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VORTEX PHOTOGRAPHY IN LIQUID HELIUM

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The study of quantized vortex lines has been advanced by the technique of vortex photography. This paper provides an overview of the information which has been acquired thus far from this method. Stationary vortex states are observed which are very similar to those predicted from theory. In addition types of dynamical phenomena are observed, such as collective oscillations and the acceleration dependence of vortex creation.

The purpose of this paper is to give a concise survey of vortex photography and its application to studies of both equilibrium and nonequilibrium vortex dynamics. The paper is organized as follows.

(1) A brief description of the theoretical structure of vortices in liquid helium [1].

(2) A summary of several different techniques that have been considered to make the quantized vortices visible.

(3) A discussion of the method of using electron bubbles as indicators of the vortex position [2]; the only technique to have produced concrete vortex images so far.

(4) A description of the kinds of information that have so far been gathered through the use of vortex photography [3].

By the late 1940s, Landau's two-fluid model of superfluid helium had gained wide acceptance as a correct phenomenological theory of the dynamics of liquid helium. Most of the then-known phenomena, such as second sound, the fountain effect, film flow and the temperature dependence of ρ_n, could be explained by Landau's work. Although Fritz London had already made the suggestion that superfluidity was a manifestation of quantum mechanics on a macroscopic scale, the fundamental constant $\hbar$ did not appear explicitly in Landau's formalism. In an attempt to develop London's ideas further, Lars Onsager, in 1949, tried to deduce what type of quantum mechanical wave function could describe a quantum fluid such as liquid helium. Proceeding from fundamental principles known in quantum theory, Onsager arrived at the conclusion that the phase of the macroscopic wave function was directly related to the flow field in the superfluid. Furthermore, he perceived that a container of rotating liquid helium should contain a distribution of vorticity in which the circulation is quantized in units of Planck's constant divided by the mass of the helium atom. His deductions were mentioned as an addendum at a low-temperature meeting and were published in a few brief lines as a supplement to *Nuovo Cimento* [4].

It is a testimony to the depth of Onsager's intellect and the obscurity of his written word that few people recognized what he had said. In fact in the early 1950s when the first experiments on rotating helium tried to detect an irrotational flow state it came as a surprise that Landau's condition, $\nabla \times v_s = 0$, did not seem to be obeyed at all. It was presumably only Onsager who knew the reason why.

In about 1955 Richard Feynman independently asked the same question about the nature of the macroscopic wave function describing liquid helium. He soon came to the conclusion that quantized line vortices should exist in that system. His paper from 1955 is really the launching pad of the field of quantized vortices [5].

Concurrent with Feynman's work, two young students at Cambridge, Henry Hall and Joe Vinen, were collaborating on an experiment to investigate the effects of rotation in liquid helium on second sound propagated in it. Since it was thought at the time that such research could yield no new information, the young men conducted the experiment while their research supervisor was out of town. They discovered that second sound was strongly attenuated when it propagated perpendicular to the rotation axis and was unattenuated when propagated along the axis. In analyzing their experiment, they made some very clever deductions about what must be going on in the superfluid helium. Before they published their ideas, Brian Pippard returned to Cambridge from a recent visit at CalTech where he had learned about Feynman's new ideas of quantized vortices. Without telling Hall and Vinen precisely what he had learned he urged them to write a paper on their work so that in the future they could decisively know what their original ideas were independent of the notion of quantized vortex lines [6]. Of course they soon recognized that their sound attenuation experiment was a verification of the existence of Onsager and Feynman's vortices. Much of the current accepted theory of vortex dynamics emerges from the foundations laid down in the theoretical work by Hall and Vinen in those early days [7].

Our current understanding of the static distribution of vortices is based on the idea of finding minima in the free energy functional of the liquid [1]. It is not difficult to show that in a cylindrical bucket of helium rotating about its symmetry axis no vortices should appear for speeds below some critical value. Above this critical angular velocity Ω_{c1} the equilibrium state should contain a single quantized vortex line located on the rotation axis. At a second critical velocity Ω_{c2} the lowest energy state of the system is that which has two singly quantized vortex lines located symmetrically in the cylinder. At higher angular velocities, a third, a fourth, and so on, vortices should appear as each state in turn becomes the state of lowest energy. Computer calculations performed at Los Alamos [8] and elsewhere have been able to predict the lowest energy configuration of vortices at quite high quantum numbers. These same calculations enable one to deduce the existence of many different metastable states which can exist at a given angular velocity. Since the barriers between all the metastable states are very large compared with thermal energies, it seems impossible to predict the precise stationary state of the helium as a function of angular velocity.

