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Simulations of Phase Slippage in an Aperture Array

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Abstract To understand the origins of synchronous and asynchronous phase slippages observed in an array of apertures connecting two reservoirs of superfluid ^{4}He, we have investigated the role of thermal fluctuations in the critical velocity and the possible effects of having an array rather than a single aperture through several model simulations. The results are compared with recent experiments carried out near the superfluid transition temperature with an array of apertures as well as those carried out at low temperatures with a single aperture.

Keywords Superfluid · Phase slip · Josephson junction

We remind the reader that a 2π phase slip is a fundamental superfluid dissipation mechanism wherein a quantized vortex is stochastically nucleated near some critical velocity v_c at an asperity at the walls of a channel/aperture. The vortex grows in size at the expense of the local fluid energy and moves across the path of the fluid. When the vortex has completely crossed the channel/aperture, the phase difference across that element decreases by 2π which corresponds to a loss of energy and a drop in aperture flow velocity $\Delta v_s \equiv v_{\text{slip}} = \kappa/\ell_{\text{eff}}$. Here, $\kappa = h/m_4 = 9.97 \times 10^{-8}$ m^2 s^{-1} is the circulation quantum and ℓ_{eff} is the effective length [1] of the aperture that takes the hydrodynamic edge effects into account.

During the past 20 years, single phase slip events, first observed by Avenel and Varoquaux [2], have been studied extensively at temperatures well below T_λ. Relevant energy barriers and statistical variations in the critical velocity have been measured [3, 4] and explained phenomenologically [5, 6]. However, the nature of phase slip dynamics very near T_λ is less well known.

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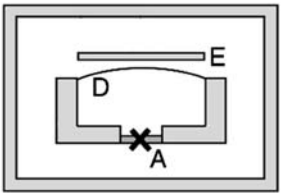
Fig. 1 Typical experimental cell schematic. D: diaphragm, E: electrode, A: aperture array (or a single aperture)



A typical experimental apparatus used to study phase slips in superfluid ^{4}He is shown in Fig. 1 and described in more detail in [7, 12, 13]. Two volumes filled with superfluid ^{4}He are separated by a diaphragm and an array of apertures (or a single aperture depending on experiments). A pressure head (and therefore a chemical potential difference) can be induced across the apertures by applying a DC step voltage between the diaphragm and the electrode thereby pulling up on the diaphragm. Motion of the diaphragm indicating fluid flow through the apertures is detected by a SQUID-based displacement sensor placed above the electrode. If the initial pull on the diaphragm is large enough, the flow velocity inside the apertures increases in time until it reaches v_c, at which point it decreases almost discontinuously by an amount v_{slip}. This process repeats, such that the flow through the apertures follows a sawtooth waveform (which can be thought of as a phase slip "oscillation").

Since vortex nucleation is a stochastic process [5], thermal fluctuations spread the critical velocity into a range Δv_c. For a single aperture, the ratio of Δv_c to the velocity drop due to a phase slip v_{slip} determines the degree of periodicity (i.e. how periodic the phase slip events are) in a phase slip oscillation driven in a way described above. As this ratio $\Delta v_c/v_{\mathrm{slip}}$ increases, phase slips become less correlated (i.e. less periodic) in time. Using single apertures (instead of an array) of size 0.042 μm × 0.12 μm, 0.18 μm × 4.2 μm, and 0.13 μm × 5.3 μm in 50 nm thick silicon nitride windows connecting two volumes of superfluid ^{4}He, Backhaus and Packard found [8] that a periodic phase slip oscillation does not exist at temperatures near 0.5 K. Although previous work [9–11] suggests that $\Delta v_c/v_{\mathrm{slip}}$ decreases with increasing temperature near T_λ, there have been few experiments done to investigate its consequences with single apertures.

In contrast, Hoskinson et al. [7, 12] recently reported a highly periodic phase slip oscillation in an array of N (=4225) apertures nominally 70 nm across, etched in a 50 nm thick membrane near the superfluid transition temperature. (Some experimental traces are shown in [12] and [13].) A further investigation [13] has revealed that the phase slip oscillation amplitude[1] which is ≈4225 times greater than that expected for a single aperture near T_λ decreases as the temperature is lowered. This indicates that phase slips occur simultaneously in all the apertures when the temperature is near T_λ but subsequently lose their simultaneity as the temperature is lowered. The exact mechanism underlying such transition is as yet unknown but possible ties to physics seen in systems such as sliding tectonic plates, magnetization of random field magnets, and disordered-pinned charge density waves have been suggested [14].

