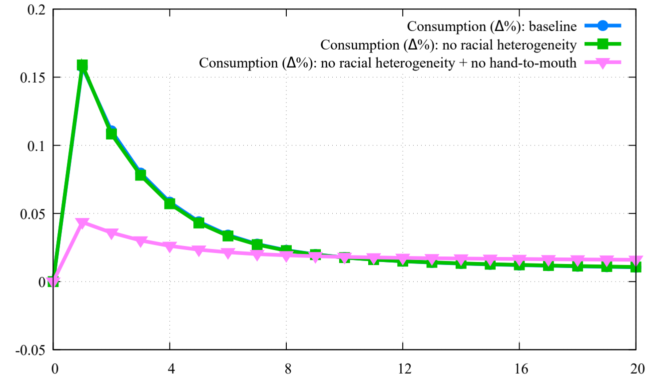
In the baseline model (first row in each panel), the overall average consumption rises by 0.16% right after a -25bp monetary policy shock, and workers on average gain equivalent of 0.025% of consumption every period. White and Asian workers enjoy smaller welfare gains. The average consumption of White and Asian workers increase by 0.12% and 0.15%, respectively, and their welfare gains are about 0.02% of flow consumption. On the other hand, Black and Hispanic workers gain more from an accommodative monetary policy shock. Average consumption of Black and Hispanic workers increase by 0.37% and 0.36%, respectively. Their welfare gains are 0.03% of flow consumption, which is 50% more than the welfare gains of White and Asian workers.

How important are racial differences in labor market risks? The second to fourth rows of each panel in Table 9 provide answers to this question. Let me make two remarks. First, the average consumption of Hispanic and Black workers respond less strongly (by about 1/3) to a rate cut if either the separation rate is assumed to be the same across all racial groups, while shutting down the heterogeneity in the job-finding rate does not change the consumption responses. This is consistent with the finding in Section 7.2 that different impulse responses to the monetary policy shock of Black and White weakens when the separation rate is assumed to be the same. Second, consequently, the larger welfare gains among Hispanic and Black workers in the baseline model all but disappear in the model with the same separation rate.

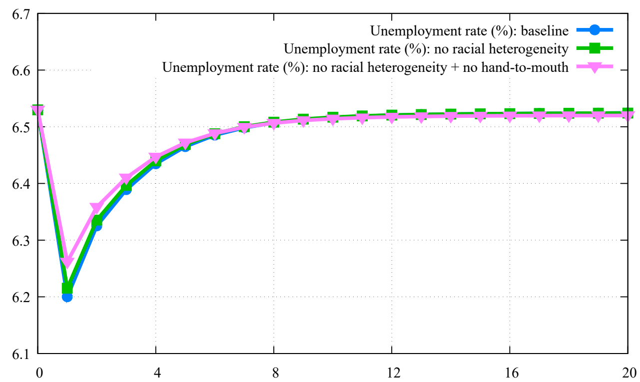
The last three alternative models (fifth to seventh rows) of each panel show how hand-to-mouth affects the racial differences in the consumption response and the welfare effects of



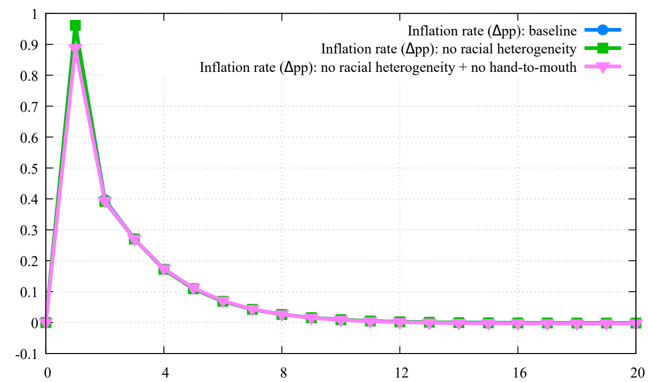
(a) Output



(b) Consumption



(c) Unemployment Rate



(d) Inflation Rate

Figure 7: Racial Inequality and Monetary Transmission

the accommodative monetary policy shock. The bottom three rows of the top panel show that the response of average consumption among Black and Hispanic workers becomes significantly muted in the models without poor hand-to-mouth. In particular, if both poor and wealthy hand-to-mouth are turned off, the consumption response becomes small and almost the same across four racial groups. However, surprisingly, differences in the consumption response do not translate into racial differences in the welfare effects. Welfare effects for four racial groups are largely unchanged and Black and Hispanic workers gain from the rate cut by about 50% more than White workers, even if either poor or wealthy hand-to-mouth is turned off. The differences in the welfare effects seem largely determined by the interactions between monetary policy and labor market risks.

8 Racial Inequality and Monetary Transmission

In this section, I investigate how macroeconomic effects of a monetary policy shock are affected by the presence of the racial heterogeneity in labor market risks and hand-to-mouth. Figure 7 shows how output (Panel (a)), consumption (Panel (b)), the unemployment rate (Panel (c)), and the inflation rate (Panel (d)) are affected by a -25bp monetary policy shock, in the baseline model economy (blue lines), as well as in the alternative model economy in which the racial heterogeneity in labor market risks and hand-to-mouth are shut down (green lines), and the alternative model economy in which hand-to-mouth is shut down, on top of shutting down racial heterogeneity (pink lines).

When the racial heterogeneity in labor market risks and hand-to-mouth are shut down, so that all workers face the same labor market risks (which are the average risks across all racial groups) and the fraction of poor and wealthy hand-to-mouth are not different across racial groups, the monetary transmission is not affected significantly. The lines in Figure 7 for this alternative model economy are almost on top of the lines for the baseline model. However, when both poor and wealthy hand-to-mouth are shut down, while keeping the common labor market risks, the monetary transmission changes significantly. The peak response of output to the accommodative monetary policy shock declines by one-third, from 0.30% to 0.19% (Panel (a)). This is because the response of consumption weakens significantly, when workers are not hand-to-mouth. Panel (b) shows that the response of aggregate consumption to the monetary policy shock declines from 0.16% to 0.04%. When aggregate consumption demand does not respond strongly to monetary accommodation, monetary transmission becomes significantly weakened. Since output doesn't need to increase to meet the aggregate demand as much as in the baseline model, the unemployment response also becomes weaker. The overall unemployment rate declines 0.27pp in the alternative model without hand-to-mouth, instead of 0.33pp in the baseline model. Since the response of the aggregate demand is weaker, the inflation rate in the model without hand-to-mouth (0.89pp) does not go up as much as in the baseline model (0.96pp).

In sum, the racial heterogeneity per se does not matter sizably for monetary transmission, but the existence of both poor and wealthy hand-to-mouth matters for the strength of the monetary transmission, since the consumption response of hand-to-mouth workers to a monetary policy shock is strong. This is consistent with Figure 6(d), which shows that consumption response of both White and Black workers becomes muted in the absence of hand-to-mouth. On the other hand, when the racial heterogeneity in labor market risks is turned off, Figure 5(d) shows that the Black-White difference in the consumption response gets smaller, but the aggregate consumption response does not change significantly, as the average consumption by White workers responds more, while the average consumption by Black workers responds less.

9 Racial Inequality over the Business Cycle

This section studies how different monetary policy rules affect different racial groups over the business cycle. Section 9.1 overviews aggregate dynamics. Section 9.2 looks at how business cycles are different across racial groups. Section 9.3 investigates heterogeneous effects of different monetary policy rules. Finally, Section 9.4 studies the effects of using the Black unemployment rate instead of the overall unemployment rate in the monetary policy rule.

