Precession of a Single Vortex Line in Superfluid ^{3}He-B

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This paper reports the discovery of a new vortex phenomenon in superfluid ^{3}He-B. A single filament of quantized vortex line precesses as a solid body around a wire extending along the axis of a cylinder. The precession frequency equals the angular velocity of the apparatus at which the presence of a single quantum of circulation minimizes the system's free energy. The period of precession is related to the circulation quantum and the dimensions of the apparatus. Thus a measurement of the precession period is an accurate determination of the quantum unit. We find $\kappa=(1.02\pm0.03)h/2m_3$, where m_3 is the mass of the ^{3}He atom.

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Superfluid helium, which displays quantized vorticity [1], is an ideal system for testing dynamical theories of vortices. However, as a result of the small size of the circulation quantum, previous experiments have usually dealt with the average motion of many vortices. In the few observations of individual vortices the geometry was essentially only two dimensional [2].

In this paper we report observations of the detailed motion of a single vortex filament in an interesting three-dimensional geometry. This experiment demonstrates that the quantitative details of the motion of ^{3}He quantized vortices can be understood by using the principles of dynamics developed for classical vortices. The experiments reveal a new type of vortex motion and also provide a new technique for determining the quantum of circulation [3,4] in ^{3}He-B.

The experimental apparatus consists of a brass cylinder (radius $R_c=1.48$ mm and length of 47 mm) filled with liquid ^{3}He-B, in which a lightly stretched wire (diameter of 16 μm) is placed slightly off axis. The apparatus is cooled to a few hundred microkelvin in a rotatable nuclear demagnetization cryostat [5].

When fluid circulation exists around the wire its lowest vibrational normal mode is split, producing a precession of the plane of vibration [3]. We measure the voltage induced across the wire as it vibrates in a perpendicular magnetic field. From the envelope of the detected voltage we find the frequency splitting of the lowest mode, which is related in an established way to the average circulation $\langle\kappa\rangle$ around the wire. Using a similar cell we recently reported [4] that, in ^{3}He-B, quantized circulation of magnitude $h/2m_3$ can be trapped on the wire for many hours. The sensitivity of the present apparatus permits detection of changes in $\langle\kappa\rangle$ as small as 1% of $h/2m_3$ in 1 s.

Vinen [3] used a similar technique to measure circulation in superfluid ^{4}He. Although the quantized values $\kappa=n(h/m_4)$ were found to be most stable, intermediate values were also observed. Vinen suggested that the intermediate levels can arise from a vortex which has detached from one end of the wire and reattached to the cylinder's inner wall, as shown in Fig. 1. If the point of attachment moves along the wire, the vortex "unzips" from the wire. Since the observed frequency splitting is determined by the weighted spatial average of the circulation, a vortex trapped only along some part of the wire will produce the same effect as fractional circulation trapped along the whole wire. Thus the unzipping process causes the observed circulation to change continuously.

This paper deals with the dynamics of a ^{3}He-B vortex filament unzipping from the wire. In both ^{4}He and ^{3}He the end of a vortex filament can "pin" to rough spots on a surface [6]. Because of the small vortex core in ^{4}He ($\xi=10^{-10}$ m), surfaces are inevitably rough enough to support pinning. As a result, vortex dynamics are very complicated in that system. However, in ^{3}He the vortex core size is approximately 3 orders of magnitude larger than in ^{4}He, and the pinning forces are considerably smaller. Standard machining techniques produce surfaces which are sufficiently smooth to prevent pinning. Thus in ^{3}He it should be possible to study dynamic pro-

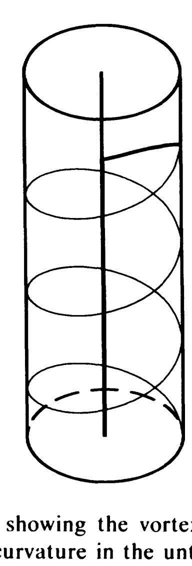

FIG. 1. A sketch showing the vortex filament as it unzips from the wire. The curvature in the untrapped vortex filament is required for stability of the solid body precession. The helical path of the tip of the filament is a result of the dissipation which accompanies the motion.

cesses without the complication of pinning.

