Available online at www.sciencedirect.com
SCIENCE DIRECT
Physica B 329–333 (2003) 62–63
PHYSICA B
www.elsevier.com/locate/physb

Numerical studies of the superfluid Shapiro effect

R.W. Simmonds[a,*], A. Marchenkov[b], J.C. Davis[a], R.E. Packard[a]

[a] Department of Physics, University of California at Berkeley, 366 Le Conte Hall, Berkeley, CA 94720, USA
[b] School of Physics, Georgia Tech, 837 State Street, Atlanta, GA 30332-0430, USA

Abstract

Although simple theoretical descriptions of the Shapiro effect focus on a voltage biased superconducting Josephson junction, experimental measurements are predominantly made using current biased systems. In an analogous *superfluid* ^{3}He system, a Josephson weak link can be predominantly *pressure* biased in accordance with the simple theory. Here, we use numerical methods to reproduce new features found in the observed "superfluid Shapiro effect". We find the proper characteristics for a pressure biased system with new additional structure resulting from a non-zero source resistance.

Keywords: Superfluid; Weak links; Josephson effect; Shapiro effect

In his original paper [1], Josephson proposed that if a constant voltage bias plus an ac voltage is applied across a superconducting Josephson junction, the supercurrents will exhibit characteristic changes. Shapiro first observed these phenomena using current-biased superconducting Josephson junctions [2,3]. Nearly 40 years later, the "superfluid Shapiro effect" has been observed using a pressure biased superfluid ^{3}He weak link array [4]. Here, we focus on numerical methods to reproduce key features in this data.

In our double-diaphragm superfluid system, we provide a constant pressure bias using the "lower" diaphragm and a previously developed feedback technique [5]. We can provide an additional AC pressure excitation using the "upper" diaphragm. In the simple model proposed here, the resultant pressure across the weak link should be

$$P(t) = P_{DC} + P_{AC}\cos(\omega t + \varphi). \quad (1)$$

Through the Josephson relations ($I = I_c \sin(\phi)$ and $\dot{\phi} = -2m_3 P/\rho\hbar$), we expect the pressure (1) will produce new currents of the form

$$I_n = I_c|J_n(\gamma)| \quad (2)$$

where $\gamma = 2m_3 P_{AC}/\rho\hbar\omega$ and J_n is the Bessel function of nth order. This leads to two types of Shapiro effects: (i) A *reduction* in the critical current of the superfluid weak link array ($n = 0$), (ii) an *increase* in the DC currents (spikes) at pressures which satisfy the condition

$$\frac{P_{DC}}{\rho} = n\frac{\hbar\omega}{2m_3} \quad (3)$$

for $n > 0$.

Measurements of the low amplitude pendulum mode oscillations when $I \approx I_c \sin(\phi)$ have confirmed the predictions of (i). To verify (ii), we obtain the I–P characteristic for different amplitudes of the AC excitation. Fig. 1 shows a unique "feature" in the DC current centered about Josephson frequencies equal to the AC excitation frequency, $\omega_J/2\pi = \omega/2\pi = 105$ Hz in accordance with (3) where $n = 1$. These additional currents are seen (in panel a) to increase and then decrease as the amplitude of the AC excitation is increased (from the bottom curve to the top curve). Panel b shows that the size of this feature varies in accordance with (2). The shape of the features found in the I–P characteristics are not merely "current spikes" as predicted by the simple theoretical model (2) first

*Corresponding author.
E-mail addresses: raywiley@socrates.berkeley.edu (R.W. Simmonds), packard@socrates.berkeley.edu (R.E. Packard).

0921-4526/03/$ - see front matter © 2003 Elsevier Science B.V. All rights reserved.
doi:10.1016/S0921-4526(02)01921-X

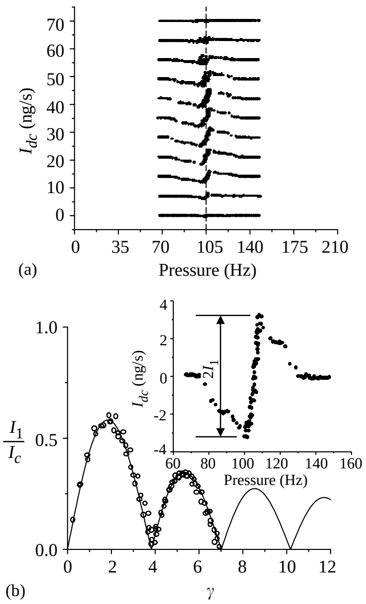

Fig. 1. (a) A plot of a series of I–P characteristics showing the current "feature". (b) A plot of I_1 as a function of γ.