9.1 Aggregate Business Cycle Dynamics

Table 10 compares business cycle statistics between the U.S. economy (1980:1-2019:4) and the baseline model economy. The volatility of output in the model is the same as in the data by

Table 10: Business Cycle Statistics: U.S. Data and Baseline Model

	U.S. Data			Baseline Model		
	S.D.(%)	Rel S.D.	Corr(Y)	S.D.(%)	Rel S.D.	Corr(Y)
Output	1.232	1.000	1.000	1.232	1.000	1.000
Consumption	0.951	0.772	0.863	1.028	0.834	0.958
Investment	6.029	4.892	0.900	1.595	1.295	0.922
Utilization	2.801	2.273	0.824	2.017	1.637	0.776
Real wage	0.596	0.484	–0.333	0.329	0.267	0.759
Inflation	0.319	0.259	0.314	1.038	0.842	0.185
UR (Overall)	10.756	8.729	–0.870	12.683	10.292	–0.490
UR (White)	11.246	9.126	–0.872	11.834	9.603	–0.494
UR (Hispanic)	11.825	9.596	–0.801	15.209	12.342	–0.487
UR (Black)	9.552	7.752	–0.773	13.002	10.552	–0.473

Note: All U.S. data are quarterly, from 1980:1 to 2019:4. Output is real GDP, consumption is real PCE, and investment is real gross private domestic investment, all of which are from the BEA. Inflation is headline PCE inflation rate from the BEA. Utilization is capacity utilization of all industries, from the FRB. Real wage is real median usual weekly earnings, from the BLS. UR is the unemployment rate, also from the BLS. All series of the data and the model are in log and detrended using the Hodrick-Prescott filter with the smoothing parameter of 1600.

construction.[19] The volatility of aggregate consumption expenditures and its correlation with output are slightly higher than the data. I could use a quadratic investment adjustment cost to adjust investment (and thus consumption) volatility, but it turns out that the consumption volatility in the baseline model is higher than in the data, without the investment adjustment cost. Investment in the baseline model is less much volatile (S.D. of 1.6%) than the data (6.0%). This is common when aggregate demand consist only of consumption and gross investment, and there is no inventory adjustments, government expenditures or imports and exports. The utilization rate in the model is slightly less volatile (2.0%) than in the data (2.8%) but both are strongly procyclical. Real wage in the baseline model is also slightly less volatile than in the data. However, while real wage is procyclical in the model (correlation with output of 0.76), it is countercyclical in the data (–0.33). This might be because I use real median usual weekly earnings as the measure of real wage, or because the nominal wage rigidity in the model is not as strong as in the data.[20] Inflation rate in the model (S.D. of 1.0%) is more volatile than the data (0.3%), although the model captures the weak procyclicality of the inflation rate in the data. Its correlation with output is 0.3 in the data and 0.2 in the baseline model.

The last four rows of Table 10 contain the overall unemployment rate, and the unemployment rate for White, Hispanic, and Black workers. As discussed in Section 2.1, the time series for the Asian unemployment rate is too short and thus excessively affected by two recent deep recessions. As for the overall unemployment rate, the volatility in the model (12.7%) is slightly

[19] See Section 4.1 for details.

[20] There is a downward pressure to median real wage in an expansionary period because workers with lower wages are more likely to find jobs in an expansion.

higher than the data (10.8%), but the model captures the fact that the unemployment rate is extremely volatile compared with output. Moreover, although the correlation between output and the unemployment rate in the model (–0.49) is weaker than in the data (–0.87), the model captures the countercyclicality. Notice that the cyclical properties of the unemployment rate are not directly targeted when the model is calibrated. Three key parameters that are important in generating the large volatility of the unemployment rate are the parameter that guarantees small profits for labor firms (ω_0), the parameter that yields real wage rigidity (ω_1), and the parameter that controls the elasticity of vacancy posting (α). ω_0 and ω_1 are calibrated to match the small profits of firms and the real wage elasticity (Hagedorn and Manovskii (2008)), while α is calibrated to match the empirical response of the Black-White unemployment rate gap to a monetary policy shock (Bartscher et al. (2021)). This approach guarantees that the unemployment rate is generally volatile, but the success that the volatility of the unemployment rate in the baseline model is close to the data is untargeted. Notice that, in the data, the unemployment rate volatility for all racial groups is about 10 times larger than output volatility. This is because the unemployment rate is logged, and thus the volatility is relative to the level of the unemployment rate. In the baseline model, the unemployment rate for all racial groups is also almost 10 times larger than output volatility.

9.2 Business Cycle and Racial Inequality

Table 11 compares cyclical properties of the unemployment rate, average income, and average consumption of four racial groups, plus output, in the baseline model, the model in which all racial groups face the same labor income risks, the model without wealthy or poor hand-to-mouth, and the model without racial heterogeneity in labor markets risks nor hand-to-mouth. There are four key takeaways. First, if there is no hand-to-mouth, output volatility declines, from 1.24% in the baseline model to 1.00% (18.8% decline). As shown in the last section, both poor and wealthy hand-to-mouth generates strong amplification of shocks. Second, if both the job-finding rate and the separation rate are set the same across racial groups, the average consumption volatility for Hispanic and Black workers are lower, but they are still higher than that of White and Asian workers, which confirms that the fraction of hand-to-mouth matters for shaping consumption volatility. The consumption volatility among Black workers declines from 1.57% to 1.14%, but it is still higher than that of White workers (0.99%), although the average income volatility is almost the same. Third, if there is no hand-to-mouth for minority racial groups, the average income volatility goes down, but average consumption volatility goes down significantly more. For Black workers, the volatility of their average income goes down from 1.51% to 0.98%, but their average consumption volatility drops from 1.57% to 0.81%. For Hispanic workers, the average income volatility decreases from 1.37% to 0.84%, but their average consumption volatility drops from 1.57% to 0.81%. Their average income volatility decreases because minority workers end up holding more assets, and enjoy higher financial income, which is more stable than labor income over the business cycle. On the other hand, their consumption volatility drops significantly because less of them are liquidity-constrained due to being either poor or wealthy hand-to-mouth. Finally, when both the racial heterogeneity in labor market risks and hand-to-mouth are turned off, all racial groups exhibit approximately the same consumption and income volatility, and the size of the decline of income and consumption volatility from the baseline model is significantly larger among

Table 11: Comparison of Business Cycle Statistics: Three Models

	(1) Baseline		(2) Same f_s & λ_s		(3) No H2M		(4) = (2)+(3)	
	S.D.%	Corr.	S.D.%	Corr.	S.D.%	Corr.	S.D.%	Corr.
Output	1.232	1.000	1.238	1.000	1.001	1.000	1.011	1.000
UR (Overall)	12.683	–0.490	12.203	–0.488	10.635	–0.464	10.313	–0.464
UR (White)	11.834	–0.494	12.203	–0.488	10.086	–0.470	10.313	–0.464
UR (Asian)	11.848	–0.497	12.203	–0.488	10.150	–0.474	10.313	–0.464
UR (Hispanic)	15.209	–0.487	12.203	–0.488	12.520	–0.458	10.313	–0.464
UR (Black)	13.002	–0.473	12.203	–0.488	10.649	–0.444	10.313	–0.464
Incm (Overall)	1.020	0.954	1.021	0.951	0.691	0.941	0.700	0.941
Incm (White)	0.972	0.914	1.037	0.921	0.642	0.929	0.705	0.941
Incm (Asian)	0.932	0.986	1.071	0.983	0.627	0.929	0.723	0.944
Incm (Hispanic)	1.356	0.958	1.042	0.971	0.842	0.972	0.673	0.937
Incm (Black)	1.512	0.953	1.043	0.974	0.976	0.938	0.677	0.938
Cons (Overall)	1.026	0.959	1.027	0.958	0.691	0.941	0.700	0.941
Cons (White)	0.920	0.950	0.989	0.953	0.673	0.941	0.702	0.941
Cons (Asian)	1.069	0.965	1.249	0.967	0.667	0.941	0.712	0.941
Cons (Hispanic)	1.571	0.979	1.150	0.971	0.729	0.941	0.686	0.940
Cons (Black)	1.571	0.963	1.140	0.970	0.805	0.938	0.687	0.941

Note: All series are in log and detrended using the Hodrick-Prescott filter with the smoothing parameter of 1600. The columns "S.D.(%)" contain the standard deviation in percent. The columns "Corr." contain the contemporaneous correlation with output.