[1] "Phase slip oscillation amplitude" is a peak to peak amplitude of sawtooth mass current oscillation resulting from a series of phase slip events in an array of apertures. Experimental procedures for measuring this amplitude is described in detail in [13].

In an attempt to understand the origins of these observed phenomena as well as some differences between results involving a single aperture and an array of apertures, we have performed several model simulations. Here we describe our simulations for several possible scenarios.

We wish to address three issues: (1) Does having an array of apertures rather than a single aperture change the effect of thermal fluctuations and therefore the periodicity of phase slip oscillations? (2) Can a decreasing $\Delta v_c/v_{\text{slip}}$ (with increasing temperature) explain why a phase slip oscillation with well-defined periodicity is observed near 2 K yet not at lower temperatures? (3) Correspondingly, can the observed decline in the phase slip oscillation amplitude at lower temperatures (suggesting a loss of simultaneity in the array) in [13] be explained by an increasing $\Delta v_c/v_{\text{slip}}$?

To answer question (1), we have performed a numerical simulation of phase slip oscillation amplitude as a function of $\Delta v_c/v_{\text{slip}}$ for an array of N apertures. It is natural to expect that the exact behavior of the array should depend on the strength of (hydrodynamic or quantum mechanical) coupling that exists among the apertures, and therefore we have picked two special cases for simulation to study some key differences. In the first case, we assume that the apertures are so strongly coupled that phase slips always occur in unison among all the apertures in the array. (In this case, the whole array is essentially orchestrated by a single aperture.) In the second case, we imagine the apertures to be completely decoupled from each other and allow phase slips to occur independently in various apertures. (Here, the array behaves simply as a collection of N independent apertures.) Our simulation reveals that the effect of thermal spread in v_c is quite different for these two cases.

The simulation procedure for N (=4225) independent apertures is as follows. We first simulate the effect of thermal fluctuation by introducing a spread in critical velocity with a Gaussian distribution with a finite width that we specify about the mean value v_c. We define $P(v_s)$ to be the probability of a phase slip event taking place at a superfluid velocity v_s. As in [9], the critical velocity here is defined to be the value of v_s at $P = 1/2$, and the width of the critical velocity distribution is set to be $(dP/dv_s)^{-1}$ at v_c. At $t = 0$, we start increasing the fluid velocity in an aperture linearly in time, and we assign[2] a critical velocity $v_{c,1}$ for the first phase slip using the statistical distribution described above.

When the fluid reaches $v_{c,1}$, the velocity is made to decrease by a fixed amount v_{slip}. After such an event, a new critical velocity $v_{c,2}$ is assigned for the next phase slip based on the given distribution. The fluid accelerates again until it reaches that new value, at which point, the fluid undergoes the second phase slip causing the velocity to drop by v_{slip}. A new critical velocity $v_{c,3}$ is assigned again for the third phase slip, and the process repeats. This is done independently for 4225 apertures, and we superimpose the simulated velocity profile from all the apertures to obtain $v_s(t)$ for the whole array. Taking the Fourier transform of this signal $v_s(t)$ and analyzing the spectral content allow us to calculate the phase slip oscillation amplitude [13].

This gives us one point in the simulated oscillation amplitude versus $\Delta v_c/v_{\text{slip}}$ plot. We then repeat this entire process for different values of $\Delta v_c/v_{\text{slip}}$. To observe the effect of stochastic Δv_c alone, the mean critical velocity v_c has been assumed

[2] In this simulation, we have set the mean value to be $\sim$1 m/sec using the data of [13].

