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QUASIPARTICLE MEAN FREE PATH AND POISEUILLE FLOW IN NORMAL LIQUID ^{3}He

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Poiseulle flow experiments in normal ^{3}He at saturated vapor pressure giving direct evidence of the effect of quasiparticle mean free path are presented. Down to 1.5 mK the data are consistent with the simple first-order correction due to the mean free path.

Several recent experiments,[1,2,3] in both the normal and superfluid phases of liquid ^{3}He have required inclusion of the influence of long quasiparticle mean free paths in the interpretation of the results. Below about 10 mK this mean free path (mfp), growing as T^{-2}, can easily become nonnegligible compared to relevant experimental dimensions. The experiments described here[4] present a direct measurement of the influence of the non-zero mfp on ordinary Poiseuille flow in liquid ^{3}He, the data yielding values for both the viscosity η, and first-order mfp correction.

In our experiment to study the mpf-induced departure from ordinary Poiseuille flow we employ a U-tube geometry consisting of two identical reservoirs of cross-sectional area A connected by a cylindrical flow tube of diameter d. A small level difference is initially established and subsequently released, the levels relaxing under the influence of gravity. Since the system is heavily overdamped the relaxation is exponential with time constant[4]

$$\tau^{-1} = (2\rho g/ZA\eta)(1 + 8c\lambda/d) \qquad (1)$$

where ρ is the fluid density, g the acceleration due to gravity, and Z the geometric flow impedance. The factor $1 + 8c\lambda d$ represents the first-order correction due to the mfp λ. The constant c is the ratio of the so-called slip length ξ, to λ. It has been estimated[5] to be 0.58, this value coming from approximate solutions to the Boltzmann equation. This correction term essentially changes the flow boundary condition to allow a non-zero velocity at the tube wall. The magnitude of this "slip" velocity is the product of ξ and the gradient of the non-slip velocity profile near the wall.

The apparatus consists of several identical reservoirs connected to a central reservoir via separate cylindrical flow tubes. Figure 1 schematically illustrates one arm of this device. The details of the construction and operation of this device have been described elsewhere.[4,6] The tubes used in this experiment have diameters of 454, 354 and 252 μm with an uncertainty of about 3 μm. The first two are 1.00 cm long while the last is 0.50 cm.

The fluid reservoirs are the annular gaps of concentric cylinder capacitors, each of height 1 cm, gap 0.020 cm, inner diameter 0.518 cm, and capacitance about 7.5 pF. One capacitor serves as a liquid-level detector while the

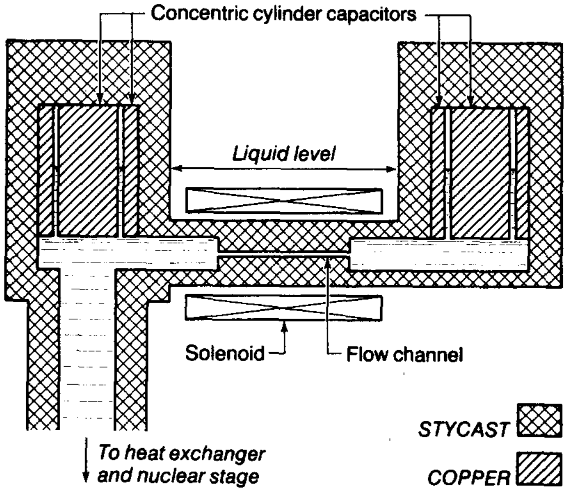

Fig. 1. Schematic of one arm of the U-tube device. The solenoid was not used in this experiment.

other is used to generate level differences which increase quadratically with applied dc potential.

The cell is thermally linked to a conventional nuclear demagnetization refrigerator. Thermometry is based on the Curie-law susceptibility of ^{195}Pt determined by pulsed NMR. We calibrate against the superfluid transition T_c where a marked change in the U-tube flow resistance occurs. We reference all our final results to the value of T_c.