Hispanic and Black workers.[21]

9.3 Monetary Policy Rule and Racial Inequality

Table 12 compares volatility of variables of interest under the baseline monetary policy rule with ϕ_y=0.125 and the accommodative monetary policy rule with ϕ_y=0.250. The columns labeled as "S.D.%" shows the standard deviation in percent of variables in the model with the accommodative monetary policy rule. The columns labeled as $\Delta\%$ show percentage change in volatility between the baseline monetary policy rule (shown in Table 11)) and accommodative monetary policy rule. The table shows the comparison of volatility for four model economies: the baseline model, the model without the racial heterogeneity in the job-finding rate and the separation rate, the model without poor or wealthy hand-to-mouth, and the model without the racial heterogeneity in labor market risks as well as hand-to-mouth.

Let me make five remarks. First, the size of the decline in economic volatility under the accommodative monetary policy rule is slightly smaller in the model without hand-to-mouth. This is because of lack of amplification of aggregate demand through hand-to-mouth workers. Second, in both the baseline model and the model without racial heterogeneity in labor

[21] Different racial groups still have different labor productivity η_s, which creates minor racial differences.

Table 12: Cyclical Properties: Baseline and Accommodative Monetary Policy Rule

	(1) Baseline		(2) Same f_s & λ_s		(3) No H2M		(4) = (2)+(3)	
	S.D.%	$\Delta\%$	S.D.%	$\Delta\%$	S.D.%	$\Delta\%$	S.D.%	$\Delta\%$
Output	1.143	–7.3	1.147	–7.3	0.943	–5.7	0.952	–5.8
UR (Overall)	11.276	–11.1	10.844	–11.1	9.580	–9.9	9.278	–10.0
UR (White)	10.530	–11.0	10.844	–11.1	9.082	–10.0	9.278	–10.0
UR (Asian)	10.539	–11.1	10.844	–11.1	9.134	–10.0	9.278	–10.0
UR (Hispanic)	13.487	–11.3	10.844	–11.1	11.271	–10.0	9.278	–10.0
UR (Black)	11.573	–11.0	10.844	–11.1	9.618	–9.7	9.278	–10.0
Incm (Overall)	0.945	–7.4	0.946	–7.3	0.657	–4.9	0.664	–5.1
Incm (White)	0.913	–6.0	0.969	–6.5	0.615	–4.3	0.669	–5.1
Incm (Asian)	0.855	–8.3	0.974	–9.0	0.602	–4.0	0.685	–5.3
Incm (Hispanic)	1.210	–10.7	0.936	–10.2	0.788	–6.4	0.641	–4.7
Incm (Black)	1.353	–10.5	0.936	–10.1	0.906	–7.1	0.645	–4.8
Cons (Overall)	0.953	–7.1	0.954	–7.1	0.657	–4.9	0.664	–5.1
Cons (White)	0.863	–6.2	0.924	–6.6	0.641	–4.8	0.667	–5.1
Cons (Asian)	0.991	–7.3	1.151	–7.9	0.636	–4.7	0.675	–5.2
Cons (Hispanic)	1.423	–9.4	1.050	–8.7	0.690	–5.3	0.652	–4.9
Cons (Black)	1.423	–9.5	1.044	–8.4	0.758	–5.8	0.653	–5.0

Note: "S.D.%" columns contain volatility in percent under the accommodative monetary policy rule (ϕ_y=0.250). All series are in log and detrended using the Hodrick-Prescott filter with the smoothing parameter of 1600. "$\Delta\%$" columns contain percentage change in volatility from the baseline monetary policy rule (shown in Table 11, with ϕ_y=0.125) and the accommodative monetary policy rule.

market risks, the size of the decline in consumption volatility is smaller than the size of decline in income volatility for all racial groups. This is because workers use saving to smooth consumption even under the baseline monetary policy rule, and thus there is less room for lowering consumption volatility when the monetary policy becomes more accommodative. Third, in both the baseline model and the model without racial heterogeneity in labor market risks, the volatility of the unemployment rate and that of the average income for four racial groups decline similarly, although levels of volatility are different across racial groups. On the other hand, the average consumption volatility for racial minorities declines less in the model without racial heterogeneity in labor market risks. Fourth, the changes in the average income volatility are different between the baseline model and the model with no hand-to-mouth workers. This is due to income composition. In the economy without hand-to-mouth, minority workers hold more assets, and thus a larger fraction of their income is from financial income, which is less sensitive to the monetary policy rule. Finally, in the model without hand-to-mouth, the average consumption volatility for racial minorities declines less than in the baseline model, and the changes in the volatility are similar across racial groups. There are two reasons. First, income is less volatile because of the difference in income composition I just discussed above. Second, workers in all racial minorities have more assets in the model without hand-to-mouth, and can use the assets to smooth consumption regardless of how accommodative monetary policy is. In other words, liquidity constraint is crucial in determining

Table 13: Monetary Policy Rule Based on Black Unemployment Rate

	(1) Baseline	(2) Overall UR $\phi_u = 0.035$	(3) Overall UR $\phi_u = 0.063$	(4) Black UR $\phi_b = 0.035$	(5) Black UR $\phi_b = 0.019$
Output	1.232	1.232	1.161	1.161	1.232
Consumption	1.028	1.028	0.971	0.971	1.028
Unemployment rate	12.683	12.649	11.500	11.480	12.636
Inflation rate	1.038	1.057	0.992	0.981	1.005
White UR	11.834	11.804	10.745	10.730	11.796
Asian UR	11.848	11.820	10.755	10.741	11.811
Hispanic UR	15.209	15.160	13.746	13.717	15.143
Black UR	13.002	12.962	11.790	11.762	12.946
White Cons	0.920	0.920	0.875	0.875	0.920
Asian Cons	1.069	1.069	1.007	1.007	1.069
Hispanic Cons	1.571	1.571	1.454	1.453	1.570
Black Cons	1.571	1.571	1.453	1.452	1.570

Note: All numbers are standard deviations in percent and in log, and detrended using the Hodrick-Prescott filter with the smoothing parameter of 1600. First column shows the standard deviations in the baseline model. The 2nd and the 3rd columns show the results of the model in which the Taylor rule includes the overall unemployment rate (with the coefficient ϕ_u) instead of output. The 4th and the 5th columns show the results of the model in which the Taylor rule includes the Black unemployment rate (with the coefficient ϕ_b) instead of output.

the consumption dynamics of racial minorities over the business cycle.

9.4 Monetary Policy Rule with Black Unemployment Rate

In this section, I conduct hypothetical experiments in which the monetary policy is based on the *Black* unemployment rate instead of the *overall* unemployment rate. What I will show is that, since the unemployment rate moves in parallel across different racial groups (see Section 2.1), the monetary policy rule based on the Black unemployment rate works virtually in the same way as the monetary policy rule with the overall unemployment rate, with a larger Taylor rule coefficient attached to the overall unemployment rate. Table 13 summarizes the results. Column (1) represents the baseline model economy. In Column (2), I first change the (logged) Taylor rule to the following:

$$\log R_t = (1-\rho_R)\log \overline{R} + \rho_R \log R_{t-1} + (1-\rho_R)[\phi_\pi(\log \pi_t - \log \overline{\pi}) + \phi_u(u_t - \overline{u})] + \log z_t^{MP} \tag{58}$$

where u_t is the overall unemployment rate in period t, and $\overline{u}$ is the steady-state (or target) unemployment rate. Basically, I replaced the output in the original Taylor rule with the overall unemployment rate. Then I calibrate the Taylor rule coefficient for the unemployment rate, ϕ_u, such that the model produces the same output volatility as in the baseline model (see the output volatility in Columns (1) and (2)). As a result, the model with ϕ_u=0.035 (shown in Column (2)) behaves virtually in the identical manner as the baseline model.