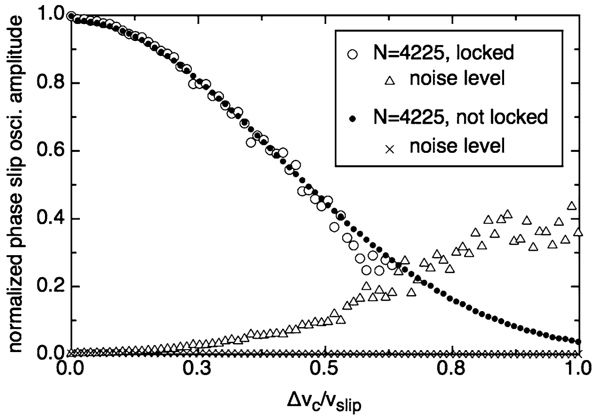


Fig. 2 Simulated phase slip oscillation amplitude vs. $\Delta v_c/v_{\rm slip}$. "Locked" means apertures are forced to slip together. "Not locked" means that phase slips can occur independently in various apertures. A locked array behaves just like a single aperture

to be the same for all the apertures. For the case of 4225 locked apertures, we simply force all the apertures to slip whenever the critical velocity is reached in some aperture.

Simulated phase slip oscillation amplitude as a function of $\Delta v_c/v_{\rm slip}$ is shown in Fig. 2. It has been normalized so that 1 is the expected amplitude for a phase slip oscillation with no spread in v_c. Open circles show the expected amplitude for 4225 apertures that are forced to slip together, and filled circles show the expected amplitude for the same number of apertures that are independent of each other. Triangle points indicate the background noise for locked apertures, and the crosses near the bottom of y-axis show the same for independent apertures. The background noise is obtained by measuring the power/Hz next to the oscillation frequency peak in the power spectrum, multiplying that by the frequency width used to measure the power in the peak, and converting that into velocity oscillation amplitude units. This particular simulation was done for the oscillation frequency of 90 Hz, with 10 kHz sampling, and the frequency resolution of 1.43 Hz. These parameters were chosen to mimic the experimental setup of [13].

What is common in the two cases simulated here is the fact that phase slip events become less and less periodic in time in a given aperture as $\Delta v_c/v_{\rm slip}$ increases. Therefore the phase slip oscillation amplitude (which is obtained from measuring the power in the peak at the oscillation frequency in Fourier space) decreases as a function of $\Delta v_c/v_{\rm slip}$. When all the apertures are required to slip together (i.e. apertures are locked due to strong coupling), this loss of periodicity in phase slip events causes the power that was in the Fourier peak at the oscillation frequency to spread out into a wider frequency band. The "background noise" then increases and it eventually washes out the decreasing peak when $\Delta v_c/v_{\rm slip} \sim 0.7$. Since the whole array is orchestrated by one aperture, this behavior is exactly the same as what one would expect from a single aperture [15]. On the other hand, when the phase slips are allowed to occur independently in various apertures (i.e. apertures are not coupled at all), the background noise seems to be reduced by a factor of $\sqrt{N}$. In the array where $N = 4225$, this factor of $\sqrt{N}$ (=65) is rather large, and this keeps the oscillation visible even when $\Delta v_c/v_{\rm slip}$ is much bigger than 0.7.

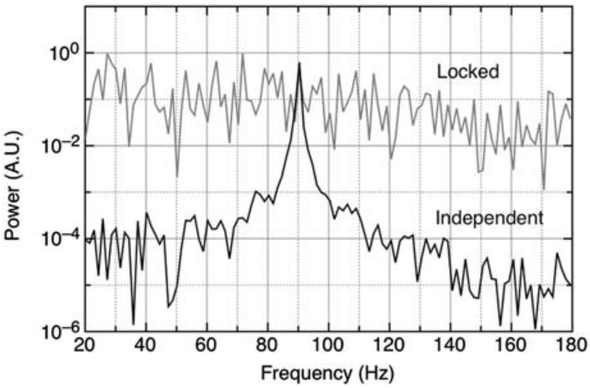


Fig. 3 The Fourier transform of phase slip oscillation when $\Delta v_c/v_{\text{slip}} = 0.7$. Two curves are for 4225 locked apertures and 4225 independent apertures

The Fourier transforms of $v_s(t)$ when $\Delta v_c/v_{\text{slip}} = 0.7$ are shown in Fig. 3 for the two cases. The Fourier peak at the oscillation frequency is lost in the noise if we lock all the apertures and make the array single-aperture-like. However, if we keep the apertures completely independent of each other, the peak becomes visible. What this shows is that if phase slips are forced to always occur simultaneously in all the apertures due to some strong coupling, then an array will be affected by thermal fluctuation just as much as a single aperture would. On the other hand, if phase slips do not have to occur simultaneously in various apertures because they are very weakly coupled, having an array of them actually suppresses the effect of Δv_c and allows one to observe a phase slip oscillation at a well-defined frequency even if it might be unobservable in a single aperture.