The Landau Fermi-liquid theory predicts that both η and λ scale as T^{-2}. Defining $\alpha = \eta T^2$ and $\beta = \lambda T^2$ Eq. (1) becomes

$$\tau^{-1} = (2\rho g/ZA\alpha)(T^2 + 8c\beta/d) \qquad (2)$$

Thus, as a function of T^2, τ^{-1} is linear with a temperature-independent offset. Figure 2 shows, a plot of the measured inverse time constant

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versus $(T/T_c)^2$ for the three tubes. The solid lines are least-square fits to straight lines $\tau^{-1} = M(T/T_c)^2 + B$. It is clear that the first-order correction adequately describes the data, and the offset caused by the mfp is readily detectable.

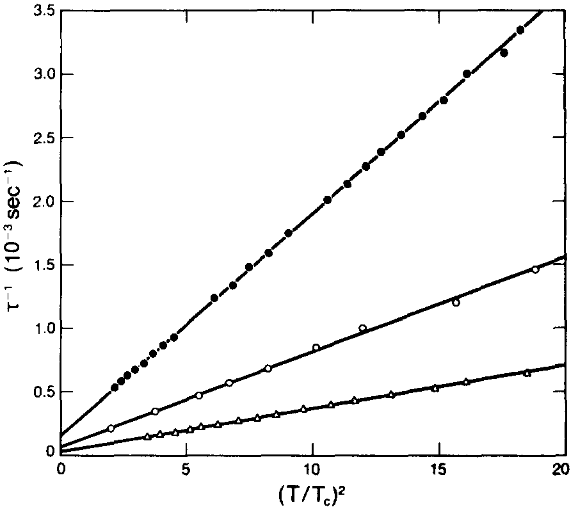

Fig. 2. Inverse time constant vs. $(T/T_c)^2$. Solid dots for 454 μm diameter tube, open dots 354 μm, open triangles 252 μm tube.

From the fitted slope M and our best estimates[6] of the flow impedance we can determine α/T_c^2, the viscosity at T_c. These values are shown in Table I along with the estimates of the total error. These error estimates include errors in tube diameters, reservoir cross-sectional area, T_c determination, and statistical errors. The weighted mean is 2.44 P and taking[7] T_c to be 1.04 mK gives $\eta T^2 = 2.64$ P-mK^2 in agreement with the previous work of Parpia et al.[1] who find 2.55 P-mK^2 at saturated vapor pressure subject to an estimated 25% systematic uncertainty.[8] We estimate[6] our systematic uncertainty in α/T_c^2 to be 5%

TABLE I. Numerical results for all three tubes

d (μm)	M (10^{-5}	B sec^{-1})	Bd/M (μm)	α/T_c^2 (P)	β/T_c^2 (μm)
454	17.52	15.38	399±19	2.47±0.10	82±4
354	7.49	6.56	310±28	2.28±0.11	67±6
252	3.32	3.75	285±23	2.64±0.14	62±5

According to Eq. (2) and the definition of the fit parameters, Bc/M should be constant for all three tubes. Table I indicates that while the results for the two smaller tubes are consistent to within error, the largest tube is not. As described previously[4] this is at least partly due to the impedance presented by the capacitor reservoirs themselves. Correcting for this effect (at most 5%) and using the value of $c = \xi/\lambda = 0.58$ as Jensen et al.[5] estimate, we can determine β/T_c^2, that is, the mean free path at T_c. These values are posted in Table I. According to Jensen et al.[5] these values should be comparable with $\lambda \simeq 90$ μm at T_c as derived from the gas-kinetic formula $\eta = p_F n\lambda/5$ and our value for η. Here p_F is the Fermi momentum and n the fluid number density.

In none of our data is there any evidence for higher-order corrections due to the mfp. In theory τ^{-1} should exhibit a "Knudsen minimum" when $\lambda \sim d$. However, in a cylindrical geometry this minimum is very shallow and actual Boltzmann equation calculations[9] place it at about $\lambda \simeq 3.3d$. For our narrowest tube (252 μm) this would imply $T \simeq 0.4$ mK, well below our lowest temperature datum of 1.8 mK.

In conclusion, we have made a direct measurement of the viscosity and first-order mean free path correction to Poiseuille flow in normal liquid ^{3}He. Presumably similar corrections must be applied to all transport measurements made in restricted geometries.[4] In particular, in the superfluid phases the rapidly increasing mfp could make such corrections of dominating importance.

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