Now, what happens if the same Taylor rule coefficient is maintained but the Black unemployment rate is used instead of the overall unemployment rate? The results are shown in Column (4), with the Taylor rule coefficient attached to the Black unemployment rate is set at ϕ_b=0.035. In the model shown in Column (4), output volatility declines by 5.8%, to 1.16%. And volatility of all the other variables declines similarly. In other words, using the Black unemployment rate as the monetary policy target instead of the overall unemployment rate, but keeping the monetary policy response coefficient the same, is equivalent to making the monetary policy rule more accommodative.

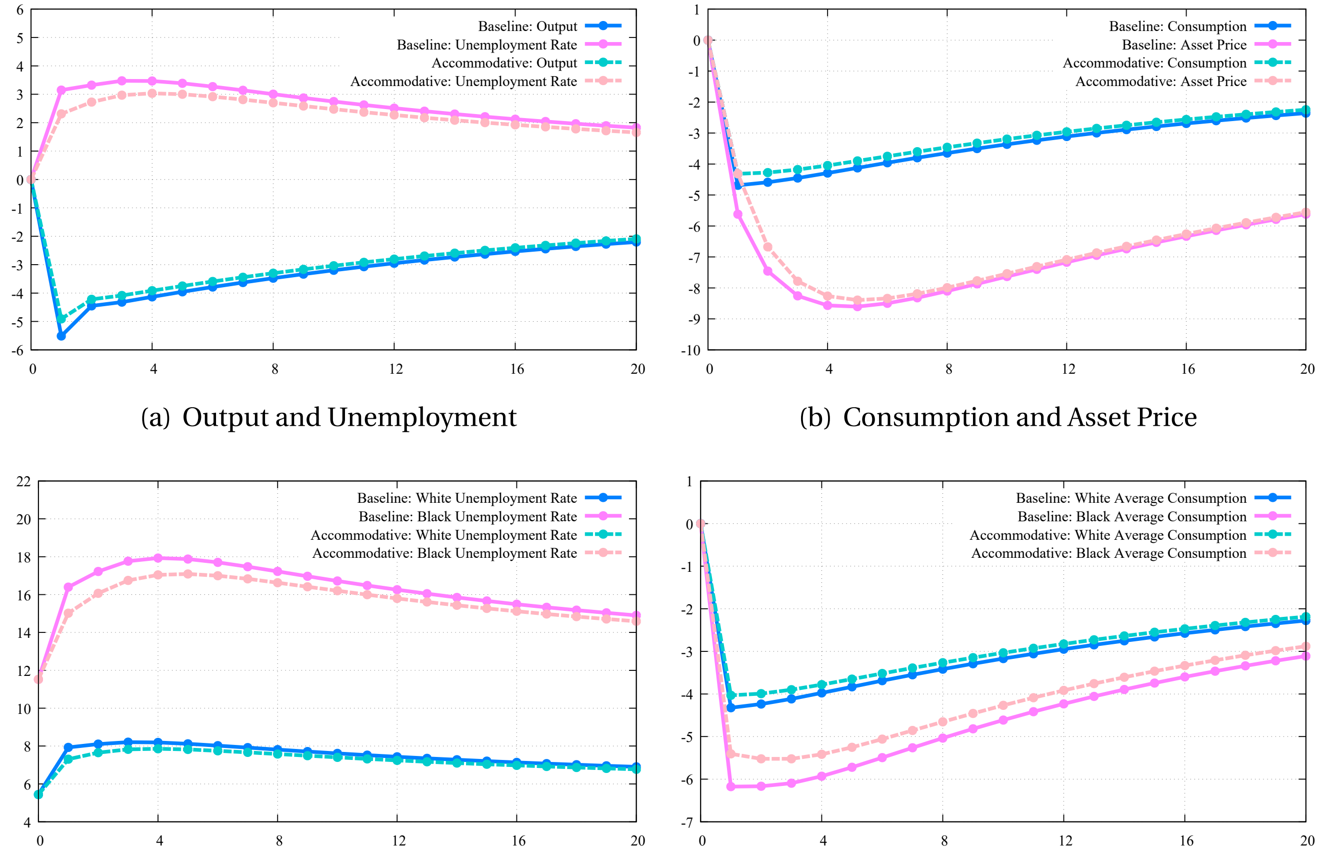
In order to make this point clearer, I conduct two additional experiments, shown in Column (3) and Column (5). In Column (5), I still use the Black unemployment rate as the policy target, but re-calibrate ϕ_b such that output volatility is back to the baseline level of 1.23. It turns out that the model with the Black unemployment rate as the monetary policy target behaves in the almost identical manner to the baseline model and the model with ϕ_u=0.035 when ϕ_b is set at 0.019. Alternatively, in Column (3), I go back to the model in which the overall unemployment rate as the monetary policy target, and re-adjust ϕ_u such that output volatility is the same as the model with ϕ_b=0.035 (Column (4)). I found that, if ϕ_u is set at 0.063, the model with the overall unemployment rate as a policy target behaves in the identical manner as the model with ϕ_b=0.035. Simply put, using the Black unemployment rate as a monetary policy target is equivalent to the case in which the monetary policy response to the overall unemployment rate is 80% (=0.063/0.035-1) stronger. This 80% number is consistent the ratio of the average Black unemployment rate to the average overall unemployment rate, which is 1.87. In other words, if the monetary authority wants to target the Black unemployment rate, that can be achieved just by raising the Taylor rule coefficient attached to the overall unemployment rate by 80%, instead of changing the Taylor rule in a more drastic manner.

10 Recession, Monetary Policy Rule, and Racial Inequality

This section investigates the role of monetary policy in affecting different racial groups in the face of a large recession. For transparency, I assume that a large unexpected negative shock to TFP causes a recession, and the size of the TFP shock is calibrated such that the overall unemployment rate peaked at 10% in the baseline model, which is what happened to the unemployment rate during the Great Recession. This approach yields the TFP shock of –2.9%. I will compare the results of the baseline model during this “Great Recession” and those of an alternative model in which the Taylor response parameter to output is twice as large (ϕ_y=0.250) as the baseline (ϕ_y=0.125). I call this monetary policy regime as accommodative or dovish monetary policy rule. The main goal of this section is to unveil how workers of different races are differently affected by these monetary policy rules during the Great Recession in the model.

10.1 Aggregate Dynamics and Monetary Policy Rule

Figure 8 shows aggregate and racial dynamics of the baseline model economy upon a large (-2.9%) negative TFP shock (the Great Recession shock). Each panel shows both the dynamics of the baseline model with ϕ_y=0.125 and the dynamics of the model in which the monetary authority follows a more accommodative (ϕ_y=0.250) monetary policy rule. According to the



(a) Output and Unemployment

(b) Consumption and Asset Price

(c) Unemployment Rate: White and Black

(d) Average Consumption: White and Black

Figure 8: Alternative Monetary Policy Rules during Recession

model with the baseline monetary policy rule (shown in solid lines in Figure 8), output declines by 5.5%, and aggregate consumption drops by 4.7% upon impact (Panels (a) and (b)). The asset price drops significantly, by up to 8.6% (Panel (b)). The overall unemployment rate gradually goes up, to reach 10.0% in the fourth quarter after the shock (Panel (a)).

Not surprisingly, under the more accommodative monetary policy rule (shown in broken lines), the economy is stimulated by stronger response of the monetary policy and the severity of the recession caused by a large negative TFP shock is mitigated. Output declines by 4.9% instead of 5.5% on impact and consumption declines by 4.3% instead of 4.7%. Asset price drops by 8.4% instead of 8.6%. The size of the decline in the asset price does not change significantly since the negative productivity shock is longer lasting than the effect of monetary accommodation. The overall unemployment rate rises, but less so, and reaches 9.6% at its peak, a 0.4pp lower than in the baseline.

