

nal we obtain values of $\chi_B''(\omega_0)/\chi_{\text{Fliq}}''(\omega_0)$ as high as 0.51, and consider it likely that at higher fields our measurements would be consistent with a BW curve properly corrected for all Fermi-liquid effects.

Our estimates of the surface pinning, bending, and field orientational energies in the BW state show that the surface and bending energies dominate the field energy below about 100 Oe and we therefore expect that θ , the angle between \hat{n} and \hat{H}_0 discussed above, should increase in a gentle arc from zero at the center of the bore to some fraction of $\frac{1}{2}\pi$ at the walls. Such a model is entirely consistent with all our observations, including the broad absorption in the B -longitudinal-resonance experiments and the smeared shape of the transverse- B -resonance signal.

The authors wish to express their gratitude to P. W. Anderson for his many helpful suggestions, to E. I. Blount and C. C. Grimes for their help in interpretation, to K. B. Jefferts for his help

with the electronics, and to W. O. Sprenger for his technical assistance. We also wish to thank R. C. Richardson and D. M. Lee for sharing their unpublished NMR results with us.

¹D. D. Osheroff, W. J. Gully, R. C. Richardson, and D. M. Lee, *Phys. Rev. Lett.* **29**, 920 (1972).

²A. J. Leggett, *Phys. Rev. Lett.* **31**, 352 (1973), and to be published (see references contained therein).

³P. W. Anderson and P. Morel, *Phys. Rev.* **123**, 1911 (1961).

⁴P. W. Anderson and W. F. Brinkman, *Phys. Rev. Lett.* **30**, 1108 (1973).

⁵D. D. Osheroff, R. C. Richardson, and D. M. Lee, *Phys. Rev. Lett.* **28**, 1885 (1972).

⁶R. Balian and N. R. Werthamer, *Phys. Rev.* **131**, 1553 (1963).

⁷D. D. Osheroff, W. J. Gully, R. C. Richardson, and D. M. Lee, in *Low Temperature - LT 13*, edited by W. J. O'Sullivan, K. D. Timmerhaus, and E. F. Hammel (Plenum, New York, 1973).

⁸D. N. Paulson, R. T. Johnson, and J. C. Wheatley, *Phys. Rev. Lett.* **31**, 746 (1973).

Thickness of the Moving Superfluid Film at Temperatures below 1°K*

Gary A. Williams and Richard E. Packard
University of California, Berkeley, California 94720
 (Received 14 January 1974)

The thickness of a flowing He II film has been measured at temperatures below 1°K. The film's thickness varies quadratically with the flow velocity, in agreement with theory. The implications of this observation are discussed with regard to similar measurements performed at higher temperatures.

In recent years attempts have been made to explain an apparent violation of basic fluid mechanics applied to a moving superfluid film. This paper presents experimental results which resolve this problem. In the first part of this Letter we briefly describe the essence of the problem. Then we present our experiment and finally make some comments on the consequences of the observed phenomena.

In 1956 Kontorovich¹ showed that in thermodynamic equilibrium, the energy terms describing a flowing film of He II are linked by the equation

$$\rho_s v_s^2 / 2\rho + gz - \beta d^{-n} = 0, \quad (1)$$

where ρ_s/ρ is the superfluid fraction, v_s is the superfluid velocity, g is the gravitational acceleration, z is the height of the film above the bulk liquid's surface, n is a constant approximately equal to 3, β is a constant describing the strength of the Van der Waals interaction between the sub-

strate and the film, and d is the film thickness. This equation implies that a flowing film should be thinner than a static film. In particular for v_s not too large the film gets thinner by the amount

$$\Delta d = \rho_s d_0 v_s^2 / 2n\rho g z, \quad (2)$$

where d_0 is the thickness of the static film. Although several experimenters^{2,3} looked for this thinning effect it was not observed, and finally Keller⁴ in an elegant experiment conclusively showed that the effect was not present (at temperatures $T > 1.12^\circ\text{K}$). Several ingenious suggestions have been made to explain Keller's experiment.⁵⁻⁸ The most convincing explanation points out that the prediction of Eq. (2) leads to an instability between the vapor and the liquid at the free surface.⁸ If the film gets thinner the vapor at the interface must be at a pressure greater than the saturated vapor pressure. As

emphasized by de Bruyn Ouboter,⁸ condensation would then occur, tending to make the film thicker. Thus the maintenance of a moving thinned film would not be an equilibrium process and the equations of Kontorovich, which are based on equilibrium conditions, would not be complete. Van Spronsen *et al.*⁹ have recently measured the flowing film's thickness in a capillary 120 m long, a geometry intended to inhibit vapor flow. They find the film thinned, in agreement with Eq. (2).