It should be noted that almost all the stationary state calculations done to date have involved strictly two-dimensional arrangements of vortices. In the real three-dimensional case, the method of images is not a useful technique to satisfy the boundary conditions. For this reason the problem of vortices which are not rectilinear, although soluble in principle, has never been completed.

It can be shown that the stable and metastable vortex patterns place the vortex cores at positions which are at rest with respect to the normal fluid [1]. Since in a rotating container the normal fluid component is in solid body motion, we draw the conclusion that the stationary vortex states are at rest in the rotating coordinate system. If the vortices are not in their stationary position, the system will dissipate energy through the mutual friction force between the vortices and the normal fluid. This force, which is just the result of the quasi-particles scattering from the normal vortex core, was introduced by Hall and Vinen in their early work [7]. If a sample of a superfluid helium is at a temperature so low that the normal fluid component is negligible, the vortices will never find the stationary states which are the local minima in the free energy.

Although a substantial number of experiments confirmed the existence of the quantized vortices [9] there remained the intriguing challenge to make the vortices in some way visible and photograph them directly. Several researchers spent time on this problem and devised some rather ingenious methods. Perhaps the first thoughts were to try to find a way to see the vortices directly with some sort of illumination. Unfortunately, the variation in the helium's index of refraction, which might be associated with the vortices, seems much too small to render the vortex cores directly visible.

Where the vortex meets a free surface a dimple will be formed in the meniscus. Although direct calculations [10] show that this dimple is very small, experimentalists have tried some very sensitive techniques to make it visible. Thus far these methods have not produced images of single vortices, although at least one experiment [11] has shown that the free surface of helium in a rotating annulus has a shape attributable to a vortex state of very high quantum number.

Another attack on this problem involved the use of some sort of neutrally buoyant solid tracer particle as a marker of the vortex core. In one experiment a mixture of solid hydrogen and deuterium formed the tracer material [12]. In another experiment small plastic microspheres would form the marker particles [13]. For various technical reasons this approach has also not been able to produce well-defined vortex photographs. Static electricity usually makes the particles coalesce.

Still another approach to this problem utilizes the interaction of second sound and vortex lines. In a clever series of experiments a group in France has succeeded in deducing several characteristics of the vortex lattice by measuring the attenuation of several different second sound normal modes in a rotating vessel [14].

My own involvement in this field started when, as a graduate student, I was seeking a research project under the supervision of Michael Sanders at the University of Michigan. At that time it was known from experiments in Rome, Chicago, and Beirut that excess electrons injected into superfluid helium would form tiny bubbles with radii approximately 15 Å. If these electron bubbles are present when vortices exist the bubbles will become attracted to the vortex core through a Bernoulli force [15]. It was known that the electrons could become trapped on the vortex lines and would remain trapped for times which became very long at temperatures below 1.5 K. It was also known that the trapped electrons were mobile in a direction along the vortex lines. It occurred to Sanders and others that the trapped electrons could possibly be used to photograph the vortex position. The basic idea was to first saturate the vortex lines with charge and subsequently apply an electric field along the vortex line to push the charge out a free surface and then accelerate it into a phosphor screen. It was hoped that each vortex line would produce a distinct spot of light on the screen.

The first experiment we attacked was to see if single vortex lines could be detected using the ion trapping technique. Previously, other investigators had been able to detect the charge from hundreds of lines. One finds empirically that only about 1000 electrons/cm can be trapped on a vortex line. Because the ion mobility is low, this is an inadequate amount of charge to be detected directly. In order to detect single vortices, a multiplying ion collector was positioned in the vapor above the liquid helium. The collector was simply a fine wire biased at a high potential and acted like a proportional counter. Due to charge multiplication in the vapor a single electron leaving the liquid surface caused about 100 ions to be collected at the wire.

To manipulate the electron bubbles in the liquid the cylindrical cell was formed by stacking several carbon resistors on the axis and drilling a hole down the center of the stack. The hole formed the cylindrical bucket and biasing the resistive walls permitted good control on the ion current.

Using this technique we could detect the presence of single vortex lines and data such as that shown in fig. 1 was produced [16]. For the first time one could see the quantum transition in the liquid helium as the system changed from a

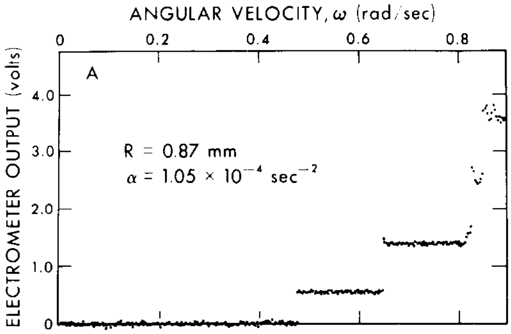

Fig. 1. A plot of charge trapped on vortices vs. angular velocity of the rotating vessel. The step-wise transitions signal the appearance of an additional vortex line.

state of no vortices present to one, two, three, etc.