10.2 Racial Heterogeneity during Recession

Bottom panels of Figure 8 show how Black and White workers are affected by the recession under different monetary policy rules, in terms of the unemployment rate (Panel (c)) and av-

Table 14: Welfare Loss from Recession under Alternative Monetary Policy Rules

	Overall	White	Asian	Hispanic	Black
(1)Baseline Model					
Baseline Monetary Policy	–2.42	–2.22	–1.85	–2.55	–2.91
Accommodative Monetary Policy	–2.25	–2.09	–1.72	–2.34	–2.67
Difference (pp)	–0.17	–0.13	–0.13	–0.21	–0.24
(2) Model with the Same Job-Finding and Separation Rate					
Baseline Monetary Policy	–2.22	–2.39	–2.18	–2.03	–2.02
Accommodative Monetary Policy	–2.06	–2.25	–2.01	–1.86	–1.85
Difference (pp)	–0.16	–0.14	–0.17	–0.17	–0.17
(3) Model with No Hand-to-Mouth					
Baseline Monetary Policy	–2.42	–2.22	–2.14	–2.60	–3.14
Accommodative Monetary Policy	–2.25	–2.06	–2.00	–2.41	–2.90
Difference (pp)	–0.17	–0.16	–0.14	–0.19	–0.24
(4) = (2)+(3)					
Baseline Monetary Policy	–2.36	–2.37	–2.39	–2.33	–2.33
Accommodative Monetary Policy	–2.19	–2.20	–2.22	–2.17	–2.17
Difference (pp)	–0.17	–0.17	–0.17	–0.16	–0.16

Note: A 2.9% negative TFP shock is assumed to cause a recession with a similar magnitude as the Great Recession. The welfare numbers are in percentage consumption equivalence variation (CEV in %), relative to the steady-state welfare.

erage consumption (Panel (d)). There are two takeaways. First, more accommodative monetary policy rule not only makes the overall unemployment rate rise less in a recession, but make the Black unemployment rate rise less relative to the White one (Panel (c)). When the overall unemployment rate hits its peak at 10.0% under the baseline monetary policy rule, the Black unemployment rate is 17.8%, and White unemployment rate is 8.2%. Under the more accommodative monetary policy rule, the overall unemployment rate peaks at 9.6%, 0.4pp lower than under the baseline monetary policy rule. Under the same policy rule, the Black unemployment rate peaks at 17.0%, or 0.8pp lower, while the white unemployment rate peaks at 7.9%, which is 0.3pp lower. In other words, the Black-White unemployment rate gap expands less under the accommodative monetary policy rule. Under the baseline monetary policy rule, the unemployment rate gap rises by 3.5pp, while it rise only by 3.1pp under the more accommodative monetary policy rule. Second, both White and Black workers benefit from the more accommodative monetary policy during a recession as their average income and average consumption decline less, but the differences are larger for Black workers. Average consumption of Black workers declines by 5.4% under the accommodative monetary policy rule, which is 0.8pp smaller than under the baseline monetary policy rule (6.2%). Meanwhile, average consumption of White workers drops by 4.3% in the baseline model and by 4.0% under the accommodative monetary policy rule, a smaller (0.3pp) decline compared with Black workers.

10.3 Welfare Implications

Finally, this section studies the diverse welfare effects of an accommodative monetary policy rule for different racial groups. Table 14 summarizes the results. The first three rows (Model (1)) compare the welfare effects, measured in percentage change in consumption equivalent variations compared with the steady-state welfare, of a recession caused by 2.9% drop in TFP (the "Great Recession" shock), in the baseline model with ϕ_y=0.125 and in an alternative model with ϕ_y=0.250. The third row shows the percentage point difference under the two monetary policy rules. The remainder of the table is for understanding what affects the heterogeneous welfare effects. The next three rows (Model (2)) show the same for the model in which the job-finding rate and the separation rate are set the same for all racial groups. The proportion of hand-to-mouth for each racial group is the same as in the baseline model. The next three rows (Model (3)) show the same for a model without poor or wealthy hand-to-mouth. Differences in labor market risks across racial groups remain the same. The last three rows (Model (4)) combine the features of Model (2) and (3).

The first row of Table 14 shows that, although all racial groups suffer from the Great Recession shock, Black and Hispanic workers suffer more. White workers on average suffer 2.2% equivalent of flow consumption, while Hispanic (2.6%) and Black (2.9%) workers suffer more. Asian workers suffer less than White workers (1.9%), because Asian and White workers are facing similar labor market risks, while White workers suffer more than Asian workers from the large decline in the asset price. Next, comparison between the first and the second row (which is shown in the third row) indicates that all racial groups benefit from the more accommodative monetary policy rule mitigating the negative effects of the Great Recession shock, but the mitigation is greater for minority groups. Under the more accommodative monetary policy (second row), Black workers' welfare loss from the Great Recession shock shrinks by 0.24pp and Hispanic workers' welfare loss declines by 0.21pp, while White workers' welfare loss shrinks less, by 0.13pp.

Next, let's compare how different racial groups are affected by the Great Recession shock in Models (2)-(4). Somewhat counterintuitively, when heterogeneity in labor market risks is shut down (Model (2)), White workers suffer more (−2.4%) than Black (−2.0%) and Hispanic (−2.0%) workers from the Great Recession shock, under the baseline monetary policy, even though the latter are more likely to be hand-to-mouth. This is because of the large decline in the asset price. White workers (and to a lesser extent, Asian workers) hold significantly more wealth, and thus suffer more from a large asset price decline. This effect overwhelms the effect of hand-to-mouth making the Great Recession more painful for Hispanic and Black workers. On the other hand, when hand-to-mouth is shut down (Model (3)), Hispanic (−2.6%) and Black (−3.1%) workers suffer more from the Great Recession shock than White workers (−2.2%) and Asian workers (−2.1%). This confirms that higher labor market risks that Hispanic and Black workers face make them suffer more in a large recession. If the welfare loss under the baseline monetary policy in Model (3) is compared with the welfare loss in Model (4), it is easy to see that the racial heterogeneity in labor market risks make Black and Hispanic workers suffer more during the Great Recession. By comparing the welfare loss from the Great Recession shock under the baseline monetary policy in Model (2) and Model (4), one can see that the drop in the asset price creates a larger welfare loss than the hand-to-mouth for Hispanic and

Black workers.

Finally, if we compare the welfare effects for four racial groups under the two monetary policy rules, the larger decline in welfare loss for Hispanic and Black workers under the more accommodative monetary policy in the baseline model (Model (1)) is due to the larger labor market risks, and not the higher fraction of hand-to-mouth. The former can be seen by comparing Model (1) and Model (2). Without the racial heterogeneity in labor market risks, the welfare effect of having the more accommodative monetary policy shrink for Hispanic (from 0.21pp to 0.17pp) and Black (from 0.24pp to 0.17pp) workers. The latter can be seen by comparing Model (1) and Model (3). When hand-to-mouth is shut down, the welfare effects of having the more accommodative monetary policy during the Great Recession do not change sizably for Hispanic and Black workers.

11 Conclusion

I build a heterogeneous-agent New-Keynesian (HANK) model with racial inequality in terms of labor market characteristics and wealth, and studies how monetary policy affects workers of difference races differently. Series of experiments highlight that the combination of higher labor market risks and a higher proportion of hand-to-mouth among Black and Hispanic workers is the key in shaping their stronger consumption response to monetary policy changes and larger welfare gains from accommodative monetary policy. I also find that using the Black unemployment rate instead of the overall as a policy target in the monetary policy rule is equivalent to making the policy rule more accommodative, because unemployment rates for all racial groups move in parallel over the business cycles.