As intriguing as this result may be, we do not know how strong the coupling among the apertures is. A hydrodynamic or quantum mechanical coupling among the apertures might be such that the behavior of the array is somewhere between the two special cases studied here. The fact that the phase slip oscillation amplitude decreases in the data of [13] suggests that phase slips might not be occurring simultaneously in all the apertures at low temperatures, and that could mean the coupling does not always force all the apertures to slip together. However, to gain a full picture, we really need a robust model of how a phase slip in a given aperture should or should not affect the rest of the array [14].

To answer question (2) ("Can the existence of highly periodic phase slip oscillation observed in an array of apertures near 2 K and nonexistence of such oscillation in a single aperture below 1 K be explained by the temperature dependence of $\Delta v_c/v_{\text{slip}}$?"), we use the data presented in [9] to find $\Delta v_c(T)$. For the array experiments of Hoskinson et al., we use $v_{\text{slip}} \sim 1$ m/sec found in [13] to see how $\Delta v_c/v_{\text{slip}}$ varies with temperature. (Apertures used in [12] and [13] are ∼30 nm in diameter and are fabricated in 50 nm thick Si–N chip.) The calculated value of $\Delta v_c/v_{\text{slip}}$ as a function of temperature for these apertures is shown in Fig. 4. For comparison, we plot what we find for the experiments of Backhaus et al. To find v_{slip} for the three different apertures used in their experiments, we use $v_{\text{slip}} = \kappa/l_{\text{eff}}$ with $l_{\text{eff}} = l + 8D/3\pi$ where l is the thickness of silicon nitride membrane and D is the diameter of an aperture. This formalism [1] corrects for the hydrodynamic edge effects (from flow field at the ends of a channel/aperture) by scaling the aperture length according to its diameter.

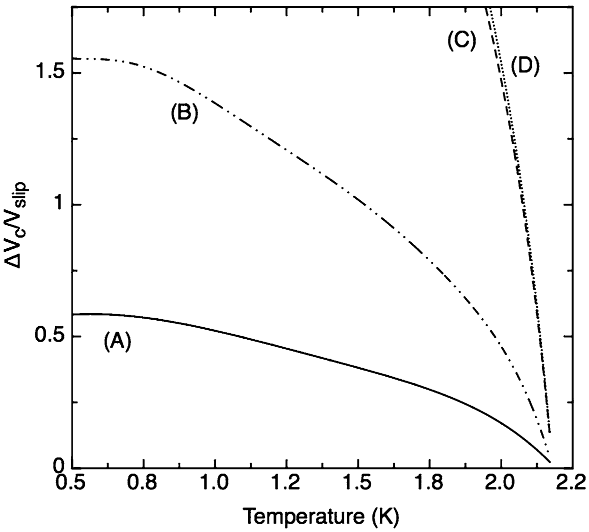


Fig. 4 $\Delta v_c/v_{\mathrm{slip}}$ as a function of temperature. (A) is for apertures used in experiments of Hoskinson et al. [12, 13]. (Aperture sizes are ∼30 nm in diameter and they are made in a 50 nm thick Si–N chip.) (B), (C), and (D) are for those used in the experiments of Backhaus et al. [8]. (Aperture sizes are (B): 0.42 μm × 0.12 μm, (C): 0.18 μm × 4.2 μm, and (D): 0.13 μm × 5.3 μm. They are fabricated in a 50 nm thick Si–N chips.) Δv_c as a function of temperature was obtained from [9]

If the apertures used in [15] are treated as circles whose radii give the same areas as the apertures, their effective lengths come out to be 265 nm, 883 nm, and 845 nm. The same analysis gives $l_{\mathrm{eff}} \sim 75$ nm for the apertures of Hoskinson et al. The effect of having smaller l_{eff} (and therefore bigger v_{slip}) is apparent in the result.

Our simulation shown in Fig. 2 indicates that for single apertures $\Delta v_c/v_{\mathrm{slip}}$ should be kept below 0.7 to observe a phase slip oscillation with some sort of periodicity. For the apertures used in experiments of Hoskinson et al., this condition seems to be always satisfied and the degree of periodicity is very high above 2 K. In contrast, for the apertures used in experiments of Backhaus et al., one would have to get very close to T_λ to have a chance of seeing phase slips highly correlated in time.

