Physica B 169 (1991) 523-524
North-Holland

Measurements of Circulation in Superfluid ^{3}He-B using a Vibrating Wire

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We have measured fluid circulation in superfluid ^{3}He-B. The apparatus consists of a straight vibrating wire which is mounted in a ^{3}He cell on a rotating millikelvin cryostat operating at temperatures in the ballistic quasiparticle regime. Circulation around the wire is found to be stable only when it takes on values $-h/2m_3$, 0 , $+h/2m_3$ where h is Planck's constant and m_3 is the mass of the ^{3}He atom.

It is widely accepted that superfluid ^{3}He is characterized by a macroscopic quantum state. Flow in such a system should exhibit quantized circulation(1). Because most theoretical models of superfluid ^{3}He are based on the concept of Cooper pairing it has been assumed that the quantum unit of circulation is $h/2m_3$. Although there is a wealth of data consistent with the BCS theory there has, until now, been no single experiment which unambiguously(2) reveals the existence of either quantized circulation or the Cooper pair in superfluid ^{3}He.

Our experimental method is essentially the vibrating wire technique used by Vinen(3), and later by Zimmermann et al (4) to measure the quantization of circulation in superfluid ^{4}He. For our investigations of superfluid ^{3}He-B, we stretched a superconducting NbTi wire of diameter 16 μm along the axis of a 50 mm long, 2.8 mm inner diameter brass cylinder . The cylinder, containing ^{3}He, is mounted in a rotating millikelvin cryostat. Its axis is parallel to the rotation axis and perpendicular to a 50 mT magnetic field.

Passing a short pulse of current through the wire creates a magnetic force impulse. It then vibrates freely at its fundamental frequency near 347 Hz. Initially the vibration is perpendicular to the magnetic field. If there is circulation around the wire, the velocity vector precesses, so the component of the wire's motion along any given direction shows a beat pattern. The emf generated as the wire vibrates in the applied field is proportional to the component of the wire's velocity perpendicular to the field. As the velocity vector precesses, the emf also shows a beat pattern.

In practice the cross section of the wire is not perfectly round. This causes beats even in the absence of circulation. If f_0 is the beat frequency due to these asymmetries, then f_c, the precession frequency due to circulation, is related to the observed precession frequency f_1 by(3,4)

$$f_c^2 = f_1^2 - f_0^2 . \qquad (1)$$

The circulation κ around the vibrating wire is(3,4)

$$\kappa = 2\pi \, \mu \, f_c / \rho_s \qquad (2)$$

where ρ_s is the superfluid density and μ is the mass per unit length of the wire.

We initially tested our vibrating wire cell in rotating superfluid ^{4}He, where we observed well-defined quantized circulation states whose quantum unit was equal to h/m_4 within the limits of known systematic error (5%). The accuracy of these measurements is limited by our knowledge of the mass per unit length of the wire. The ^{4}He measurement is a calibration of the device on the assumption that the quantum of circulation in ^{4}He is exactly equal to h/m_4. This calibration indicates that $\mu = 1.54 \times 10^{-6}$ Kgm^{-1}.

To measure a single quantum of circulation accurately the damping of the wire's vibration must be sufficiently small that several beats can occur before the amplitude becomes too small to measure. This condition places a practical upper limit of about T/T_c=0.2 on the ^{3}He temperature. In this temperature range the superfluid is in the ballistic quasiparticle regime. During the experiment the cryostat must rotate at speeds of order 1 radian s^{-1} while remaining at these temperatures. The Berkeley rotating millikelvin cryostat is described in a paper in these proceedings(5).

The experiment was carried out in superfluid ^{3}He-B at a pressure of 17.8 bar and at temperatures below 0.4 mK. Figures 1a and 1b each show the emf induced across the vibrating wire as a function of time. On the time scale of the figures the individual oscillations cannot be resolved. The envelope of these oscillations is seen to be a combination of low frequency beats, due to the wire's precession, and the decay of the amplitude of its vibration.