In a typical measurement our experimental cell first rotates faster than 0.5 rad/s so that one unit of quantized circulation appears around the wire [4]. The cryostat is then brought to rest and the circulation becomes metastably trapped.

After some time, typically several hours, the measured circulation begins to decrease. The dynamic process of interest occurs while the measured circulation changes. Figure 2 shows a typical transition, in which the observed circulation changes, over a period of hours, from $\langle\kappa\rangle = h/2m_3$ to zero. The most striking feature is the small amplitude oscillation of fixed frequency which accompanies the decaying circulation. This oscillation is the signature of the dynamic vortex process.

We have found several ways to understand the observed oscillation. One approach is based on the premise that all forces within the fluid must sum to zero. There is an effective force F_1 arising from the decrease in energy of the fluid due to a virtual displacement of the trapped segment along the wire. In magnitude this is equal to the energy per unit length of the circulation trapped on the wire,

$$F_1 = \frac{\rho_s \kappa^2}{4\pi} \ln \frac{R_c}{R_w}, \quad (1)$$

where ρ_s is the superfluid density, κ is the circulation of the vortex segment on the wire, R_w is the radius of the wire, and R_c is the radius of the cylinder.

This force can be balanced by a net Magnus force which exists if the segment of free vortex filament precesses around the cylinder. If the filament undergoes solid body rotation with period T, we can integrate the Magnus force per length, $\rho_s(\boldsymbol{\kappa}\times\mathbf{v})$, from the surface of the wire to the boundary of the cylinder. This yields

$$F_M = \rho_s \kappa \pi (R_c^2 - R_w^2)/T, \quad (2)$$

where T is the period of the motion. This total vertical Magnus force F_M is independent of the shape of the vortex filament.

If we equate these two forces and solve for the period we obtain

$$T = 4\pi^2 (R_c^2 - R_w^2)/\kappa \ln(R_c/R_w). \quad (3)$$

This formula is the fundamental equation describing this newly discovered mode of vortex motion [7].

We can also understand Eq. (3) from energy considerations. There is some critical angular velocity of the container ω_c at which the free energy of the system is lowered (relative to the vortex-free state) by the presence of a single quantum of circulation around the wire [3]. At this angular velocity the free energy $F = E - \omega L$ is zero, so $\omega_c = E_1/L_1$. Here E_1 is the kinetic energy of the fluid and L_1 is the angular momentum, both calculated assuming one unit of quantized circulation along the wire.

The precession of the filament with respect to the stationary normal fluid must be dissipative [see Eq. (5) below]. The energy reservoir which is depleted by the dissipation lies in the quantized circulation trapped on the wire. As the filament precesses, the length of trapped vorticity on the wire is decreased. The trajectory is thus a helical path, as shown in Fig. 1.

The pitch of the helix is determined by the dissipation and depends on ω, the angular velocity of the container. If $\omega < \omega_c$ the length of trapped vortex decreases as the system is driven towards the vortex-free state. Conversely if $\omega > \omega_c$ the length of trapped vortex should increase as the system is driven toward the state of vorticity trapped along the entire wire. If $\omega = \omega_c$ the system is in a state of equilibrium where the helix pitch is zero. At this rotation speed the precession frequency of the filament equals that of the container and no dissipation occurs. We conclude that the precession period must be $T = 2\pi/\omega_c = 2\pi L_1/E_1$. By a straightforward calculation this gives Eq. (3).

Equation (3) can also be derived rigorously from the kinematic principle that the vortex filament moves at the local superfluid velocity. Analytic and numerical calculations based on the vortex dynamic equations confirm [8] that the configuration shown in Fig. 1 indeed undergoes solid body rotation at the period given by Eq. (3). This approach also yields the shape of the precessing filament, indicated schematically in Fig. 1.

As mentioned earlier, the wire is off axis in the cylinder by ΔR. Thus to compute the period with Eq. (3) we must use an average value for the parameter R_c^2. A first-order correction is $R_c^2 = R_{c0}^2 + (\Delta R)^2/2$, where $R_{c0} = 1.480 \pm 0.006$ mm is the measured inside diameter of the experimental cylinder and $\Delta R = 0.35 \pm 0.05$ mm (see below).