If condensation from the vapor is indeed the mechanism which inhibits the film thinning, one should measure the flowing film's thickness at temperatures so low that vapor effects would be negligible. The remainder of this paper describes such an experiment.

To measure the film thickness we attach a cell, similar to Keller's, to the mixing chamber of a dilution refrigerator. Figure 1 shows the basic cell which consists of two concentric annular regions 1 and 2. At the top of the inner annular region 1 is a cylindrical capacitor C1 with the following dimensions: height $H = 0.33$ cm, diameter $D = 1.45$ cm, and plate separation $S = 0.076$ mm. The walls of region 2 form the plates of a second cylindrical capacitor C2 with dimensions $H = 1.05$ cm, $D = 2.71$ cm, $S = 0.50$ mm. The annular regions are partially filled with liquid helium which enters the cell through the capillary tube at the top of the cell. The Bakelite umbrella U ensures that no drops fall directly on the narrow gap of C1. If this is not done, C1 will completely fill with liquid, held in place by surface tension. Region 1 fills via film flow from region 2. Wherever possible, parts are made from copper to ensure uniform temperature throughout the cell. Temperatures were measured using a calibrated Speer resistor.

C1 and C2 are simultaneously measured with independent capacitance bridges (General Radio No. 1615-A). Because of the finite dielectric constant of liquid helium,¹⁰ a measurement of C2 yields the liquid level in region 2 and a measurement of C1 gives the average film thickness in C1. Using a phase-sensitive detector we could resolve a change in film thickness of 2 \AA with a time constant of 300 msec. Flow was induced from region 1 to region 2 by applying a dc potential across C2. The electric field in C2 decreases the hydrostatic pressure in the liquid and causes the fluid in C2 to rise to a higher level as liquid flows out of region 1 via the film (at a constant critical velocity). The inertia of

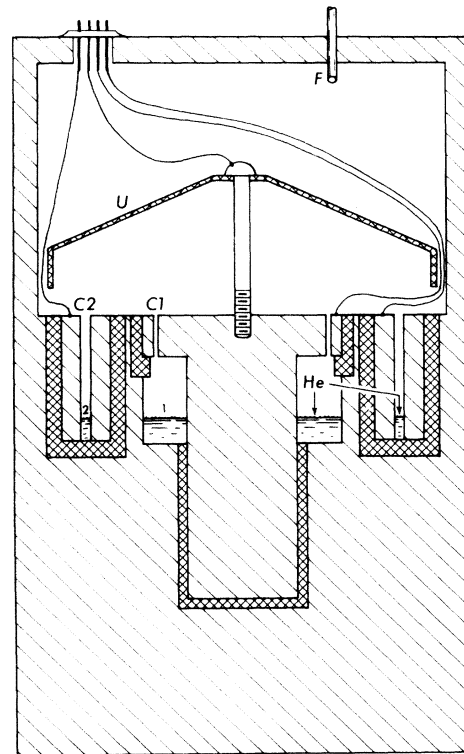


FIG. 1. A cross sectional view of the film-flow apparatus described in the text. The cross-hatched areas are Bakelite and all other materials are metal.

the moving superfluid causes the level in C2 to overshoot the equilibrium height and the system exhibits weakly damped U-tube oscillations. According to Eq. (2) the film thickness should at first decrease while the fluid flows into C2. Furthermore, as the levels in regions 1 and 2 oscillate at frequency ω , the film thickness, which depends on v_s^2 , should vary periodically at twice this frequency.

This is exactly the behavior we observe. Figure 2 is a typical recording showing both the level in region 2 and the film thickness in C1 as a function of time. At $t = 0$, a 500-V bias is removed from C2. The level in C2 at first decreases linearly, with a critical velocity of 28 cm/sec. During this period of constant flow, the film gets thinner by 56 \AA and then remains constant. When the flow rate slows, the film rapidly gets thicker, returning to its original thickness at the instant the flow into C2 stops. As U-tube oscillations progress we see the film thickness change at twice the frequency, the film reaching maximum thickness at points of zero flow. There is an asymmetry in the thickness