Figure 1a shows a typical signal obtained when the ^{3}He is cooled into the superfluid phase without rotation. In this case the observed beat frequency f_0 (which is $1/t_0$, where t_0 is the time between two successive zeroes of the beat pattern) is due only to the wire's asymmetry. Figure 1b shows the typical beat pattern when the rotation speed of the cryostat Ω has been ramped from 0 up to about 1 rad. s^{-1} and then down to 0 again. Here, the beat frequency $f_1 = 1/t_1$ is higher. This observed difference in beat frequencies is used with Equations 1 and 2 to calculate the circulation around the wire. We find that the circulation has magnitude $h/2m_3$. Circulations of this magnitude have been observed to remain unchanged for at least 14 hours. No other circulations persist for more than a minute.

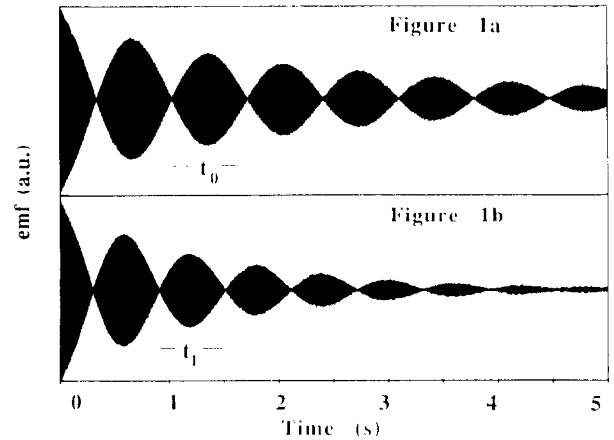

Figure 1a. The free decay of oscillations with no circulation present.

Figure 1b. The free decay of oscillations with 1 quantum of circulation around the wire.

Figure 2 shows the results of a series of measurements in which Ω is ramped from 0 up to a maximum speed Ω_{max} and then back to 0, with a constant angular acceleration and deceleration. After each ramp ends, the circulation κ around the wire is measured at $\Omega = 0$. We repeat the measurement of κ for 10 minutes in order to investigate the stability of the trapped state. Positive and negative Ω_{max} correspond to clockwise and anticlockwise rotation of the cryostat, respectively.

Three different regimes can be identified:

1) $\Omega_{max} < \Omega_{max}{}^c$, where the critical angular velocity $\Omega_{max}{}^c$ is typically about 0.5 rad s^{-1}: the circulation is stable and zero.

2) $\Omega \sim \Omega_{max}{}^c$: the circulation is unstable and has been observed to fluctuate for periods of up to several hours.

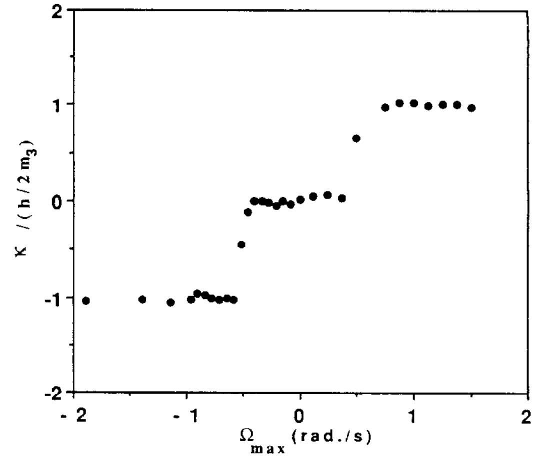

Figure 2. The average value of circulation (in units of $h/2m_3$) measured for 10 minutes after the cryostat has been ramped to Ω_{max} and back to 0. The data for positive Ω was first taken. Then the cell was warmed up above T_c to remove any vorticity. The data for negative Ω was then taken after cooling down again.

3) $\Omega_{max} > \Omega_{max}{}^c$: the circulation is stable. Its magnitude shows small fluctuations within 5% of $\kappa = h/2m_3$.

We distinguish the quantum number n=+1 state from the n=-1 state by rapidly changing the magnetic field, as described by Zimmerman et al.(4) Free energy arguments indicate that in He-B, and for a 16 μm wire, trapped circulation states with greater than one quantum are not favoured.

In conclusion, we have demonstrated that stable circulation around a small diameter wire in superfluid He-B is quantized in units of $h/2m_3$.

We wish to acknowledge the excellent machining skills of Mr. Tom Pederson, who built much of the Berkeley rotating cryostat. This work was supported by the National Science Foundation under grant #DMR 88-19110.

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