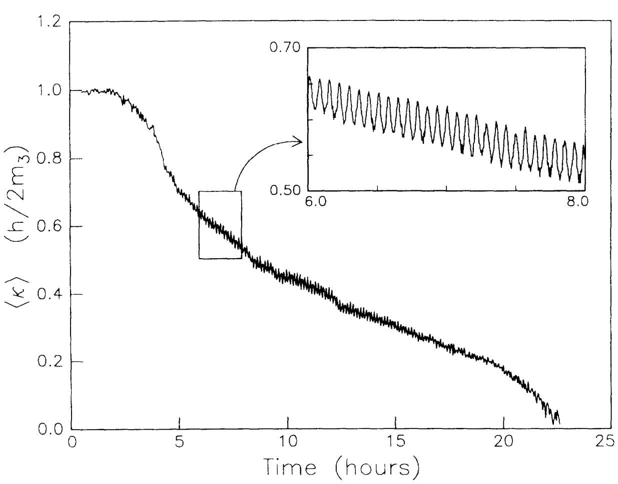

FIG. 2. Effective circulation as a function of time. Straight lines are drawn between adjacent points to help the eye. This data set was stopped when circulation had attained the baseline level, $\langle\kappa\rangle = 0$.

We have determined the average radius of the wire R_w by several methods involving measurements of the mass/length μ. Our most reliable method of finding μ is measuring the precession frequency of the wire when ^{4}He quantized circulation is trapped on it. These measurements agree with direct weighing and visual determinations. We find $R_w = 8 \pm 0.8$ μm, where the systematic error arises from an uncertainty in the precision of the manufacturer's estimate of the density.

Inserting the measured values of R_c, ΔR, and R_w into Eq. (3) predicts $T = 259 \pm 8$ s. The oscillations displayed in Fig. 2 have a measured period of 253 ± 1 s. The good agreement between experiment and theory validates Eq. (3).

This is the first precise measurement of the critical angular velocity ω_c. Other measurements of equilibrium critical velocities are hampered by effects of pinning and general metastability problems [9]. The measurement of the vortex precession period can also be used to determine the circulation around the filament. An error analysis of the complete experiment, assuming Eq. (3) is exact, yields $\kappa/(h/2m_3) = 1.02 \pm 0.03$.

The precession phenomenon is detectable because the wire is off axis. As the vortex precesses, its length changes to accommodate the distance from the wire to the cylinder's wall. When the free filament lengthens, the vortex trapped on the wire shrinks so that the total energy remains constant. Thus the precession of the free filament around the off-axis wire is accompanied by a modulation, at the same frequency, of the length of vorticity trapped on the wire. This is the mechanism responsible for the modulation in $\langle\kappa\rangle$ seen in Fig. 2. Energy conservation applied to the measured oscillation amplitude implies that $\Delta R = 0.35 \pm 0.05$ mm. This value is consistent with the fact that the wire enters the cylinder by passing along the inside surface of a hollow hermetic connecter tube of radius 0.4 mm.

We expect the amplitude of the observed modulation to reflect the position of the point of attachment, z, being largest when the position is near the center of the wire, the antinode of the fundamental vibration mode. The effective circulation $\langle\kappa\rangle$ is related to z by [3]

$$2\pi[z/\lambda - \langle\kappa\rangle/(2m_3/h)] = \sin 2\pi z/\lambda . \quad (4)$$

Here λ is the length of the wire. Figure 3 is a plot of the observed modulation amplitude $\Delta\langle\kappa\rangle$ as a function of the attachment point as deduced from Eq. (4). As expected, $\Delta\langle\kappa\rangle$ is largest when the filament is near the center of the wire, although the measured points do not follow the theoretical curve precisely.