This explains why phase slips were found to be completely uncorrelated in the experiments of Backhaus et al. which were done at temperatures below 1 K. This result suggests that if they had gone very close to the transition temperature, they might have been able to observe a phase slip oscillation with a well-defined frequency. However, it is possible that the system might have transitioned into a Josephson regime [12] where a sinusoidal current-phase relation eliminates dissipative phase slips. On a similar note, this result also suggests that Hoskinson et al. might have been able to see a phase slip oscillation with a well-defined frequency even with a single aperture. However, one could argue here that the mass current though a single aperture of such small size would have been too small to detect.

To answer question (3) ("Can the observed decline in the phase slip oscillation amplitude at lower temperatures in [13] be explained by an increasing $\Delta v_c/v_{\mathrm{slip}}$?"), we have used the curve (A) in Fig. 4 to turn Fig. 2 into a plot of simulated oscillation amplitude as a function of temperature. For comparison, we present the experimental data from [13]. Figure 5a shows the measured mass current oscillation amplitude I_{slip} along with the expected value for the fully synchronous case I^N_{slip}. Background noise is calculated the same way as it is for the simulation. The experimental data shows that the phase slips occur simultaneously among different apertures when $T_\lambda - T \approx 10$ mK but they lose their simultaneity as the temperature is

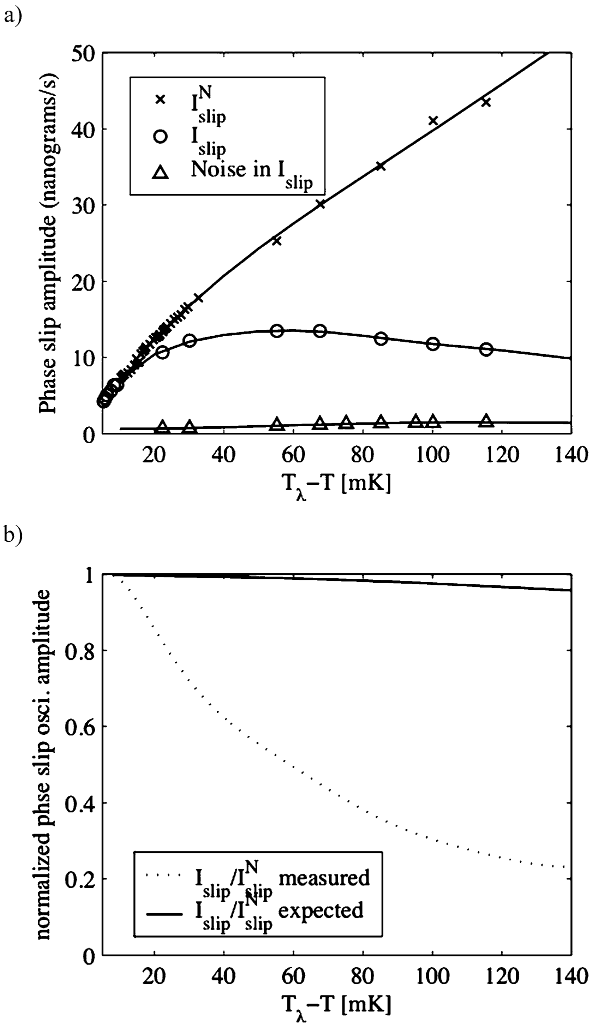


Fig. 5 **a** Measured phase slip oscillation amplitude (see footnote 1) and the noise in the peak. The temperature dependence for the expected value I^N_{slip} comes from the increasing superfluid density with decreasing temperature. The *dotted line* in (**b**) is the normalized phase slip oscillation amplitude which was obtained using the fits to the data shown in (**a**). A *solid line* (**b**) is the normalized oscillation amplitude (simulated) which takes the effect of thermal width in v_c

lowered. A dotted line in Fig. 5b is $I_{\mathrm{slip}}/I^N_{\mathrm{slip}}$, which is an observed oscillation amplitude normalized so that the expected value I^N_{slip} is taken to be 1. (Fitted curves to the data in Fig. 5a were used to produce this plot.) A solid line in Fig. 5b is the simulated $I_{\mathrm{slip}}/I^N_{\mathrm{slip}}$ with thermal fluctuation in v_c.

