Direct verification of the relation $v_s = \frac{\hbar}{m} \bigtriangledown \phi$

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Abstract. We describe an experiment in which we induce a heat-driven superfluid flow in a straight tube and monitor the phase difference across the tube's ends with a superfluid ^{4}He quantum interference device (SHeQUID). We quantitatively verify the relation $v_s = (\hbar/m) \bigtriangledown \phi$. We also demonstrate the linearization of a SHeQUID using the heat injection method.

Superfluid is described by a macroscopic wavefunction of the form $\Psi = \sqrt{\rho_s} e^{i\phi}$, where ρ_s is the superfluid density and ϕ is the quantum mechanical phase. Applying a momentum operator ($\hat{p}\Psi = -i\hbar \bigtriangledown \Psi = p\Psi$) with $p = mv_s$ immediately gives [1] the relation

$$v_s = \frac{\hbar}{m} \bigtriangledown \phi, \tag{1}$$

where this velocity is interpreted as that of the superfluid component in the framework of Landau's two-fluid model [2]. Although this relation has been used to understand many physical phenomena such as the existence of vortices/quantized circulation etc., direct verification has been difficult for the lack of an appropriate phase-measuring device. In an experiment reported here, we generate a known superfluid velocity in a straight tube and measure the phase difference across the tube's ends with a superfluid ^{4}He quantum interference device (SHeQUID) [3]. By so doing, we quantitatively confirm Equation 1, verifying the direct relation between the superfluid velocity and the phase gradient of the condensate macroscopic wavefunction.

Our experimental apparatus is shown in Figure 1a and described in more detail elsewhere [4]. The topmost tube (of length $l(\approx 2.5cm)$ and cross-sectional area $\sigma(\approx 3.8 \times 10^{-2} cm^2)$) contains a resistive heater (R) at one end and a thin roughened Cu sheet (S) at the other that serve as a heat source and a sink respectively. When power $\dot{Q}$ is applied to the heater, the normal component flows away from the heat source with velocity v_n while the superfluid component flows towards it with velocity [1]

$$|v_s| = \frac{\rho_n}{\rho \rho_s T s \sigma} \dot{Q}, \tag{2}$$

where ρ and ρ_n are the total and normal fluid densities, T is the temperature, and s is the specific entropy (per unit mass). Equation 1 suggests that uniform v_s should correspond to a uniform phase gradient $\bigtriangledown\phi_{heat}$ along the top tube. We monitor the phase difference $\Delta\phi_{heat} = l \bigtriangledown \phi_{heat}$ by configuring the top tube as part of a superfluid ^{4}He interferometer loop.

The output of a SHeQUID is a combined oscillation amplitude from two weak link junctions, which exhibits interference depending on the relative phase differences $\Delta\phi \equiv \Delta\phi_1 - \Delta\phi_2$ that exists between them (where $\Delta\phi_1$ and $\Delta\phi_2$ are the phase drops across the two weak

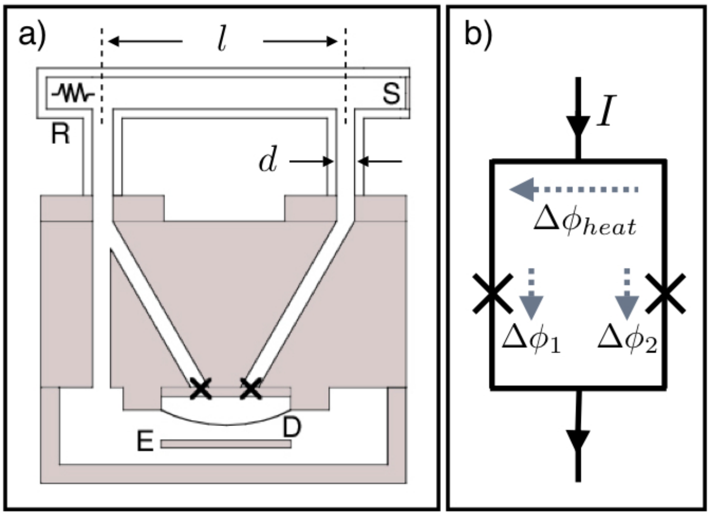

Figure 1. a) Experimental apparatus. Crosses indicate the superfluid weak link junctions. Flexible diaphragm (D) and electrode (E) are used to drive the mass current oscillations. Two pipes connected to the top arm have a width $d(\approx 2.2mm)$. In writing $\Delta\phi_{heat} = l \bigtriangledown \phi_{heat}$, l becomes uncertain by this amount d, which dominates the systematic uncertainty in this experiment. b) Equivalent circuit.