Dissipation controls the unzipping time for the vortex transition. If this time is determined by mutual friction [10] (i.e., quasiparticles scattering off the vortex line) we can use energy arguments to relate the friction parameter

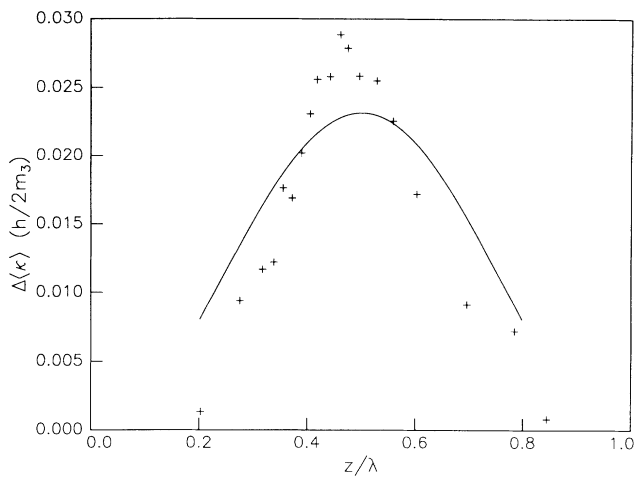

FIG. 3. Oscillation amplitude $\Delta\langle\kappa\rangle$ as a function of position of the attachment point. The solid line is the simplest theoretical curve.

D to the parameters of the experiment. This yields

$$D = \frac{F}{v_L - v_N} = \frac{3\pi\rho_s v_z R_c}{\ln(R_c/R_w)} , \quad (5)$$

where ρ_s is the superfluid density, v_z is the velocity of the point of attachment, F is a mutual friction force per unit length, and $v_L - v_N$ is the velocity of the vortex with respect to the normal component of the liquid. We find v_z to be typically 3×10^{-7} m/s. Equation (5) then suggests that D is approximately [11] 6×10^{-8} kg/m s.

A theoretical estimate [12] predicts that in the low-temperature limit (at a pressure of 14 bars) $D \sim (7.7 \times 10^{-5})(\Delta/kT)^{1/2} e^{-\Delta/kT}$ kg/m s. This expression agrees with the value derived from Eq. (5) when $\Delta/kT = 8.2$.

Our thermometry used a platinum NMR probe which was not in good thermal contact with the liquid in the cylinder at temperatures below $T/T_c = 0.3$. From the increase in the Q of the vibrating wire below the temperature at which the Pt thermometer saturated, we believe that the liquid in the cylinder was near $0.2T/T_c$. At this temperature $\Delta/kT \sim 10$, which is in reasonable agreement with the value deduced above. However, this may be fortuitous.

At the low temperatures of this experiment the Q factor of the wire should scale [13] as $e^{-\Delta/kT}$. In two instances where v_z changed by a factor of 5, there was no corresponding change in the recorded Q factor of the wire. This suggests that energy loss in the system, either for the wire itself or for the precessing filament, is not due solely to bulk scattering processes.

We have also tested this vibrating-wire apparatus with superfluid ^{4}He. In ^{4}He, the $N = 1$ state does not leave the wire while the apparatus is stationary, for times as long as 12 h. Transitions from $N = 2$ to $N = 1$ do occur, but we detect no oscillations in $\langle\kappa\rangle$ and the transition time, at temperatures where the wire's Q factor is comparable to that in the ^{3}He experiment, is about 2 orders of magnitude shorter than in ^{3}He. We suspect that in ^{4}He the vortex filament becomes strongly pinned to rough spots on the wall, which inhibits the precession motion. The vortex then becomes highly distorted as in a vortex mill [14], eventually being torn off the wire.

To complete our discussion there are several points worth mentioning. The precession occurs with respect to an absolute inertial frame, i.e., the fixed stars. Simultaneous rotation of the cylinder with respect to that frame should decrease or increase the observed period, depending on the sense of rotation. Rotating the cylinder should also cause the dissipation rate to vary, eventually driving the vortex back onto the wire if the cylinder's rotation is in the same direction as, but greater than, the precession rate of the vortex filament. The precession, depending as it does on the circulation quantum and the physical dimensions of the container, is in some sense a quantum pendulum where the restoring force is determined by $h/2m_3$. The precession is quite slow, completing only about two cycles in the time required to read this paper.

In conclusion, we have discovered a novel precessional motion of a vortex filament connected to a wire. We are able to predict accurately the precession frequency and modulation amplitude. The phenomenon provides a new measurement of the quantum of circulation in ^{3}He-B and of the critical velocity for vortex stability. The phenomenon could possibly be used to determine the mutual friction coefficient at low temperatures.

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