This simulation reveals that the thermal width in the critical velocity plays a relatively small role with the apertures of [12] and [13] if the phase slip oscillation is driven above 2 K. There is a slight decrease in the oscillation amplitude, which comes from a slight loss of periodicity in individual apertures. However, the effect is not as drastic as the measured decline shown in this plot. Therefore, the observed variation in the phase slip oscillation amplitude is more likely to be due to some other effect that is more dominating in this particular temperature regime. As suggested in [13], one such candidate is the superfluid healing length which changes from ∼10 nm to ∼1.5 nm in the temperature range plotted in Fig. 5. (The superfluid healing length decreases with decreasing temperature.) As the healing length shortens and more nanometer size asperities in aperture walls become exposed, different apertures might start to have different critical velocities. That can bring the overall oscillation amplitude down for the whole array while maintaining well-defined periodic oscillations in individual apertures.

In summary, to understand the existence of a highly periodic phase slip oscillation near 2 K and the decline of the oscillation amplitude with decreasing temperature, we explored the role of thermal fluctuation in the critical velocity and the effect of having an array of apertures rather than a single aperture. Using a numerical simulation, we find that the role that Δv_c plays in the single aperture experiments of Backhaus et al. is large, but it is negligible in the array experiments of Hoskinson et al. This difference is a consequence of different aperture dimensions used in those experiments and has nothing to do with the existence of an array. The simulation reveals that the effect of thermal fluctuation is not large enough to explain the observed decline in the phase slip oscillation amplitude seen in the data of [13]. As for the array dynamics, we find that if the apertures are independent of each other, having many apertures can further suppress the effect of Δv_c. However, it still remains a mystery if such an independent behavior is allowed in a quantum mechanical system with a single macroscopic wavefunction governing the whole array. A more robust model of how a phase slip in a given aperture should or should not affect the rest of the array is needed to fully understand this matter [14]. We hope this study will shed some light on the possible mechanisms underlying the observed phenomena and the nature of phase slip dynamics in a multiply connected geometry.

References

1. P.M. Morse, *Vibration and Sound, Acoustical Society of America*. American Institute of Physics (1981)
2. O. Avenel, E. Varoquaux, Phys. Rev. Lett. **55**, 2704 (1985)
3. J. Steinhauer, K. Schwab, Y. Mukharsky, J.C. Davis, R.E. Packard, Phys. Rev. Lett. **74**, 5056 (1995)
4. J. Steinhauer, S. Backhaus, R.E. Packard, Phys. Rev. B **52**, 9654 (1995)
5. E. Varoquaux, O. Avenel, Physica B **197**, 306 (1994)
6. E. Varoquaux, M.W. Meisel, O. Avenel, Phys. Rev. Lett. **57**, 2291 (1986)
7. E. Hoskinson, R.E. Packard, T.M. Haard, Nature **433**, 376 (2005)
8. S. Backhaus, R.E. Packard, Phys. Rev. Lett. **81**, 1893 (1998)
9. W. Zimmermann Jr., C.A. Lindensmith, J.A. Flaten, J. Low Temp. Phys. **110**, 497 (1998)
10. C.A. Lindensmith, J.A. Flaten, W. Zimmermann Jr., in *Proceedings of the 21st International Conference on Low Temperature Physics*, Prague, 1996, ed. by S. Danis, V. Gregor, K. Zaveta, p. 131
11. E. Varoquaux, O. Avenel, Y. Mukharsky, P. Hakonen, in *Quantized Vortex Dynamics and Superfluid Turbulence*, vol. 571, ed. by C.F. Barenghi, R.J. Donnelly, W.F. Vinen (Springer, Berlin, 2001), p. 36
12. E. Hoskinson, Y. Sato, I. Hahn, R.E. Packard, Nat. Phys. **2**, 23 (2006)
13. Y. Sato, E. Hoskinson, R.E. Packard, Phys. Rev. B **74**, 144502 (2006)
14. D. Pekker, R. Barankov, P.M. Goldbart, Phys. Rev. Lett. **98**, 175301 (2007)
15. S. Backhaus, Ph.D. thesis, University of California at Berkeley, 1997