Ballistic effusion of normal liquid ^{3}He through nanoscale apertures

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We have measured mass transport of normal liquid ^{3}He through an array of submicron diameter apertures in a thin membrane. As the temperature is decreased we observe the crossover from viscous flow to ballistic effusion transport by quasiparticles. In this ballistic regime the quasiparticle mean free path is large compared to both the aperture diameter and the membrane thickness, and the flow conductance is temperature independent. At lowest temperatures, this experiment provides an analog of the electronic ballistic point contact for neutral Fermi liquids. The measured conductance is in quantitative agreement with theory.

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As nanotechnology has progressed there has been increased interest in ballistic electron transport in the quantum limit.[1] The overarching theme of this area of research is to understand how noninteracting, degenerate Fermions, traveling ballistically through nanoscale regions of space, can be used to create useful devices. Electrons are, however, not the only Fermions, which can exhibit novel quantum transport phenomena. In this paper we describe measurements in normal liquid ^{3}He, which exhibit pure ballistic mass transport through nanoscale apertures.[2] The present work may be viewed as a first step toward reaching the quantum transport regime for neutral matter, where the particle's Fermi wavelength is comparable to the aperture dimension.[3]

In addition to studying ballistic quantum transport, the experiment described herein is also a test of the consistency of Landau's prediction[4] that at sufficiently low temperatures, normal (as opposed to superfluid) liquid ^{3}He, which is a complex system of strongly interacting Fermions, will behave as an ideal gas of weakly interacting ''quasiparticles.'' These quasiparticles obey Fermi statistics and their number per unit volume is equal to the number of ^{3}He atoms. Many successful tests of this theory have been performed in the so-called hydrodynamic limit where the quasiparticle's mean free path, l, is very short compared to relevant apparatus dimensions.

A number of theoretical and experimental investigations have also been pursued in the opposite regime, where l is large compared to at least one characteristic dimension, d. Until recently, the research was concentrated on transport phenomena in long narrow channels and slits with transverse dimensions much smaller than the interquasiparticle mean free path. Rice[5] studied theoretically several transport coefficients in this limit, introducing phenomenologically the scattering of quasiparticles from the walls of such channels. Experimental verification of Rice's predictions for the coefficients of spin diffusion[6] and thermal conductivity[7] was performed in thin (about 1 mm) slabs of Vycor porous glass (average pore diameter $\sim$70 Å). These measurements have shown qualitative agreement with theoretical predictions, while the quantitative discrepancies were associated with the unknown nature of interaction between quasiparticles and channel surface.

This interaction remained the dominant theme in subsequent mass flow experiments. Rice's theory treated three different flow regimes in long narrow channels: hydrodynamic or Poiseuille ($l \ll d$), intermediate ($l \sim d$), and Knudsen ($l \gg$ d). The effect of the quasiparticle mean free path on Poiseuille mass flow was detected experimentally by Eisenstein *et al.*[8] Subsequently, extensive studies in the regime when the quasiparticle mean-free-path affects flow properties (''slip'' regime) have been performed in pure ^{3}He and ^{3}He–^{4}He solutions.[9–12] The effect of scattering changes by plating surfaces with thin layers of ^{4}He was studied as well.[13] Theoretical and experimental advances in the area were reviewed comprehensively by Smith[14] and, more recently, by Einzel and Parpia.[15] These reviews concluded that in order to achieve quantitative rather than qualitative agreement between theory and experiment, better understanding of the quasiparticle-surface interaction is required, as well as better surface preparation and characterization.[16]

All the described flow experiments can be classified as diffusion transport, since the transport properties are dominated by the nature of scattering of the carriers by the conduction path surface and/or bulk defects. In contrast, effusion flow refers to the transport of mass through an infinitesimally thin aperture in a container wall in the limit where the mean free path is large compared to all three dimensions of the aperture. More specifically, each particle ballistically impinging on the aperture passes out of the container without disturbing the equilibrium within the remaining particles.[17] In this regime, the details of particle (or quasiparticle) scattering on the walls of a container are irrelevant for the mass flow properties. In this paper, we report the direct observation of effusion mass transport by quasiparticles in normal liquid ^{3}He.