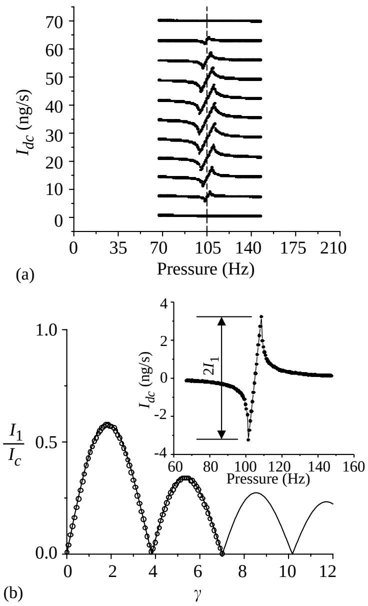

Fig. 2. (a) A plot of the prediction for the I–P characteristic. (b) A plot of I_1 as a function of γ.

derived by Shapiro [2]. Of course, these features are far from being the "steps" found in superconducting current biased experiments. These results imply that the pressure across the weak link is *not* simply given by Eq. (1).

Consider a situation where the pressure source includes a ohmic resistance Z. The resistance Z appears in series with the weak link so that the solution for the phase difference $\phi(t)$ must come from

$$\dot{\phi} = \omega_Z \sin(\phi) - \omega_J - \gamma\omega\cos(\omega t + \varphi), \qquad (4)$$

where $\omega_Z = 2m_3 I_c Z/\hbar$. Eq. (4) can be solved numerically using a 4th order Runge–Kutta technique. Once $\phi(t)$ is known, the resulting additional DC currents are given by

$$I_n = I_c \langle \sin(\phi) \rangle. \qquad (5)$$

Numerical simulations show that as the magnitude of the resistance Z is increased, the current spikes transform into a tilted "S"-shape like that found in the data. Panel a in Fig. 2 shows the numerical results when $n = 1$, $\omega/2\pi = 105$ Hz and we have chosen $\omega_Z/2\pi = 11$ Hz so that the slope of the central slant in the feature is nearly equal to that of the experimental data. We find an impressive agreement between the data and the resulting shape of the prediction made using Eqs. (4) and (5). The size of this feature increases and decreases with γ in a consistent way with the theoretical prediction (2) as shown in panel b.

The slope of the feature is a result of the difference between the DC pressure applied across the whole system and that found across the weak link. The time average of $\omega_Z \sin(\phi)$ in Eq. (4) shifts the DC pressure across the weak link, $\propto \langle \dot{\phi} \rangle$, from the applied value, $\propto \omega_J$. If we were to plot the DC current as a function of $\langle \dot{\phi} \rangle$ the feature would appear vertical because of the locking condition, $\langle \dot{\phi} \rangle = \omega$, during the increasing DC currents.

We have been able to reproduce the experimental data very well using this simple theoretical model. Unfortunately, the value obtained for Z is at least three times larger than known sources of dissipation. None the less, it is clear that the affect of a source resistance Z is to *dynamically* alter the pressure across the weak link. Even in this wonderful superfluid ^{3}He system, it is difficult to provide a *perfect* pressure source or bias.

We thank S. Vitale, S. Pereverzev, A. Loshak, and S. Backhaus. Sponsors: NSF (DMR-9807761), NASA (NAG8-1432), ONR (N00014-94-1-1008).

References

[1] B.D. Josephson, Phys. Lett. 1 (1962) 251.
[2] S. Shapiro, Phys. Rev. Lett. 11 (1963) 80.
[3] C.C. Grimes, P.L. Richards, S. Shapiro, J. Appl. Phys. 39 (1968) 3905.
[4] R.W. Simmonds, A. Marchenkov, J.C. Davis, R.E. Packard, Phys. Rev. Lett. 87 (2001) 035301.
[5] S. Backhaus, R. Packard, Czech. J. Phys. 46 (1996) 2743.