Online Appendix: A Macroeconomic Model of Price Swings in the Housing Market

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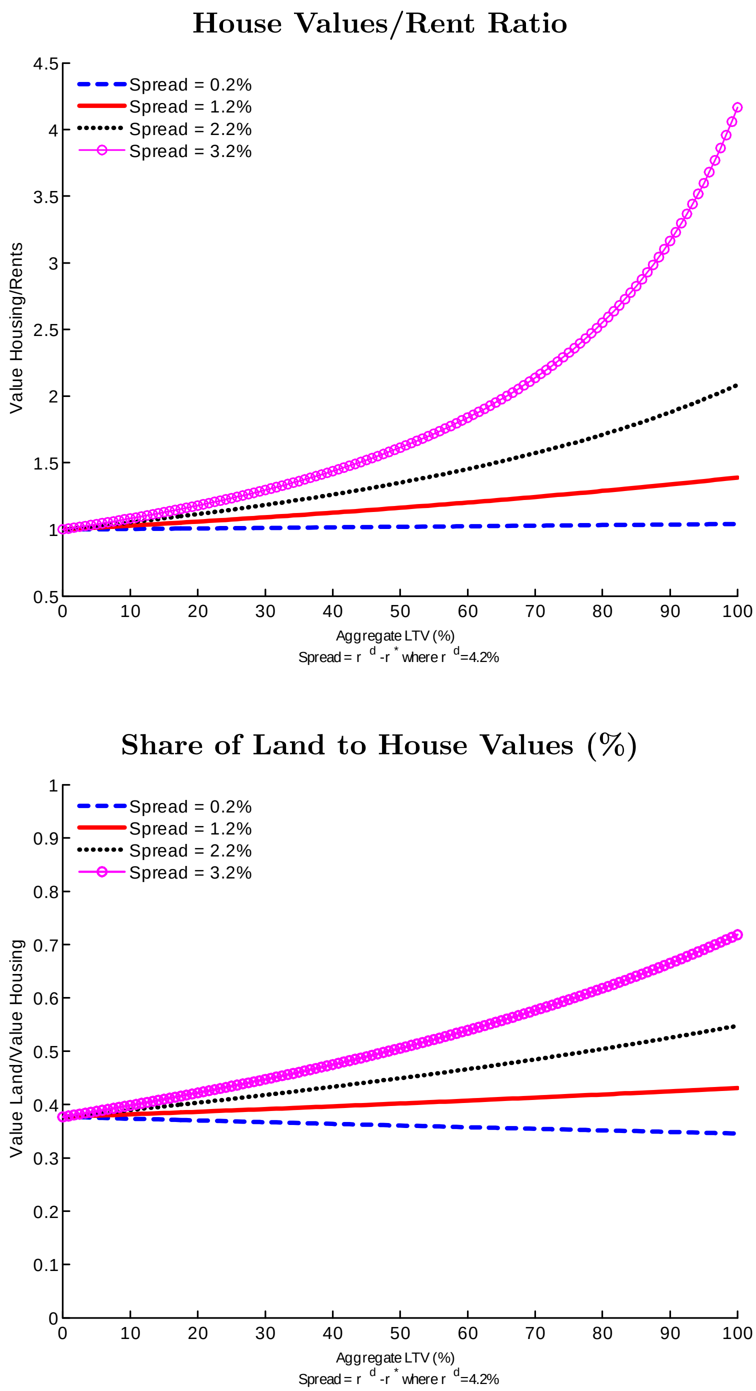
Contents

1 Appendix A: Steady State: Sensitivity to Housing Finance

This Appendix explores the sensitivity of house values to persistent changes in financial variables.