I had been at Berkeley for over a year working in ultraviolet spectroscopy when Sanders telephoned me and offered to give me the rotating cryostat which I had used in my thesis research. Since at the time I found the world of vortices more fascinating than the world of UV, I readily accepted his offer and we began to think more about trying to execute the vortex photography idea.

To perform electron optics in the space above liquid helium one needs to maintain the temperature below about 0.4 K so that the helium vapor does not interfere with the electron trajectories. After constructing a rotating ^{3}He refrigerator, I was joined in the project by Gary Williams, a graduate student at Berkeley. Our initial experiments pointed out several difficulties which we had not completely anticipated. The greatest nuisance was that the electrons could only be accelerated through about 500 V before electrical breakdown occurs. The phosphor screen was deposited on the end of coherent fiber optics which led to room temperature. We found that with the 500 V accelerating potential each vortex line produced a signal at room temperature of only about 10 or 100 photons. Thus we had to construct an image intensifier system capable of recording film images of a few photons.

Although resolution tests proved that the electro-optical system could adequately map low-temperature electron patterns onto our film, the pictures which were recorded showed only a uniform blur with no distinct dots to mark the individual vortex positions. Subsequent work convinced us that undamped vortex motion was the source of the image blurring. It had been somewhat anticipated that at the temperatures at which the experiment was performed there might not be sufficient normal fluid present to damp nonequilibrium vortex motion.

To create damping at these low temperatures one can add a small amount of ^{3}He to the sample. The ^{3}He will act as a finite amount of normal fluid even at extremely low temperatures and will therefore slow random vortex motion [17]. Unfortunately, since the ^{3}He also raises the vapor pressure of the liquid a rotating dilution refrigerator had to be constructed so that the experiments could be performed at temperatures of about 50 mK; a temperature low enough so that even the ^{3}He atoms have a very small partial pressure in the vapor. All the vortex photography which has been performed in our laboratory to date has employed a dilute solution of ^{3}He in ^{4}He and a rotating dilution refrigerator.

Although the first vortex photographs were recorded directly on film [18], all of the data which will be discussed here was recorded using a low light level TV camera, a video storage device, and a single-frame movie camera. Fig. 2 schematically illustrates the experimental system. The complete apparatus, except the TV monitor and movie camera, is rotating with the dilution refrigerator. Thus the images recorded are those that an observer would see if he was at rest in the rotating coordinate system.

It takes about 10 s to fully charge the vortices and create a single vortex picture. Thus, every 10 s we can monitor the vortex pattern in the helium. Over 300 000 individual vortex pictures have been recorded so far. Most of the pictures were taken by Dr. Ed Yarmchuk and the features of the data which I will now discuss are largely due to his analysis of the data [3].

When individual photographs are examined

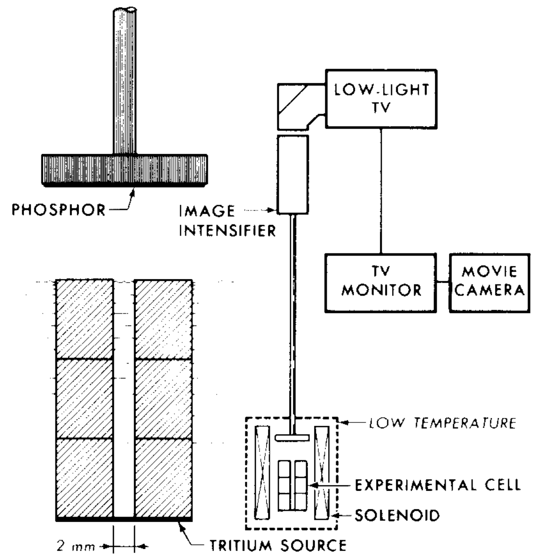

Fig. 2. A block diagram of the apparatus. The cylindrical vessel is a 2 mm diameter hole drilled in a stack of three carbon composition resistors. It is 22.7 mm in height. Voltage differences applied across each resistors section produce acial electric fields for the manipulation of the ions. A tritiated titanium foil forms the bottom surface of the vessel and serves as the ion source. A 700 V potential difference is applied between the phosphor screen and the top of the vessel for acceleration of the electrons. The solenoid produces a magnetic field of 0.5 T which prevents defocusing of the accelerated electrons.