In our experiments liquid ^{3}He is pushed through an array of nominally 100 nm square apertures made in a nominally 50 nm thick silicon nitride (SiN) membrane[18] by applying a chemical potential difference between the two sides. The crossover from hydrodynamic viscous flow to effusion occurs when the mean free path is comparable to the diameter of the aperture, d. One can estimate the characteristic crossover temperature, T_{cr}, by equating the temperature dependent mean free path[19] $l = 6.8 \times 10^{-11} T^{-2} m$ to d. We find $T_{cr} \sim 26$ mK when $d \sim 100$ nm. For temperatures well above T_{cr} the flow impedance of an aperture is proportional to the liquid's viscosity which scales as T^2. In contrast, well below T_{cr} the impedance should be determined by ballistic transport

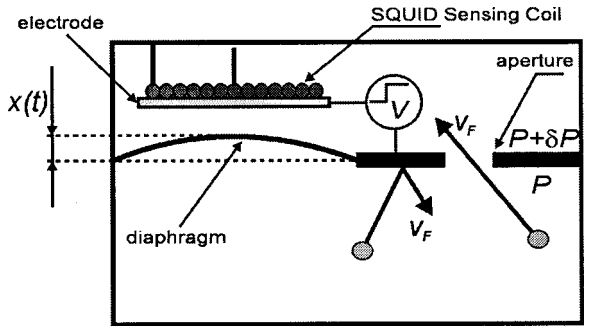

FIG. 1. A schematic diagram of the experiment. The ^{3}He filled box is divided by a partition containing a flexible metalized diaphragm and a nano-aperture array. We indicate the presence of only one aperture. Voltage applied between the electrode and the diaphragm creates a pressure difference across the apertures. The SQUID sensing coil monitors the displacement of the diaphragm.

and the flow impedance should be temperature independent.

The experimental apparatus, which is shown schematically in Fig. 1, has been described elsewhere.[20,21] The cell contains two volumes filled with liquid ^{3}He, which are separated from each other by a metalized flexible diaphragm[22] and a SiN window which contains nanoapertures. A SQUID displacement sensor monitors the position of the diaphragm, the SQUID output voltage being linear in the average diaphragm displacement $x(t)$. A fixed electrode adjacent to the diaphragm permits electrostatic manipulation of the pressure difference between the two sides.

The liquid temperature is varied in the range between 1 mK and 100 mK by a nuclear demagnetization refrigerator. Over this temperature interval, the quasiparticle mean free path spans the range from much smaller than the aperture diameter to much larger. The mass currents passing through a single 100 nm aperture are very small. Consequently we use a 65×65 array (i.e., total $N=4225$) of apertures. All the measurements are made near zero ambient pressure. Temperature is measured by a platinum NMR thermometer calibrated at the superfluid transition temperature, $T_c\approx0.93$ mK.

In equilibrium the effusion fluxes across the two sides of the apertures are equal and there is no net mass transfer. The flow through the apertures is initiated by applying a step voltage between the metalized diaphragm and the adjacent rigid electrode. The electrostatic force on the diaphragm creates a differential pressure head, which changes the effusion mass fluxes between the two sides. The resultant current permits the initial pressure head to relax as the diaphragm moves toward its final equilibrium position.

For small displacements, the diaphragm acts as a simple spring with an effective spring constant that relates the pressure on the diaphragm to its displacement from equilibrium. By knowing the position of the diaphragm, $x(t)$ (from the output of the SQUID circuit) we can infer the pressure difference across the apertures using Hooke's law:

$$\delta P(t)=-\frac{k_{\rm eff}}{A}x(t), \tag{1}$$

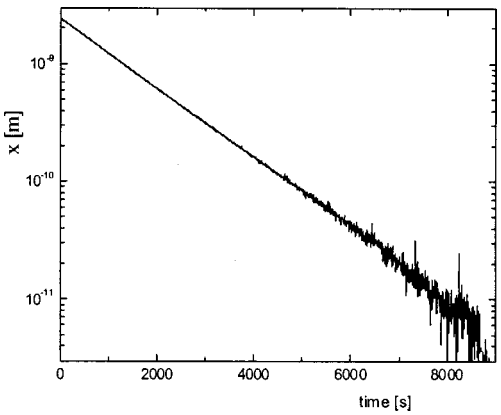

FIG. 2. Relaxation of the flexible diaphragm after the application of an initial pressure step. The relaxation is almost perfectly exponential. The main error in determining the characteristic relaxation time τ comes from a slowly drifting SQUID baseline signal. The drift of a SQUID baseline was recorded for about one hour before the application of the pressure step and for approximately the same period of time after the relaxation. Subsequently, the baseline was fit with a second order polynomial and subtracted from the recorded signal. At each temperature, 4 to 10 relaxation curves were taken.

where $k_{\rm eff}$ is the diaphragm's stiffness constant and $A=1.3\times10^{-4}$ m^2, its area. We determine $k_{\rm eff}$ both from the resonance of vacuum oscillations and from direct deflection measurements made by applying voltages to the fixed electrode. For this membrane, $k_{\rm eff}=3.6\times10^3$ N/m.

We determine the current through the apertures by considering mass conservation for the system,

$$I(t)=\rho A\frac{dx(t)}{dt}, \tag{2}$$

where ρ is the density of liquid ^{3}He. Thus dx/dt yields I. Following the convention in electron transport we define the conductance, G, as the ratio of mass current to the chemical potential difference which drives the current

$$G\equiv\frac{I}{\delta P/\rho}=-\frac{\rho^2A^2\dfrac{dx(t)}{dt}}{k_{\rm eff}x(t)}=-\frac{\rho^2A^2}{k_{\rm eff}}\frac{\dot{x}}{x}. \tag{3}$$

For a constant conductance we expect to see an exponential relaxation to equilibrium $x(t)=x_0e^{-t/\tau}$, where the relaxation time constant is inversely proportional to G:

$$G=\frac{\rho^2A^2}{k_{\rm eff}}\frac{1}{\tau}. \tag{4}$$

The exponential response is confirmed in Fig. 2 which shows $x(t)$.

Using Eq. (4) and measuring the relaxation time τ we determine the conductance as a function of temperature down to the superfluid transition temperature $\sim$1 mK. Figure 3 shows the temperature dependence of the measured conductance. At temperatures above about 20 mK the conduc-

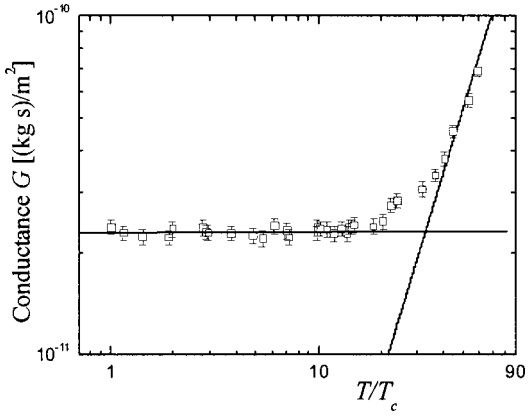

FIG. 3. The measured conductance of the array of 4225 apertures. The line on the right (i.e., higher temperatures) is the function $G_{\rm hyd}=2.14\times10^{-14}(T/T_c)^2$ kg s/m^2 characteristic of hydrodynamic viscous flow for a degenerate Fermi liquid. Below about 20 mK, the conductance is temperature-independent as expected for the effusive flow regime. The horizontal line is $G=2.16\times10^{-11}$ kg s/m^2. In this figure, each point is an average of 4 to 10 measurements (relaxations).

tance rises as T^2 whereas below this temperature G is temperature independent. This is the general expected trend since in the hydrodynamic regime above 20 mK the conductance should be inversely proportional to viscosity and thus vary as T^2 (see Ref. 19).