Figure A.1: Steady State House Values and Housing Finance



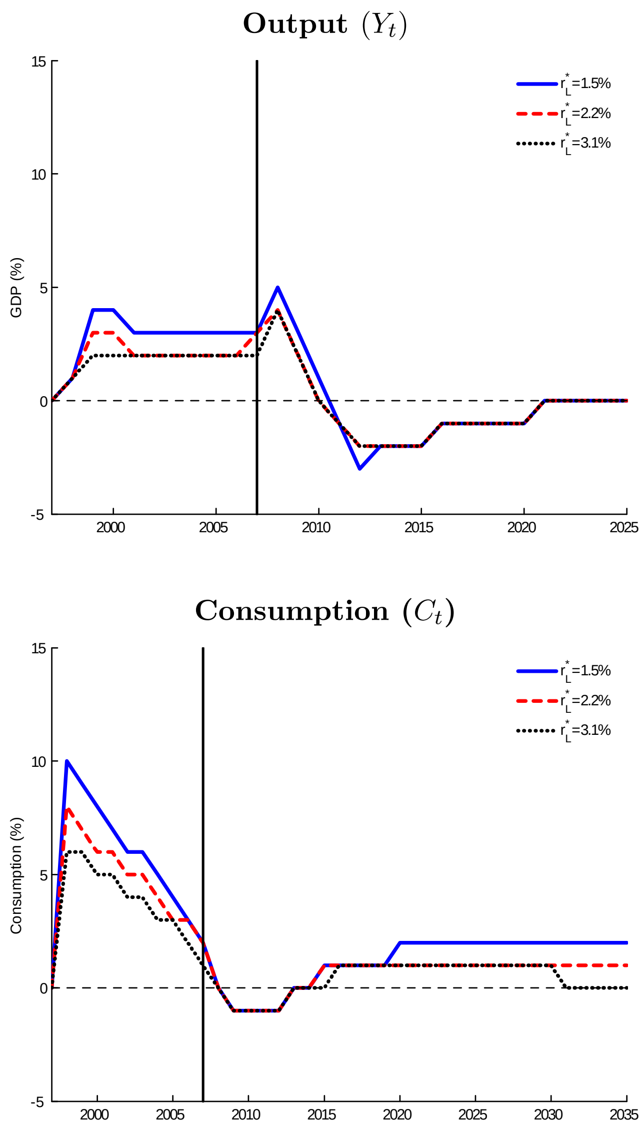
Source: Model-simulated data.

The left panel of Figure A.1 shows house values relative to rents, or the price-rent ratio in the calibrated economy for different loan-to-value ratios and spreads of interest rates $(r^d - r^*)$. The top panel of Figure A.1 shows the contribution of land to house values.

2 Appendix B: Perfect Foresight: Macroeconomic Aggregates

Here we report the level of macroeconomic aggregates in the perfect foresight case.

Figure B.1: Macroeconomic Aggregates



Source: Model-simulated data.