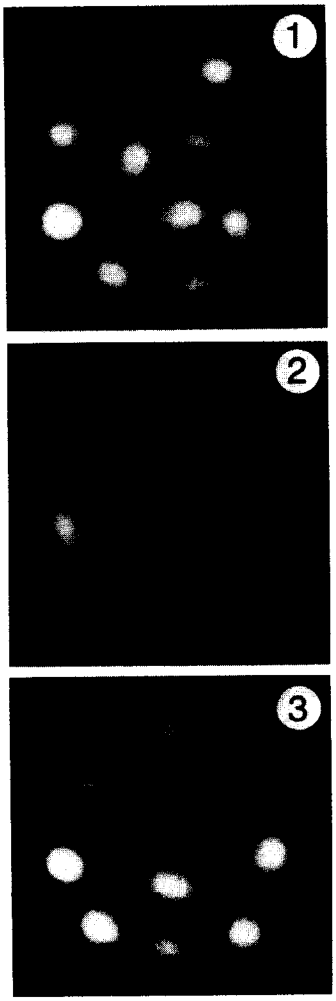

Fig. 3. A sequence of multiple exposure prints. Each print corresponds to 10.6 min of filming (60 frames) at a rotation speed of 0.59 rad/s. Note the slight changes in the array configuration between photographs 1 and 3.

the information on vortices is degraded somewhat by the presence of ion flashes and other dark current signals originating from the image intensifier tube. To improve the signal-to-noise many sequential photographs are superimposed on a multiple exposure picture. This photographic integration process enhances the image of the stable vortex pattern at the expense of the random noise Some typical time averaged pictures are shown in fig. 3. Each picture is a multiple exposure made from 60 sequential photographs. The container is rotated at a constant speed through the experiment. Several features are readily apparent. It appears that there are 11 stable vortices in the system and they are arranged in a quite symmetric pattern consisting of an outer ring with 8 and a center ring with 3. The middle picture shows blurring, indicative of the pattern being weakly bound in the azimuthral direction. This is not surprising since the system is degenerate for rotations about the symmetry axis. The fact that stable patterns exist probably indicates that the ends of the lines are pinned somewhat on the bottom or walls of the container. Comparing the top and bottom picture shows that the inner ring of 3 vortices has shifted with respect to the outer ring.

One also notices in these pictures that the patterns are not exactly symmetric. This distortion cannot be attributed to any known defect in the electro-optical system and is believed to be a genuine reflection of the positions of the vortices. Presumably this loss of symmetry arises because the lines are not perfectly rectilinear but in fact have their lower ends pinned to the side wall or bottom of the container. In fact the fundamental weakness of this photographic method is precisely the lack of knowledge of the position of the lower end of the vortex. Quantitative comparison with two-dimensional theory is hindered by this limitation. At best one can say that when the vortices do show quantitative agreement with two-dimensional calculations that they were probably almost entirely rectilinear. Of course this is only speculation.

Fig. 4 shows the low quantum number arrays which are predicted to be either stable or metastable. This figure was produced from numerical calculations by Campbell and Ziff [8]. Also in the figure are multiple exposure prints made from actual samples of rotating liquid helium. At first glance it appears that the agreement between theory and experimental is quite good. A clear look, however, shows some lack of