Going forward, I consider this paper as the first step to understand how different racial groups are affected differently by monetary policy, and there are many other dimensions of racial differences that can be investigated in future research. Let me list five. First, I do not consider the labor force participation decision here in the model, but as shown in Section 2.1, there is a persistent differences in the labor force participation rate across racial groups. Second, introducing multiple assets, in particular housing, into the model is an interesting avenue to proceed, as housing is the most important assets for majority of households, and there is a large racial difference in terms of the homeownership rate. Third, an interesting dimension of racial differences that is not in the current paper is the difference in consumption basket, and thus difference in the average inflation rates that different racial groups face. This is emphasized by Lee et al. (2021). Fourth, racial inequality in terms of access to credit is also considered an important issue. This could be an important extension as access to credit, which is abstracted from the current paper, affects the ability to smooth consumption over time. Finally, it is known that there are more singles and single parents among Black households, which could weaken their ability to absorb shocks to income. This could also be an important channel that affects the racial differences in terms of the efficacy of monetary policy.

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Appendix

A Additional Facts about Racial Inequality

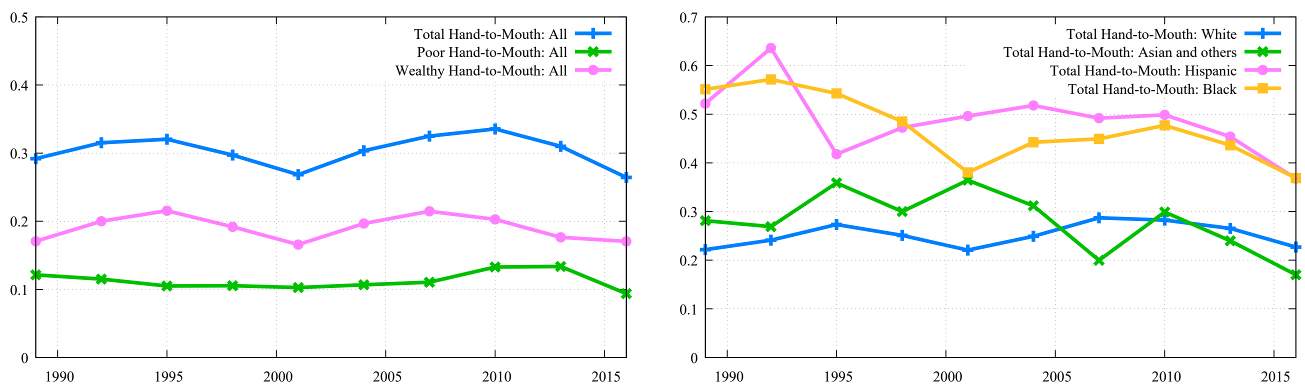
Table A.1: U.S. Wealth Distribution for Four Racial Groups: Alternative Measures

	Overall	White	Asian	Hispanic	Black
Alternative Measures of Hand-to-Mouth					
% with Non-Positive Net Worth	10.8	7.9	10.8	18.7	21.8
% with Non-Positive Total Wealth	10.8	7.4	11.0	25.2	20.3
% with Total Wealth $\leq$ 1-Week Earnings	15.7	11.1	16.1	33.9	28.5
Alternative Measures of Wealth Holding					
Mean Net Worth	529,315	656,489	527,478	143,386	115,538
Relative to White	80.6	100.0	80.3	21.8	17.6
Median Net Worth	110,527	159,885	101,924	18,824	20,687
Relative to White	69.1	100.0	63.7	11.8	12.9
Home Ownership Rate	66.5	73.5	56.8	45.3	46.1
Vehicle Ownership Rate	86.7	91.0	83.2	79.1	69.2

Note: The source is the the Survey of Consumer Finances (SCF). I use the average of 1989 to 2016 waves (10 waves, since the SCF is available every three years). I use the Extract Public dataset. Following Kaplan et al. (2014), households whose head is between 22 and 79 years old, and their non-financial income is strictly positive, are included. Since the SCF over-samples wealthier households, I use the sample household weights provided by the SCF. With the Extract Public dataset, Asians are bunched together with all the other (other than White, Hispanic, or Black) racial groups. Dollar amounts are shown in 2010 dollars.

The upper block of Table A.1 contains the fractions of hand-to-mouth based on alternative definitions. In the first line, I define hand-to-mouth as households whose net worth is zero or negative. Net worth is a similar but more comprehensive measure than the total wealth which I employ in Section 2. On top of all the items included in the total wealth, net worth includes other managed financial assets (annuities and trusts), other misc financial assets, net equity of vehicles (value of vehicles minus the outstanding value of car loans), value of businesses, other misc non-financial assets, net of education loans and other installment loans, and other debt. Overall, 10.8% of households have zero or negative net worth position. Among the White, the fraction is lower, at 7.9%. The fraction among Asians is exactly the same as the overall fraction (10.8%). Among Hispanic (18.7%) and Black (21.8%) households, more households have zero or negative net worth position. If I use zero or negative total wealth position to define hand-to-mouth, the fractions are similar to the previous case. Overall, again, 10.8% of households are hand-to-mouth, by holding zero or negative total wealth position. The fraction based on total wealth position is higher Hispanics (25.2% compared with 18.7%), but similar for other racial groups. If I define hand-to-mouth as total wealth less than half of non-financial income per pay period (2 weeks), the fraction of hand-to-mouth is obviously higher. Overall, 15.7% of households are hand-to-mouth, compared with 10.8% when zero is used as the threshold. Not surprisingly, all racial groups exhibit a higher fraction of hand-to-mouth. The fraction for White, Asian, Hispanic, and Black households are 11.1%, 16.1%, 33.9%, and 28.5%, respectively.

The lower block of Table A.1 contains alternative measures of wealth holding. Both mean and median net worth holding for four racial groups are as unequally distributed as total wealth. The last two rows show the fraction of households with housing, and that with vehicles. In general, minority groups exhibit a lower homeownership rate, which shows up as smaller illiquid wealth holding as well as smaller total wealth holding for minority groups. The homeownership rate for White households is 73.5%, while the homeownership rate for Asian, Hispanic, and Black households are 56.8%, 45.3%, and 46.1%, respectively. Vehicle ownership is also higher among White households compared with minority households. For White households, the vehicle ownership rate is 91.0%, while it is 83.2% for Asian households, 79.1% for Hispanic households, and 69.2% for Black households.



(a) Hand-to-Mouth: Overall

(b) Total Hand-to-Mouth: Four Racial Groups

Figure A.1: Proportion of Hand-to-Mouth: 1989-2016

While I show the average of the 10 waves of the SCF in Tables 3 and A.1, Figure A.1 shows the time series of the overall fraction of total, poor, and wealthy hand-to-mouth (Panel (a)) and the fraction of total hand-to-mouth for four racial groups (Panel (b)), both from 1989 to 2016. Panel (a) confirms the finding of Kaplan et al. (2014); the fraction of hand-to-mouth households remained stable throughout the period covered by the SCF. As for the fraction of hand-to-mouth households for each racial group, the fraction remained stable, or slightly increasing among White households. On the other hand, for the three minority groups (Hispanic, Black, and Asians and others), there is no discernible trend, but the fraction of total hand-to-mouth is lower in the recent years compared with earlier years.

B Note on the Definition of Hand-to-Mouth

This Appendix summarizes the definition of poor and wealthy hand-to-mouth, following Kaplan et al. (2014). According to their definition, a household is poor hand-to-mouth if one of the following two holds:

$$a \leq 0 \quad \text{and} \quad 0 \leq m \leq \frac{y}{2} \tag{A.1}$$

$$a \leq 0 \quad \text{and} \quad m < 0 \quad \text{and} \quad m \leq -\underline{m} + \frac{y}{2} \tag{A.2}$$

The first case is when a household has a positive liquid asset position. a and m are household's illiquid and liquid wealth holding, respectively, and y is household's income in pay period. y is divided by half because y could be received anytime during the period. $a < 0$ rarely happens in the data. It happens only if a house price decline makes the home equity negative. The second case is when a household has a negative liquid asset position. Then a household is assumed to be able to borrow up to $-\underline{m}$. If the liquid asset position is less than the borrowing limit plus half of income in pay period, the household is considered liquidity constrained as the household is too close to the borrowing limit.