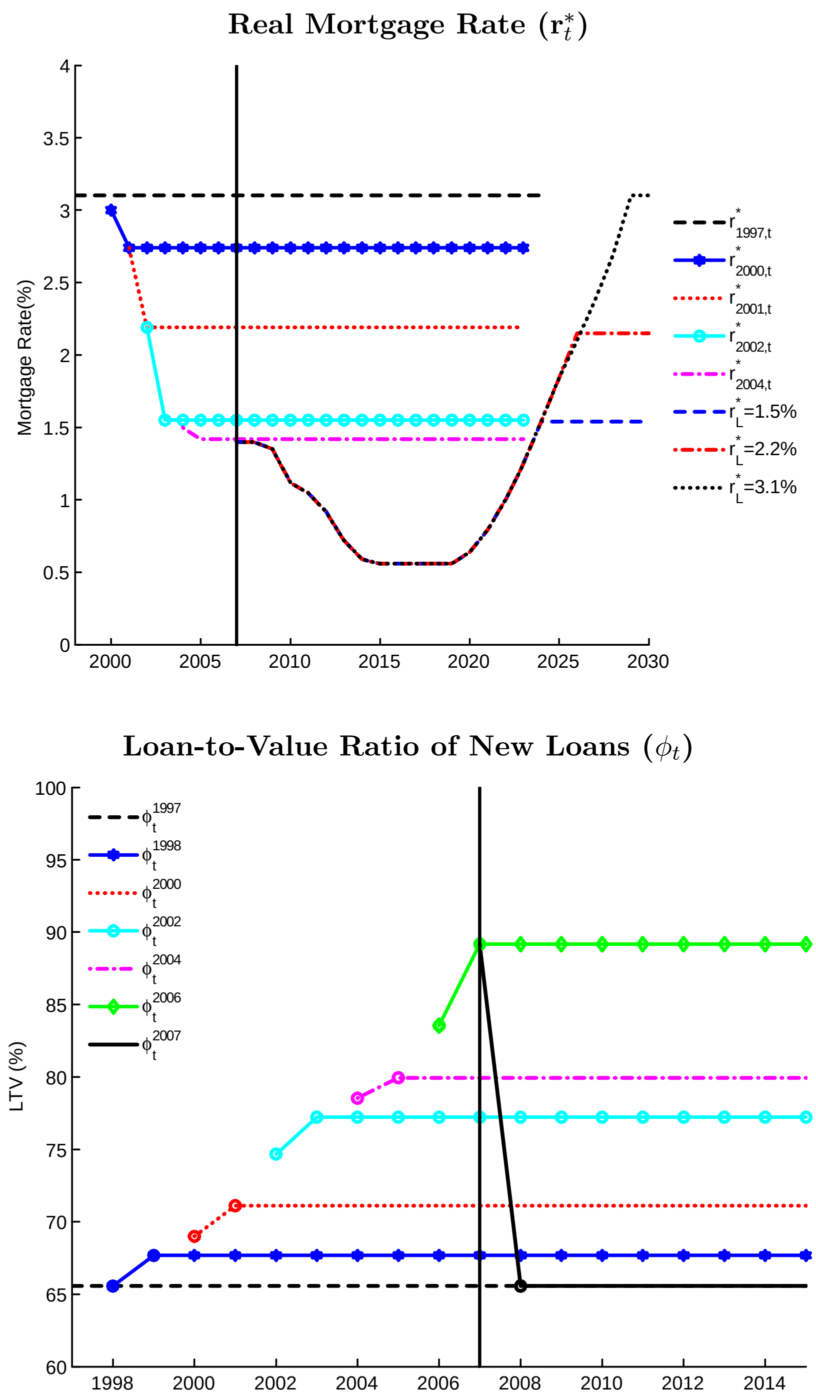
The initial and final steady states are not determined by conditions in housing finance, hence, the different simulations converge to the same level of production, $Y^* = C^* + \delta_s S^* + r_L^* \phi V^*$, but the spending allocation varies across experiments.

3 Appendix C: Shocks to Expectations

3.1 Timing of News about Financial Variables

Figure C1 depicts the timing of news about financial variables described in Section 6.5

Figure C.1: Time Path of Financial Variables



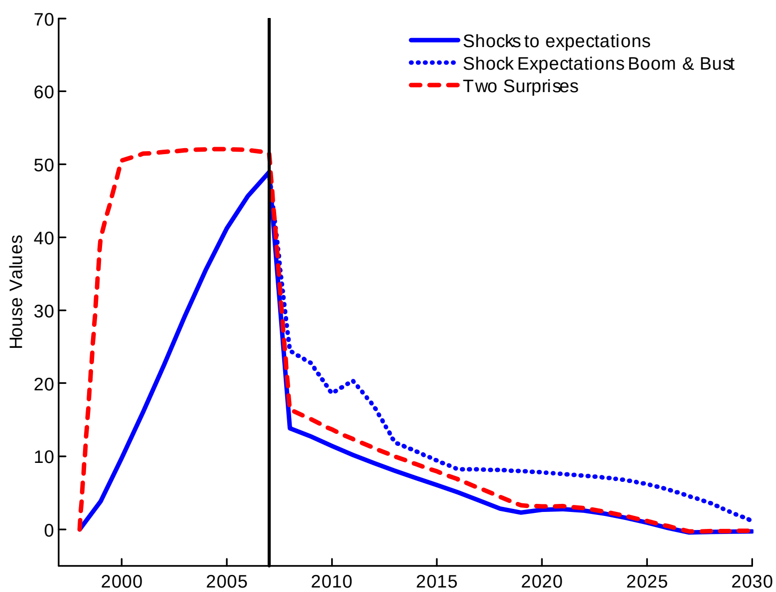
Source: Authors' calculations.