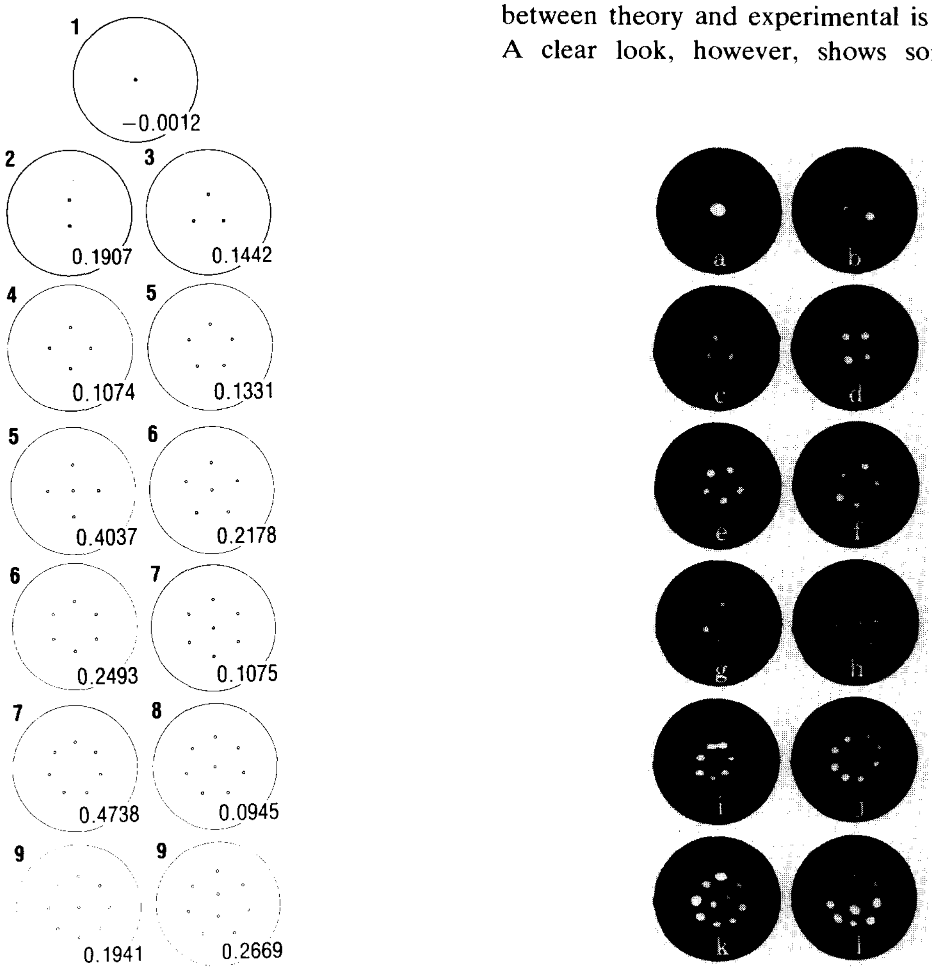

Fig. 4. On the left are displayed the predicted stable and metastable arrays of quantized vortices. The number below each array is a dimensional free energy characteristic of that pattern. On the right are displayed the stationary patterns observed in the experiments.

symmetry in some of the patterns. This is probably due to vortex pinning on the walls. These pictures were produced from samples of superfluid which were being rotated at a constant speed. Inspection of the movie films played at ordinary viewing speed allows one to deduce whether a stable pattern is present. In those cases where a stable pattern appeared to be present a multiple exposure print was made. Most trials at constant angular velocity showed stable patterns.

It should be emphasized that the actual pattern which appears at a given angular velocity is very history-dependent and the existing calculations cannot predict the appearance or disappearance of a particular pattern. For a fixed number of vortices there may be several metastable arrangements possible. Usually only the lowest free energy configuration is observed, but occasionally the sample will make transitions between two different states with the same number. Fig. 5 shows a sequence of prints where the helium undergoes a change from one pattern of 6 vortices to another pattern of 6. Also included in the figure are the two patterns which calculations suggest should exist. It is not known what type of fluctuations drive the system from one state to another since thermal fluctuations seem much too small to overcome the barriers between different states. We speculate that random mechanical vibrations in the system are the actual driving force.

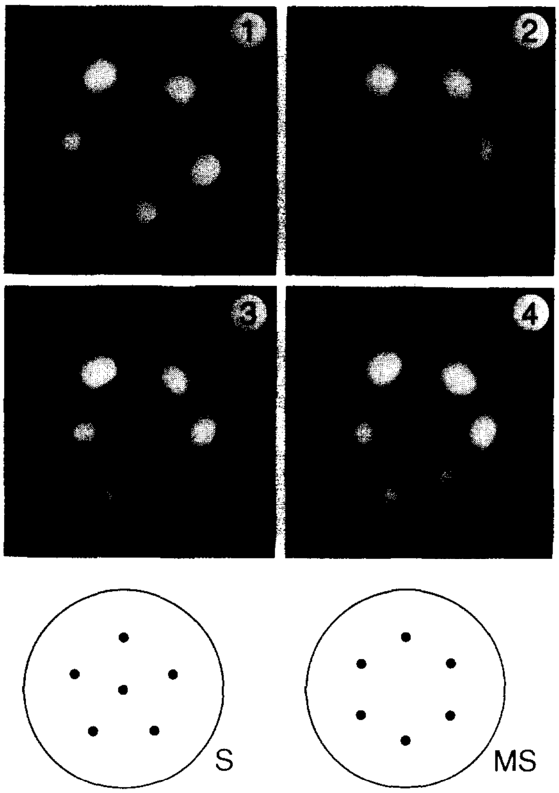

Fig. 5. Photographs 1 through 4 are seqential multiple exposure prints. There are 30 frame exposure and each corresponds to 6.7 min of filming at 0.47 rad/s. Below these are shown the predicted S and MS configurations for a rectilinear array of six vortices as given in ref. 8.