links). When the two junctions are identical, the total oscillation amplitude can be written [3] as $I_t \propto |cos(\Delta\phi/2)|$. For a closed loop drawn through two junctions and the heat current tube (see Figure 1b for an equivalent circuit), we can write the accumulated phase as $\Delta\phi_{heat} + \Delta\phi_1 - \Delta\phi_2 = 2\pi n$ and set $n = 0$ since the flow velocity is kept well below the critical velocity to create quantum vortices. Using this relation with Equations 1 and 2, we find that the uniform phase gradient associated with superfluid flow implies that the SHeQUID output should modulate as

$$I_t \propto |cos(\pi \frac{m_4}{h}[\frac{l}{\sigma}\frac{\rho_n}{\rho\rho_s Ts}\dot{Q}])|. \quad (3)$$

Figure 2 shows the measured SHeQUID amplitude as a function of heat input in the top tube. The solid line is a fit using a more general function (described elsewhere [3]) for the SHeQUID output for two weak link junctions with unequal critical currents. The periodic variation in the SHeQUID output as a function of $\dot{Q}$ shows that there is indeed a uniform phase gradient across the topmost tube such that $\bigtriangledown\phi \propto v_s$.

The distance between two adjacent maxima seen in Figure 2a is the power that leads to a 2π phase change across the heat current tube. Figure 2b shows these powers ($\dot{Q}_{2\pi}$) measured at different temperatures. From Equation 3, we expect $\dot{Q}_{2\pi} = (h/m_4)\beta(T)$, where $\beta(T) \equiv (\sigma/l)(\rho\rho_s Ts/\rho_n)$. We have computed $\beta(T)$ with published data on ρ_s, ρ_n, ρ, and s, and the designed values of l and σ. We plot this function and multiply it by a constant to fit the data in Figure 2b. The best multiplication factor is $(9.1 \pm 0.9) \times 10^{-8} m^2/sec$, which agrees with the expected value of $h/m_4 = 9.97 \times 10^{-8} m^2/sec$ within the systematic uncertainty. This result demonstrates the fundamental relation linking the macroscopic wavefunction picture and the two-fluid description of superfluid helium.

Having demonstrated the heat current technique that injects phase variations into a SHeQUID, we can now use it as a feedback element to nullify an external phase shift. This linearizes the intrinsically nonlinear interference relation underlying the device and essentially

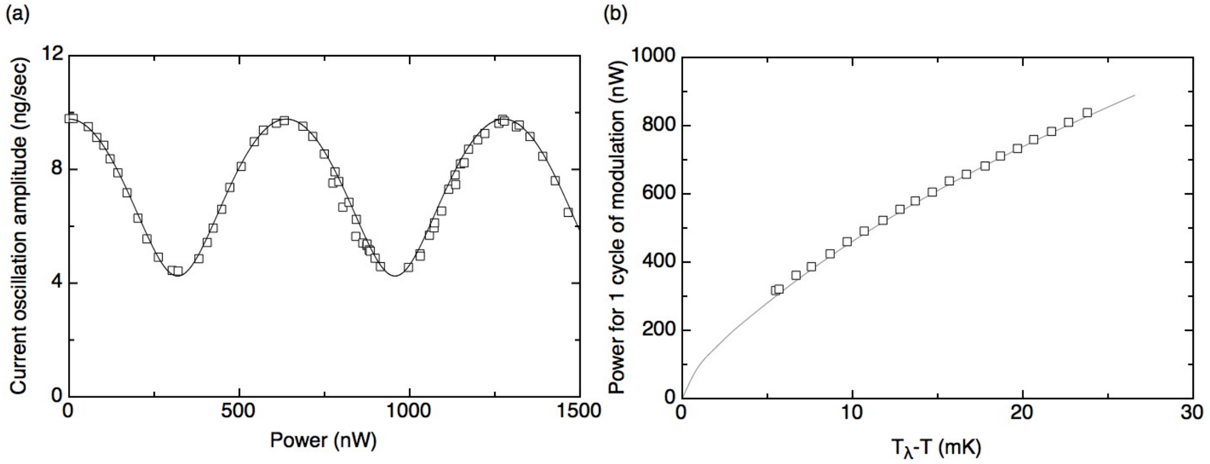

Figure 2. a) Measured SHeQUID output as a function of $\dot{Q}$. These data are taken at $T_\lambda - T \approx 16mK$. b) Power needed to cause the SHeQIUD output to move from one maximum to the next.

gives a flux locked interferometer. As a proof of principle experiment, we have used the so-called Sagnac phase shift [5] as the external phase shift here.