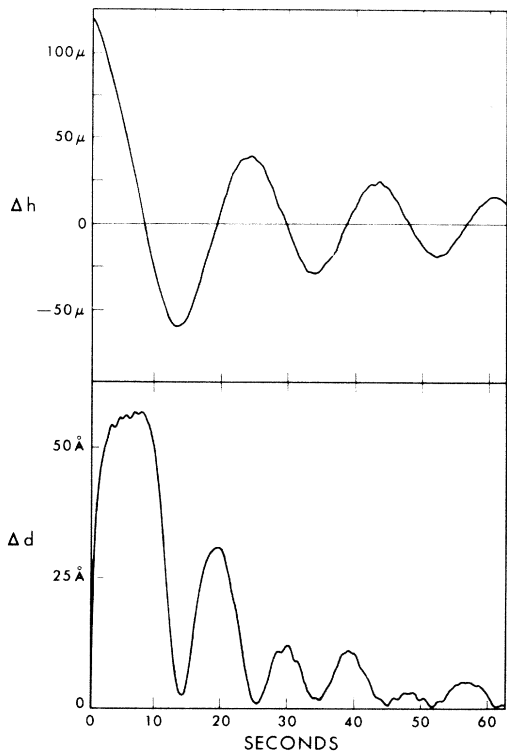


FIG. 2. Typical recording displaying simultaneously the decrease in film thickness Δd and the liquid level Δh in capacitor C2. The temperature during this trace was $T = 0.26^\circ\text{K}$.

change with respect to flow direction that we do not understand. As seen in Fig. 2, the thickness change is much less when the U-tube oscillation flow is in the same direction as the preceding critical flow.

The film velocity v_s in C1 is calculated from the slope of C2 versus time and the known cell geometry and measured film thickness. The static film thickness d_0 was measured by observing the change in C1 during our initial filling process. (For this cell with $z \approx 0.8$ cm the thickness was $d_0 = 630 \text{ \AA}$.) Inserting this value of v_s in Eq. (2) we calculate the expected film thinning, $(\Delta d)_{\text{calc}}$. Changes in C1 give us an observed value $(\Delta d)_{\text{obs}}$. From 25 filling sequences we find that for the constant flow period $(\Delta d)_{\text{calc}} \approx (\Delta d)_{\text{obs}}$ within 30%. From measurements made during the weakly damped U-tube oscillations we find $(\Delta d)_{\text{obs}}$ varies quadratically with v_s for flow in a given direction. These observations verify Eq. (2) in the low-vapor-pressure regime. The agreement between $(\Delta d)_{\text{calc}}$ and $(\Delta d)_{\text{obs}}$ is not always consistent, differing (infrequently) by as

much as a factor of 2. One possibility is that there may have been fluid reservoirs, other than region 1, which could contribute to the observed level rise in 2 and lead to an overestimate of the film velocity v_s . Differential contraction between the copper and Bakelite could open cracks of sufficient volume for this to occur. Another possible source of inconsistency arises from the observation of Keller and Hammel¹¹ who have shown that the region of flow dissipation is not localized but spread out over the surface of the inner reservoir and that the flow is in general not reproducible. Equation (2) is not strictly valid in the presence of vorticity or dissipative flow.

We observed the film-thinning phenomenon in the range $0.26 < T < 0.9$ and found no apparent temperature dependence. We attempted to observe this phenomenon at higher temperatures but electrical breakdown (presumably in the vapor) precluded the use of bias voltages large enough to observe the effect.¹² Our observations, coupled with Keller's, support the view of de Bruyn Ouboter⁸ that the free surface of He II, in the high-vapor-pressure regime, will *not* depend on the velocity field v_s . This has several consequences.

In particular the meniscus of rotating He II will have a shape which depends on vapor pressure as well as v_s .¹³ In this case the relevant force is gravity rather than the Van der Waals force, and so the effect will not be as pronounced as in films.

Another comment is that the previously not understood observation of Bianconi and Maraviglia¹⁴ may be explained by our results.¹⁵ These authors, investigating U-tube oscillations using ions as a probe of the moving film, reported observing currents which had a square-law dependence on v_s . Electrons can form trapped states outside a helium film. The dependence of film thickness on v_s would move the trapped charge near a substrate collector in a manner which would produce the current observed by Ref. 14. Indeed, trapped electrons may provide a sensitive probe of film thickness.

A final comment is that in superfluid film flow above 1°K , condensing vapor would provide a dissipative mechanism.⁸ This would prevent the observation of persistent currents in films, as found experimentally.¹⁶ This may be the explanation of this previously not understood phenomenon. We will be testing this hypothesis soon.

In summary we restate that Eq. (2) is found

valid at low vapor pressures. The surface contour calculated by Kontorovich is a nonequilibrium state which is detectable only in the limit of vanishing vapor.

We gratefully acknowledge helpful conversations with K. de Conde, D. Goodstein, S. Ma, S. Putterman, F. Reif, I. Rudnick, and T. M. Sanders. We thank P. L. Richards for the generous loan of his capacitance bridge.

*Work supported by the National Science Foundation.

¹V. M. Kontorovich, Zh. Eksp. Teor. Fiz. **30**, 805 (1956) [Sov. Phys. JETP **3**, 770 (1956)].