Since the distance scale of these stationary patterns is in principle set by the quantum of circulation it should be possible to measure that fundamental parameter by knowing the overall magnification of the system. A rather simple expression exists which relates the quantum of circulation to the radius of a symmetric ring of vortices [19]. In those film sequences, where such symmetric rings are apparent, one can experimentally determine the quantum of circulation by measuring the vortex line positions and solving the equation for h/m. In our photographs neither the rotation axis nor the center of the cylindrical vessels is known precisely. Thus the simplest array to compare to the theory is a simple two-vortex line system.

In one experiment the vessel was accelerated from rest to a final angular velocity of 0.364 rad/s and maintained at that speed for approximately 1 h. This procedure was repeated many times and a two-line vortex array appeared in almost every such trial. Fig. 6 shows plots of the vortex separation as the function of film frame number for two such trials. It is apparent that there is a different average vortex separation in the two different arrays. The cause of this irreproducibility is not known and is rather typical of the uncertainties in the experiment. The uncertainty in locating the vortex line position is of the order

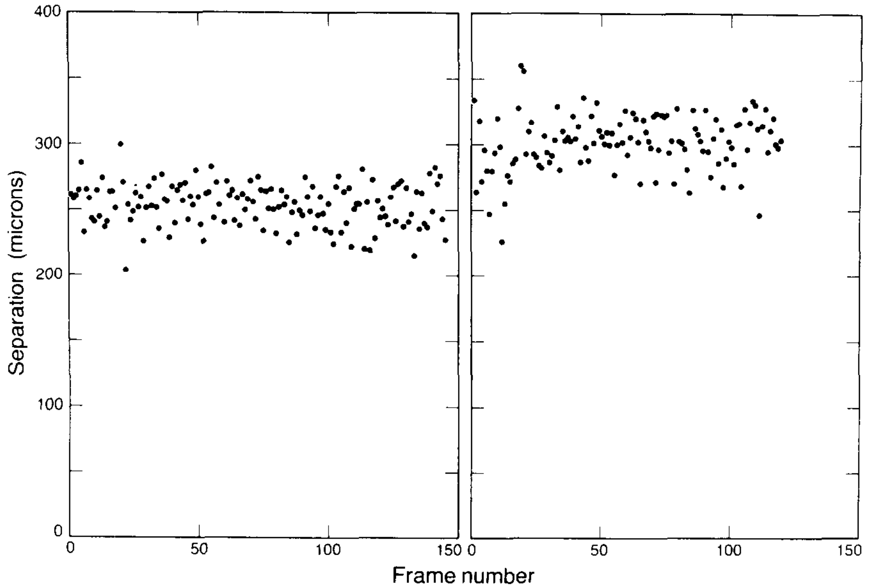

Fig. 6. The separation of the vortices in a two-line array plotted vs. film frame number for two successive spin-up trials. The rotation speed was 0.364 rad/s for both. The time between film frames was 10.6 s.

of 10 μm which is also the same as the fiber diameter in the image conduit (11.5 μm). If the average vortex separation in each trial is computed for 18 spin-up trials, the graph of fig. 7 results. The horizontal line is the computed separation based on the expected value of h/m. The error bars on each point are small compared to the scatter between points, and the points certainly do not lie on the line within the experimental accuracy. One would suspect that the unknown shape of the vortex line below the free surface influences the vortex position and causes a vortex separation on the surface close to but not exactly equal to the value expected for rectilinear vortices. The qualitative agreement between two-dimensional theory and the experimental observation is good but our lack of knowledge concerning what is occurring below the free surface prevents a quantitative comparing with any existing theory. This illustrates a fundamental limitation in this vortex photography method.

When the vortex movie films are played at ordinary viewing speeds, one occasionally sees dynamical motion in which well-defined vortex trajectories can be determined. The criteria for a dynamical study of vortex motion is that the vortices move only a short distance between each

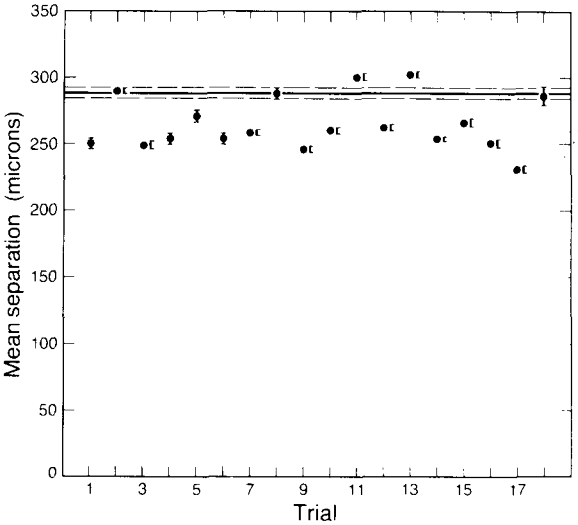

Fig. 7. The time averaged separations of the vortices in two-line arrays produced by independent spin-ups to a rotation speed of 0.364 rad/s. The data are plotted vs. the spin up trial number (chronologically ordered). The solid line is the predicted separation for rectilinear vortices and the dashed lines indicate the uncertainty in this prediction, as explained in the text.