3.2 Sensitivity Analysis: The Timing of News about Financial Variables

This section in the Appendix explores how sensitive house values are to the particular path of shocks to expectations. We present the predictions of the model for house values in two scenarios. In the first scenario, labeled “Shock to Expectations: Boom and Bust,” we assume that news about the changes in financial conditions during the post-2008 period arrive as expectational shocks. The second experiment, labeled “Two Surprises,” assumes that there is a “boom” shock in 1998 and delivers a path of mortgage rates and loan-to-valio ratio constraints consistent with the observed path until 2007 because households initially believed the shock was permanent. The second surprise happens in 2007. At this point, the households learn that the path of the for housing finance variables behaves is as in the main body of the paper.

The results of these two alternative specifications of expectations as well as the baseline case with shocks to expectation are in Figure C.2. The predictions are relatively close, taking into account how radically different the implied expectations are. We find this robustness test somewhat reassuring.

Figure C.2: Housing Values and the Arrival of Information

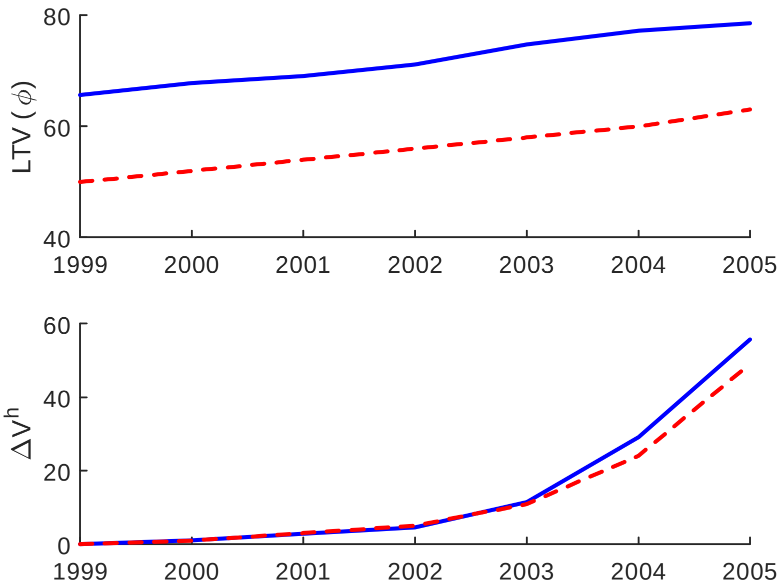


Source: Model-simulated data.

3.3 Sensitivity Analysis: LTV Constraints and Housing Values

This section in the Appendix shows the sensitivity of housing appreciation to different paths of LTV constraints under the assumption that agents expectations are shocked every period.

Figure C.3: LTV Constraints and Housing Values



Source: Model-simulated data.

For the parth of mortgage rates described in Figure C.1, the top panels in Figure C.3. display the dynamics of the realized LTV and the bottom panels the implied path of house values. Notice that in the early periods of the boom, the LTV constraint is relaxed but the mortgage rates are high resulting in a similar response of house prices. As mortgages rates fall, the difference in housing appreciation changes. For this particular parametrization the gap is around 25 percent. This finding is consistent with Figure A.1. and it highlights the nonlinearities of house prices that result from declines in the cost of borrowing.

4 Appendix D: Proof of Proposition 1

Proof: Simple computations show that a steady state is the solution to the following system of equations:

$$p^\ell = \frac{r^d + \delta_s - \phi(r^d - r^*)}{r^d - \phi(r^d - r^*)} \frac{1 - \alpha_s}{\alpha_s} \left(\frac{S}{L}\right)^{1+\mu}, \tag{1}$$

$$c(S, \phi, r^*) = \left[\frac{r^d + \delta_s - \phi(r^d - r^*)}{1 + r^d} \frac{\alpha_c}{\alpha_s(1 - \alpha_c)} S^{1+\mu} G(S, L)^{\rho-\mu}\right]^{\frac{1}{1+\rho}}, \tag{2}$$

$$V = V^1(S, \phi, r^*) = S\left[1 + \frac{1 - \alpha_s}{\alpha_s} \frac{r^d + \delta_s - \phi(r^d - r^*)}{r^d - \phi(r^d - r^*)} \left(\frac{S}{L}\right)^{1+\mu}\right], \tag{3}$$

$$V = V^2(S, \phi, r^*) = \frac{Y - c(S, \phi, r^*) - \delta_s S}{\phi r^*}. \tag{4}$$