In the effusion limit, the conductance can be calculated by considering the number of particles that strike a container wall per unit area, per unit time, i.e., the total flux of particles striking the wall. Due to the pressure head there are more quasiparticles striking the wall on one side than on the other so there is a net flux though the aperture. Taking the difference of the effusion number fluxes on opposite sides of the apertures gives the mass current through a single aperture

$$I=I_2-I_1=\tfrac{1}{4}\,\delta(np_F)S=\tfrac{1}{3}\,\delta n\,v_F m_3 S, \tag{5}$$

where δn is the excess quasiparticle number density due to the differential pressure head, v_F is the Fermi velocity, m_3 is the mass of a ^{3}He atom, and S is the area of a single aperture. The factor of $\frac{1}{4}$ comes from averaging over all ballistic directions. The final expression can be obtained by differentiating the expression for the Fermi momentum $p_F=h(3n/\pi^2)^{1/3}$.

We can calculate δn from the density of quasiparticle states only near the Fermi surface, $N(0)$, because these are the only quasiparticles which carry the net mass current,

$$\delta n=N(0)\,\delta\mu=\frac{m_3^* p_F m_3}{\pi^2\hbar^3\rho}\,\delta P, \tag{6}$$

where $\mu=m_3P/\rho$ is the chemical potential, $N(0)=m_3^*p_F/\pi^2\hbar^3$, and m_3^* is the quasiparticle effective mass.

Dividing the current by $\delta P/\rho$ and using the ideal gas expression for the Fermi momentum yields the *temperature independent* effusion conductance which can be expressed by the formula

$$G_{\rm eff}=\left(\frac{1}{3\pi^2}\right)^{1/3}\frac{m_3^{4/3}\rho^{2/3}NS}{\hbar}, \tag{7}$$

where N is the number of apertures. Since the density is well known,[23] Eq. (7) leaves only the aperture area, S, to compare to our observed low temperature conductance limit. SEM pictures of the individual apertures have a resolution of about 20 nm. On this scale the nominally square apertures appear somewhere between circular and square. Assuming round apertures we find an effective diameter of 116 nm whereas a square aperture would have an effective side length of 117 nm. By comparison, these same apertures, assumed to be circular, were found to have effective diameters of 100 nm in Josephson effect experiments.[24] Considering that the exact shape of the apertures is uncertain and that the effusion calculation, Eq. (7), has left out effects of finite wall thickness, the agreement between Eq. (7) and the experiment is very good indeed.

In the hydrodynamic regime the conductance for an aperture is

$$G_{\rm hyd}=\frac{\rho^2}{Z\eta}, \tag{8}$$

where the flow impedance Z has dimension $(\text{Length})^{-3}$. For a round aperture of radius r in a wall of zero thickness we can write $Z=\beta/r^3$ where β is a constant of order unity. Using a tabulated value of η, and taking a value of 65 nm for r, the fit to the hydrodynamic regime in Fig. 3 implies $\beta=1.2$. Again, considering that we are ignoring both the finite length of the aperture and its precise shape this is a very reasonable value for the hydrodynamic impedance.

It is appealing to ask if effusion measurements such as these can be used to more quantitatively test the Landau model of liquid ^{3}He. Much of the systematic uncertainty in the present experiment could be reduced if the aperture diameter d, were made considerably larger. For example, if $d\sim1000$ nm, the flow will be completely effusive at 4 mK, still well above the superfluid transition. In these larger apertures, using improved microscopy the geometry uncertainty would be on the order of 1%. Furthermore, the finite thickness of the aperture (50 nm) would play a negligible role. Then the area factor in Eq. (7) would be a known parameter and the measured conductance could be closely compared to the kinetic theory prediction.

At the opposite size extreme it would be fascinating to observe the ballistic conductance in apertures or tubes whose dimensions approach the Fermi wavelength ~0.8 nm. By analogy with effects observed in electron transport, at that limit the ballistic conductance might be expected to be quantized in units of $2m_3^2/h$.

In summary we have determined the conductance of normal ^{3}He through an array of small apertures. We find that at the predicted crossover temperature the conductance changes from the predicted hydrodynamic high temperature limit ($G\sim T^2$) to the temperature independent low temperature ballistic effusion limit. The quantitative agreement between the predicted and observed magnitude of the ballistic con-

ductance gives hope that future experiments will add effusion to the menu of properties used to test the Landau theory. Finally, an important step has been taken down the road toward the possible observation of quantum conductance in neutral matter.

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