Rotation flux through the interferometer loop ($\vec{\Omega} \cdot \vec{A}$ where $\vec{\Omega}$ is the angular velocity vector of the earth, and $\vec{A}$ is the area vector of the interferometer loop) induces a phase shift given by $\Delta\phi_{Sagnac} = (4\pi m_4/h)\vec{\Omega} \cdot \vec{A}$. Figure 3a shows the interference pattern exhibited by the SHeQUID when we reorient the apparatus about the vertical, thus changing the angle between the interferometer loop and the earth's spin axis. With both the Sagnac phase shift and the heat current phase shift picked up by the SHeQUID, we can rewrite Equation 3 as

$$I_t \propto |cos(a\vec{\Omega} \cdot \vec{A} + b\dot{Q})|, \tag{4}$$

where $a \equiv 2\pi m_4/h$ and $b \equiv (\pi m_4/h)(l/\sigma)(\rho_n/\rho\rho_s T s)$. We can now use the heater power to cancel any change in rotation flux so that $a\vec{\Omega} \cdot \vec{A} + b\dot{Q}$ remains constant. The interferometer can then be maintained at any fixed output desired, and the flux is locked. This allows us to bias the device at the point of the greatest slope and operate there without having to trace many interference cycles arising from large changes in rotation flux. It also provides a linear measure of the change in rotation flux since $|\vec{\Omega} \cdot \vec{A}| = b\dot{Q}/a$. Figure 3b shows the SHeQUID output with power applied to the heater nullifying the Sagnac phase shift. Within the noise level of the experiment, the SHeQUID is flux locked. Figure 3c shows the heater power injected to keep $a\vec{\Omega} \cdot \vec{A} + b\dot{Q}$=constant. Clearly, $\dot{Q} \propto \vec{\Omega} \cdot \vec{A}$, and the output is linearized. We find that this feedback technique works well for injected heater power up to what corresponds to ~250 flux quanta. As the heater power is increased further, we observe vortex crossing in the heat current tube [7] followed by a rapid onset of quantum turbulence, making it impossible to make reliable measurements of external phase shifting influences such as rotation.

In summary, we have directly verified the relation $v_s = (\hbar/m) \bigtriangledown \phi$ by driving the superfluid flow in a straight tube and measuring the phase difference across the tube's ends with a superfluid matter wave interferometer. We have also used the heat current injection method to linearize the output of a SHeQUID so that this class of interferometer can be widely used in basic and applied sciences. More details on these experiments can be found in refs [4] and [6].

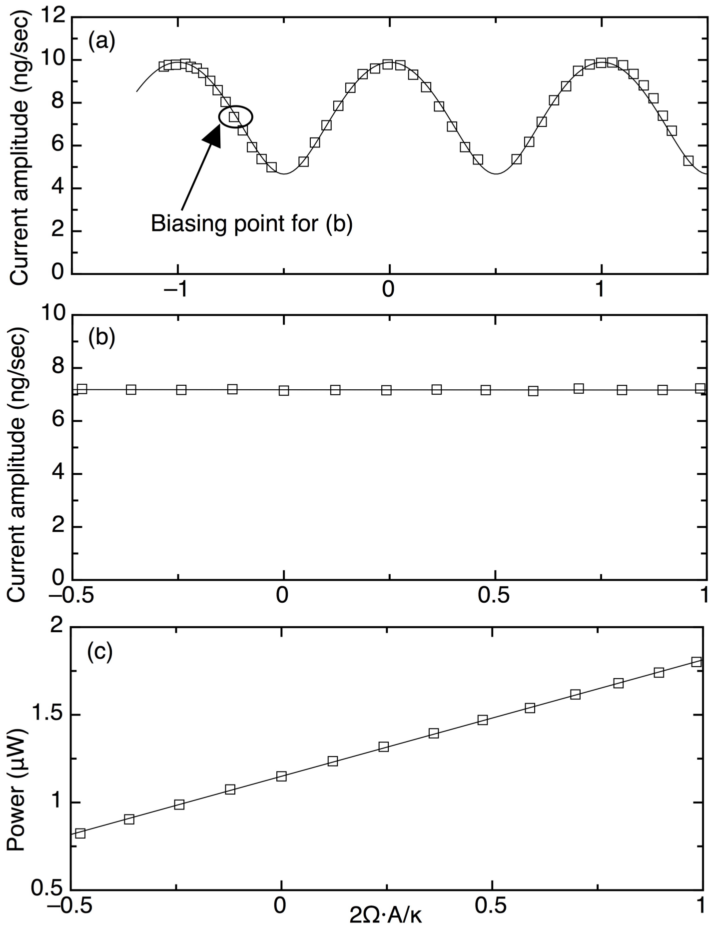

Figure 3. a) SHeQUID output modulation from changes in the earth's rotation flux. b) Modulation compensated by injected heater current thus making the SHeQUID output independent of the rotation flux. c) Feedback heater power needed for a given value of rotation flux to maintain the SHeQUID output constant.

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