Similarly, a household is a wealthy hand-to-mouth if one of the following two holds:

$$a > 0 \quad \text{and} \quad 0 \leq m \leq \frac{y}{2} \tag{A.3}$$

$$a > 0 \quad \text{and} \quad m < 0 \quad \text{and} \quad m \leq -\underline{m} + \frac{y}{2} \tag{A.4}$$

Total hand-to-mouth is the sum of poor hand-to-mouth and wealthy hand-to-mouth. Kaplan et al. (2014) set y to be two-week of earnings, based on the pay frequency in CEX from 1990 to 2010. According to their calculation, during the period, 32% of respondents are paid weekly, 52% are paid bi-weekly, and the rest are paid at a lower frequency. In terms of $\underline{m}$, Kaplan et al. (2014) set the borrowing limit as one-month equivalent of non-financial income as their baseline case. They also try alternative case with one-year equivalent of non-financial income and self-reported borrowing limit in the Survey of Consumer Finances (SCF). According to their calculation (Table 3 of their paper), between 1989 and 2010 in SCF, 31.2% of households are hand-to-mouth. Among those, about 1/3 (12.1% of total) are poor hand-to-mouth, and 2/3 (19.2% of total) are wealthy hand-to-mouth.

In the model constructed in the paper, since there is no liquid debt, only the first condition for both poor and wealthy hand-to-mouth is used. In order to be consistent with the definition of poor hand-to-mouth of Kaplan et al. (2014), I set the second grid (first grid represents zero assets) to be equal to two-week equivalent of earnings. By doing it, the threshold on average between the first grid (zero assets) and the second grid is half of two-week equivalent of earnings. In other words, the first grid captures those with equal or less than half of two-week equivalent of earnings on average, which is consistent with the definition of poor hand-to-mouth in Kaplan et al. (2014). When I calculate the fraction of hand-to-mouth for each racial group, shown in Table 3, I use the same definition.

As for the wealthy hand-to-mouth, I assume that there is an i.i.d. shock with probability π_h^s. With probability $1 - \pi_h^s$, a worker of type-s is not hit by the wealthy hand-to-mouth shock, and their liquidity constraint is the standard one ($a_{t+1} \geq 0$). With probability π_h^s, a worker is hit by the wealthy hand-to-mouth shock, and the liquidity constraint becomes $a_{t+1} \geq (1 - \delta_h)a_t$. δ_h is calibrated to be median credit card limit divided by median total wealth. In the SCF, credit card limit for each household is defined as equivalent to one-month of earnings, following Kaplan et al. (2014). I assume that δ_h is common across all racial groups. The i.i.d. probability of wealthy hand-to-mouth shock, π_h^s, is calibrated such that the proportion of wealthy hand-to-mouth ($a > 0$ and hit by the shock) is equal to the data for each type-s. The fraction of wealthy hand-to-mouth for each racial group is reported in Table 3.

The assumption that households with positive amount of illiquid asset can use the value of illiquid asset up to the amount of median credit card limit can be considered tight, as households probably could use the value of illiquid asset as well in case they need more liquidity. However, notice that, even with the liquidity constraint for wealthy hand-to-mouth which can be considered tight, the aggregate MPC implied by the model is at the lower end of available estimates.

C Equations Characterizing the Equilibrium

In this appendix, I organize the equations characterizing the equilibrium of the model so that the model can be solved with the first-order (linear) perturbation method developed by Schmitt-Grohé and Uribe (2009). In particular, we need to organize the equations characterizing the solution of the model in the following manner:

$$\mathbb{E}_t \mathbf{f}(\mathbf{x}_t, \mathbf{x}_{t+1}, \mathbf{y}_t, \mathbf{y}_{t+1}) = 0 \tag{A.5}$$

where $\mathbf{x}_t$ is a size-n_x vector of state variables in period t, meaning that $\mathbf{x}_t$ are predetermined at the beginning of period t. $\mathbf{y}_t$ is a size-n_y vector of control variables, which are not determined at the beginning of period t, but determined before period $t+1$. Denote $n = n_x + n_y$. $\mathbf{f}$ is a function that characterizes the equilibrium, and has to be a function which takes $2n$ variables ($\mathbf{x}_t$, $\mathbf{x}_{t+1}$, $\mathbf{y}_t$, and $\mathbf{y}_{t+1}$) and maps into n conditions.

What should be $\mathbf{x}_t$ and $\mathbf{y}_t$ in the model developed in this paper? Let's start with $\mathbf{x}_t$. First, shocks z_t^{TFP}, z_t^{MEI}, and z_t^{MP} are included. Second, other variables predetermined at the beginning of period t are k_t, i_{t-1}, and R_{t-1}. Finally, type distribution of heterogeneous workers m_t is a part of $\mathbf{x}_t$. How do we store the type distribution? I use the simplest method and store the distribution of wealth holding by n_a-grid histograms. This is also used by the bare-bone version of the algorithm proposed by Reiter (2009). A type distribution can be stored by a vector of length $n_s \times n_b \times n_p \times n_e \times n_a$. Notice that, since the wealthy hand-to-mouth shock, h, is i.i.d., there is no need to keep track of the type distribution in terms of h, allowing to reduce the dimension of the type distribution. Moreover, the probability measure at one of the asset grids (I use the lowest grid point) for each of type-s is not necessary since this can be backed up using the measure of type-s workers (which is fixed). In the end, $\mathbf{x}_t$ is a vector of length $n_x = 6 + n_s \times n_b \times n_p \times n_e \times n_a - n_s$.

Let's move on to $\mathbf{y}_t$. Aggregate variables that are not predetermined are the following $16+3\times n_s$: y_t, c_t, i_t, ℓ_t, n_t, p_t^a, d_t, mc_t, τ_t, δ_t, x_t, w_t, r_t, r_t^k, p_t^i, π_t, $u_{s,t}$, $v_{s,t}$, and $f_{s,t}$. Moreover, the optimal consumption function by heterogeneous workers and the value of labor firms are a part of $\mathbf{y}_t$. Using the same grids as those for storing the distribution of assets, the optimal consumption function can be stored by $n_s \times n_b \times n_p \times n_e \times n_a \times n_h$ points. Notice that we need to store the optimal decision rule for each realization of the wealthy hand-to-mouth shock h. The value of labor firms can be stored by $n_s \times n_p$ points. In sum, $\mathbf{y}_t$ is a vector of length $n_y = 16 + 3 \times n_s + n_s \times n_b \times n_p \times n_e \times n_a \times n_h + n_s \times n_p$.