recorded image. It has been particularly interesting to study collective oscillations of the vortex arrays because they lend themselves to tractable theoretical analysis.

The simplest oscillation to analyze is an azimuthal mode of two vortex lines which seems to systematically occur during slow acceleration trials when a second vortex appears to join a pre-existing single vortex state. After an initial transient in which the vortex motion is too fast to follow, the two vortices settle down, making a decaying azimuthal oscillation like that shown in fig. 8. These data display the time evolution of an orientation angle with respect to the equilibrium position. The data can be fit to decaying sinusoids and a frequency and a decay constant may be extracted. The oscillation frequency can be determined with an uncertainty of 1% and the fitted decay time can be determined with an uncertainty of about 20%. In the three oscillation sequences shown the frequencies are 0.100, 0.095, and 0.083 rad/s and the decay times are 400, 380, and 370 s. We are unable to explain why the differences in measured frequency are substantially greater than the measurement uncertainty. We can only guess again that a structure below the surface is influencing the system. Several different and more complicated types of oscillations have been recorded for vortex arrays with 3 and 4 vortices present [3].

When 3 vortices are present there are several different types of normal mode motion. Our analysis technique is to digitize the vortex positions using a film scanner and then numerically separate center of mass motion from azimuthal motion and from distortive motion. An example of this distortive motion for a three-line array is shown in fig. 9. In this particular example, the

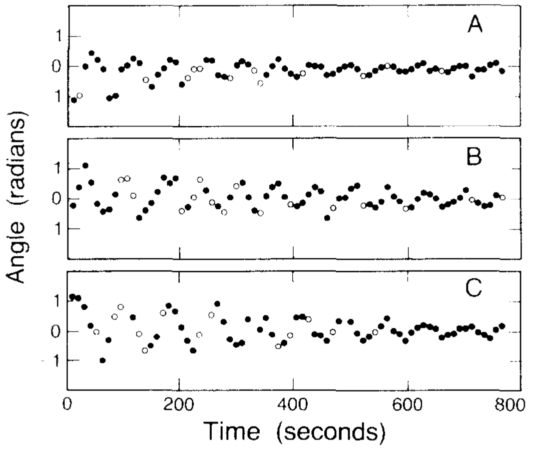

Fig. 8. The angular orientation of two-line arrays plotted vs. time for three different spin-ups, all to a rotation speed of 0.364 rad/s. The open circles are points which have been determined on the basis of a single vortex image, as discussed in the text.

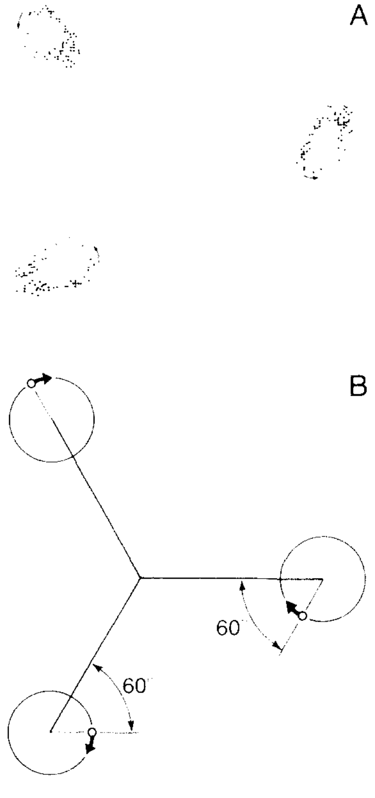

Fig. 9. The measured distortive motion of a three-line array (A), and the predicted normal mode motion for rectilinear vortices (B). Each point in A is the average of ten film frame image locations after aximuthal and displactive motion have been subtracted out (see text). The arrows in A and B indicate the sense and the relative phase of the orbital motion. The data shown is for four complete cycles of motion. The predicted normal mode motion (B) includes the effects of a finite cylindrical boundary, and the orbits are elliptical rather than circular. However, the effect is too small to be seen in this plot.

oscillation persisted undamped for 22 h and was not the result of any transient event. Presumably, a mechanical disturbance in the system, probably occurring at the rotation frequency, drove this particular normal mode.