²P. W. F. Gribbon and L. C. Jackson, Can. J. Phys. **41**, 1047 (1963).

³K. A. Pickar and K. R. Atkins, Phys. Rev. **178**, 389 (1969).

⁴W. E. Keller, Phys. Rev. Lett. **24**, 569 (1970).

⁵E. R. Huggins, Phys. Rev. Lett. **24**, 573 (1970).

⁶D. L. Goodstein and P. G. Saffman, Proc. Roy. Soc., Ser. A **325**, 447 (1971).

⁷S. J. Putterman and I. Rudnick, Phys. Today **24**, No. 8, 39 (1971).

⁸R. de Bruyn Ouboter, J. Low Temp. Phys. **12**, 3

(1973).

⁹E. Van Spronsen, H. J. Verbeek, R. de Bruyn Ouboter, K. W. Taconis, and H. Van Beelen, Phys. Lett. **45A**, 49 (1973).

¹⁰C. E. Chase, E. Maxwell, and W. E. Millett, Physica (Utrecht) **27**, 1129 (1961).

¹¹W. E. Keller and E. F. Hammel, Phys. Rev. Lett. **17**, 998 (1966).

¹²Taking into account the excess flux of atoms striking the thinned-film surface, we calculate a characteristic time to reach a steady-state thickness,

$$\tau = (\rho_l d_0 / 3\gamma \rho_v g z) (2\pi kT/m)^{1/2},$$

where γ is the accommodation coefficient, ρ_l is the liquid density, ρ_v is the vapor density, k is Boltzmann's constant, and m is the mass of a helium atom. At $T = 1.0^\circ\text{K}$, $\tau \approx 1$ sec.

¹³Presumably this would also apply to a recent calculation of the free surface near a vortex line by K. C. Harvey and A. L. Fetter, J. Low Temp. Phys. **11**, 473 (1973).

¹⁴A. Bianconi and B. Maraviglia, J. Low Temp. Phys. **1**, 201 (1969).

¹⁵This point was brought to our attention by Professor T. M. Sanders.

¹⁶F. Wagner, J. Low Temp. Phys. **13**, 185 (1973).

Amplitudes and Level Spacings of Bound States in Superconducting Cd†

S. L. Colucci, W. J. Tomasch, and Hyung Joon Lee

Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556

(Received 28 January 1974)

Bound- and virtual-state structure has been observed in the tunneling characteristics of thick Cd films deposited over Pb. We observed amplitude effects displaying a temperature dependence not previously reported. An important point is made concerning basic spacings of bound and virtual levels. Amplitudes and level spacings appear to contain retrievable proximity-effect information. Taking nonlinear effects into account, a value of $v_F = 1.30 \times 10^8$ cm/sec is obtained for the c direction.

Oscillatory structure in the tunneling density of states $\rho_S(\omega)$ of thick ($d_1 \approx 1 \mu\text{m}$) superconducting films (M_1) results from interference between degenerate quasiparticle states coupled by a pairing-potential perturbation ($\delta\Delta = \Delta_2 - \Delta_1$) due to the proximity of a thinner second metal (M_2).¹⁻⁷ This Letter deals with the case in which M_2 is a superconductor with energy gap $2\Delta_2 \gg 2\Delta_1 \geq 0$. A quasiparticle wave incident upon the interface M_1/M_2 suffers partial reflection when it has energy $\omega > \Delta_2$ and total reflection when $\omega < \Delta_2$ (no states available in M_2). In either event, interference produces standing-wave resonances associated with energies $\omega = \omega_n$. For sufficiently long mean free paths $l_1 = d_1/\gamma$, resonances contribute

peaks to ρ_S corresponding to bound-state ($\omega < \Delta_2$) and virtual-state ($\omega > \Delta_2$) levels. For $\gamma \gg 1$, a reasonably detailed theory due to Wolfram⁵ predicts an average level separation $\langle \delta\omega \rangle \approx hv_F/2d_1$, where v_F is the renormalized Fermi velocity. This is the same result as for virtual states when $\delta\Delta < 0$ and $\gamma \sim 1$.^{3,5,6} Leaving this limit, however, Wolfram predicts a strong harmonic contribution which adds a second series of peaks ($\omega < \Delta_2$, $\omega > \Delta_2$), so that $\langle \delta\omega \rangle \approx hv_F/4d_1$.

Early attempts to observe bound levels ($\Delta_1 > 0$) employed In (M_1) and Pb (M_2).⁷ These yielded very weak structure, possibly because of interdiffusion of In and Pb. In at least one instance, however, stronger structure was observed re-