The $n = n_x + n_y = 22 + 3 \times n_s + n_s \times n_b \times n_p \times n_e \times n_a - n_s + n_s \times n_b \times n_p \times n_e \times n_a \times n_h + n_s \times n_p$ equations included in $\mathbf{f}(.)$ are as follows:

$$\log z_{t+1}^{TFP} = \rho_{TFP} \log z_t^{TFP} + \epsilon_{t+1}^{TFP} \tag{A.6}$$

$$\log z_{t+1}^{MEI} = \rho_{MEI} \log z_t^{MEI} + \epsilon_{t+1}^{MEI} \tag{A.7}$$
$$\log z_{t+1}^{MP} = \rho_{MP} \log z_t^{MP} + \epsilon_{t+1}^{MP} \tag{A.8}$$
$$k_{t+1} = (1-\delta_t)k_t + i_t z_t^{MEI} \tag{A.9}$$
$$\ell_t = \int \mathbb{1}_{e=1} p\eta_s \, d\, m_{t+1} \tag{A.10}$$
$$i_{(t-1)+1} = i_t \tag{A.11}$$

$$\begin{aligned} \log R_t = (1-\rho_R)\log \overline{R} + \rho_R \log R_{t-1} \\ + (1-\rho_R)[\phi_\pi(\log \pi_t - \log \overline{\pi}) + \phi_y(\log y_t - \log \overline{y})] + \log z_t^{MP} \end{aligned} \tag{A.12}$$

$$\tau_t \int \mathbb{1}_{e=1} w_t p\eta_s \, d\, m_{t+1} = \int \mathbb{1}_{e=0} \min(\phi_0 w_t p\eta_s, \phi_1 \overline{wp\eta}_s) \, d\, m_{t+1} \tag{A.13}$$
$$y_t = c_t + i_t + z_t^{MEI} i_t \frac{\psi_i}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 + \sum_s \kappa_s v_{s,t} + \frac{\psi_1}{2}(\pi_t - \overline{\pi})^2 y_t + \psi_0 \tag{A.14}$$
$$\overline{a} = \int a \, d\, m_{t+1} \tag{A.15}$$
$$c_t = d_t + w_t \ell_t \tag{A.16}$$
$$w_t = \omega_0 \overline{x} + \omega_1(\log x_t - \log \overline{x}) + \omega_2(\log \pi_t - \log \overline{\pi}) \tag{A.17}$$
$$z_t^{MEI} p_t^i = 1 + \frac{z_t^{MEI}\psi_i}{2}\left[3\frac{i_t^2}{i_{t-1}^2} - 4\frac{i_t}{i_{t-1}} + 1\right] - \mathbb{E}_t \frac{p_t^a}{p_{t+1}^a + d_{t+1}} z_{t+1}^{MEI} \psi_i \frac{i_{t+1}^2}{i_t^2}\left[\frac{i_{t+1}}{i_t} - 1\right] \tag{A.18}$$
$$y_t = z_t^{TFP}(k_t n_t)^\theta \ell_t^{1-\theta} \tag{A.19}$$
$$r_t^k = mc_t z_t^{TFP} \theta (k_t n_t)^{\theta-1} \ell_t^{1-\theta} \tag{A.20}$$
$$x_t = mc_t z_t^{TFP}(1-\theta)(k_t n_t)^\theta \ell_t^{-\theta} \tag{A.21}$$
$$[y_t - \psi_1(\pi_t - \overline{\pi}) y_t \pi_t + (mc_t - 1)\epsilon_p y_t] + \mathbb{E}_t \frac{p_t^a}{p_{t+1}^a + d_{t+1}} [\psi_1(\pi_{t+1} - \overline{\pi}) y_{t+1} \pi_{t+1}] = 0 \tag{A.22}$$
$$\delta_t = \delta_0 n_t^{\delta_1} \tag{A.23}$$
$$r_t^k = p_t^i \delta_0 \delta_1 n_t^{\delta_1 - 1} \tag{A.24}$$
$$R_t = \mathbb{E}_t \pi_{t+1} \frac{p_{t+1}^a + d_{t+1}}{p_t^a} \tag{A.25}$$
$$p_t^i = \mathbb{E}_t \frac{p_t^a}{p_{t+1}^a + d_{t+1}} \left[r_{t+1}^k n_{t+1} + (1 - \delta_0 n_{t+1}^{\delta_1}) p_{t+1}^i\right] \tag{A.26}$$
$$r_t + \int \iota_s a_t \, m_{t+1} = \frac{d_t}{p_t^a} \tag{A.27}$$

The following gives $n_s \times n_b \times n_p \times n_e \times n_a \times n_h$ equations characterizing the optimal consumption function.

$$a_{t+1} = \begin{cases} \max\{((1+r_t+\iota_s)p_t^a a_t + (1-\tau_t)w_t p_t \eta_s - c_t)/p_t^a, \underline{a}_t\} & \text{if } e_t = 1 \\ \max\{((1+r_t+\iota_s)p_t^a a_t + \min(\phi_0 w_t p_t \eta_s, \phi_1 \overline{wp\eta}_s) - c_t)/p_t^a, \underline{a}_t\} & \text{if } e_t = 2 \end{cases} \tag{A.28}$$

where

$$c_t = \left[\beta_t \mathbb{E}_t \frac{(1 + r_{t+1} + \iota_s) p^a_{t+1}}{p^a_t} c^{-\sigma}_{t+1}\right]^{-1/\sigma} \tag{A.29}$$

$$\underline{a}_t = \begin{cases} 0 & \text{if } h_t = 1 \\ (1-\delta_h) a_t & \text{if } h_t = 2 \end{cases} \tag{A.30}$$

Using the type distribution at the beginning of period t, m_t, optimal decision rules, and transition probabilities of shocks, the type distribution can be updated to $\hat{m}_{t+1}$. Since m_{t+1} is a part of $\mathbf{x}_{t+1}$, we have:

$$m_{t+1} = \hat{m}_{t+1} \tag{A.31}$$

This gives $n_s \times n_b \times n_p \times n_e \times n_a$ conditions. But n_s conditions can be dropped since it can be backed up by the fixed measure of each s-type. $u_{s,t}$, $v_{s,t}$, and $f_{s,t}$ are characterized by the following equations for each s:

$$u_{\bar{s},t} = \int \mathbb{1}_{e=2} \mathbb{1}_{s=\bar{s}} d\, m_t \tag{A.32}$$

$$\kappa_s = \mu v^{\alpha-1}_{s,t} u^{1-\alpha}_{s,t} \sum_p \pi_{p|s,e=2} J_{s,p,t} \tag{A.33}$$

$$f_{s,t} = \mu v^{\alpha}_{s,t} u^{-\alpha}_{s,t} \tag{A.34}$$

Finally, the following recursive definition of the firm's value gives $n_s \times n_p$ equations.

$$J_{s,p,t} = (x_t - w_t) p \eta_s + \mathbb{E}_t \frac{p^a_t}{p^a_{t+1} + d_{t+1}} (1 - \lambda_s) \sum_{p'} \pi_{p'|p,1,1} J_{s,p',t+1} \tag{A.35}$$

D Variables in the Steady State

By imposing steady-state conditions to equations characterizing the equilibrium, steady-state variables can be characterized. They are summarized in Table A.2. I eliminate time scripts to denote variables in the steady state.

Table A.2: Steady-State Values and Conditions

Variable	Value	Condition
z^{TFP}	1.0000	From law of motion
z^{MEI}	1.0000	From law of motion
z^{MP}	1.0000	From law of motion
k	45.984	$= (k/y\ z^{TFP} n^{\theta})^{1/(1-\theta)} \ell$
ℓ	1.321	$= \int \mathbb{1}_{e=1} p \eta_s \, d\, m$
i	0.6898	$= \delta k$
R	1.0138	$= \pi(1+r)$
y	3.8320	$= z^{TFP}(kn)^{\theta} \ell^{1-\theta}$
c	2.8882	$= d + w\ell$
n	1.0000	By assumption
p^a	47.586	$= d/r$
d	0.4164	$= rk - \sum_s \kappa_s v_s$
mc	0.9500	$= 1 - \frac{1}{\epsilon_p}$
τ	0.0216	From the government budget constraint
δ	0.0150	$= \delta_0$
x	1.9290	$= mc(1-\theta) z (kn)^{\theta} \ell^{-\theta}$
w	1.8711	$= \omega_0 x$
r	0.0088	$= r_k - \delta$
r^k	0.0238	$= mc\theta z (kn)^{\theta-1} \ell^{1-\theta}$
p^i	1.0000	From the first order condition of investment firms
π	1.0050	$= \overline{\pi}$
f_s	Table 6	From Cajner et al. (2017)
u_s	Table 6	$= \lambda_s/(f_s + \lambda_s)$
v_s	Table 6	$= (f_s/\mu)^{1/(\alpha-1)} u_s$

Note: Quarterly frequency.