Calculations exist for the motion of similar arrays and the lower half of the figure shows the type of orbits which are predicted for such an array. Because of the uncertainties inherent in this method one would not expect a quantitative agreement between the observation and the calculation. Actually the analysis of these films is rather complex and is described more fully in another paper [3]. We only mention this type of motion here to give a flavor of the dynamical effects which one can observe in the vortex photography technique.

As a final topic in this survey of vortex photography I would like to describe a few experiments which help to determine how a particular vortex state evolves in time. In one type of experiment the cell is very slowly accelerated from rest and we measure, using the film, how many vortex lines are present as a function of time. Fig. 10 shows the result of eight such trials. In each graph is shown a plot of the angular velocity versus time, the equilibrium number of vortices expected from theory as a function of time, and the actual number of observed vortex lines as a function of time. A few trends are clearly visible in the data. Most striking is the fact that the number of vortices present is always less than the equilibrium number. This is not surprising because there are substantial energy barriers between states of different vortex numbers. Also noticeable is the fact that it is more probable to have transitions involving the appearances of a single vortex line rather than multiple vortices. And finally, there is the somewhat surprising result that no transitions occur when the angular acceleration is equal to zero. We know of no model to explain these last two observations.

These graphs, considered together with the

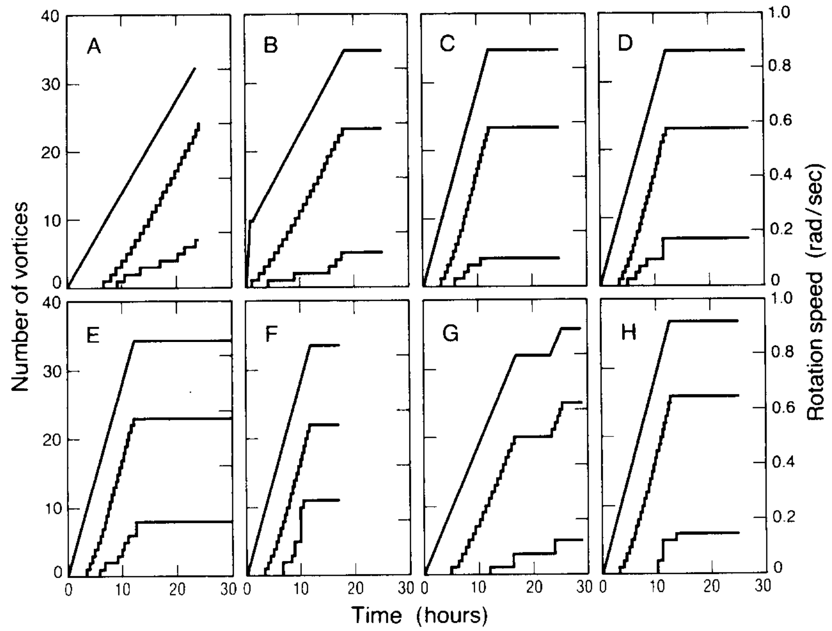

Fig. 10. The results of eight slow acceleration trials. For each trial the upper curve gives the rotation speed (right scale), the middle curve gives the predicted equilibrium number of vortices (left scale) and the lower curve gives the observed vortex number (left scale). All are plotted vs. time.

photographs showing static vortex arrays and the dynamical studies already mentioned, give one a fairly good qualitative picture of the nature of rotating superfluid helium. The superfluid in rotation at low angular velocities contains an array of vortices which is very similar in structure to the symmetric arrays that describe an equilibrium rectilinear system of vortices. The particular array is history-dependent and is determined by angular accelerations, mechanical disturbances as well as boundary conditions and unknown nucleation process. Whether or not this vortex photography technique can yield more quantitative and understandable information will probably hinge on the experimentalists' ability to insure the two-dimensional character of his vortex sample.

Acknowledgement

It is a pleasure to acknowledge and thank the many people who have helped in this research over the past 14 years. As mentioned above, T.M. Sanders introduced the author to this field and provided the seminal ideas for the method. The earliest vortex images were produced through the efforts of Gary Williams. The bulk of the data and analysis discussed in this paper was provided by Ed Yarmchuk. Stimulating and helpful conversations as well as technical assistance were donated generously over the years by K. De Conde, J. Eisenstein, M.J.V. Gordon, S. Garrett, and G. Swift. All the work at Berkeley has been supported by the Division of Materials Research of the National Science Foundation.

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