It is useful to exploit the recursive nature of the economy to understand the effect of some shocks. In particular, equations (3) and (4) can be used to pin down (V, S). Given this, equation (2) determines the level of non-housing consumption and equation (1) gives the price of land. Simple inspection shows that the functions $V^1(S,\phi,r^*)$ and $V^2(S,\phi,r^*)$ are continuously differentiable and satisfy

$$\lim_{S\to 0} V^1(S,\phi,r^*)=0,\ \lim_{S\to\infty} V^1(S,\phi,r^*)=\infty,\ V^1_S>0,\ V^1_\phi>0,\ V^1_{r^*}<0$$
$$\lim_{S\to 0} V^2(S,\phi,r^*)=\frac{Y}{\phi r^*},\ \exists S^H(\phi,r^*),\text{ such that } V^1(S^H,\phi,r^*)=0 \text{ and}$$
$$V^2_S<0,\ V^2_{r^*}<0.$$

Given the continuity of $V^1(S,\phi,r^*)$ and $V^2(S,\phi,r^*)$ and their monotonicity, there is a unique point in (V,S) at which they intersect, and this result holds even at the boundary when $r^*=r^d$ and $\phi\in\{0,1\}$. Given this point, there are unique values of c and p^ℓ that satisfy equations (2) and (1). First, consider the effect of a decrease in r^*. This change shifts the $V^1(S,\phi,r^*)$ and the $V^2(S,\phi,r^*)$ functions up and unambiguously increases the value of the housing stock, V. In order to determine the impact on the equilibrium quantity, note that

$$r^*\phi\delta_s\le(r^d-\phi(r^d-r^*))(r^d+\delta_s-\phi(r^d-r^*))$$

holds for all $\phi\in[0,1]$ and $r^*\le r^d$; this, in turn, implies that

$$\mid\frac{\partial V^2}{\partial r^*}\mid_{S=S^*}\le\mid\frac{\partial V^1}{\partial r^*}\mid_{S=S^*},$$

and, hence, that $\partial S/\partial r^*\le 0$. Second, an increase in ϕ shifts the $V^1(S,\phi,r^*)$ function up and has an ambiguous effect on $V^2(S,\phi,r^*)$. A sufficient condition for such an increase to lower both V and S is that $\partial V^2/\partial\phi\le 0$. It is possible to show that

$$\frac{\partial V^2}{\partial\phi}=-\frac{V^2}{\phi}+\frac{c(S,\phi,r^*)}{(1+\rho)\phi r^*}\frac{r^d-r^*}{r^d+\delta_s-\phi(r^d-r^*)}$$

and, hence, that

$$sign[\lim_{\frac{1}{1+\rho}\to 0}=\frac{\partial V^2}{\partial\phi}]=sign[\lim_{r^d-r^*\to 0}=\frac{\partial V^2}{\partial\phi}]=sign[\lim_{\phi\to 0}=\frac{\partial V^2}{\partial\phi}]<0.$$

It follows that if the mortgage relevant interest rate is close to the market rate (i.e., r^d-r^* close to zero), the loan-to-value ratio is very low (i.e., ϕ close to zero); or if non-housing and housing consumption are extremely complementary goods, an increase in the loan-to-value ratio can result in a decrease in the value of housing and in the quantity consumed.

5 Appendix E: Numerical algorithms

5.1 Perfect foresight

The perfect foresight case can be generated using the Fortran program "perfect foresight.f" The code is in Fortran and can be compiled and run using any Fortran90 compiler. It requires using Absoft's IMSL Fortran 7.0 numerical routines (for the nonlinear solvers, the user should be able to use alternative algorithms is IMSL is not available, but we have not tested alternatives).

The idea of the algorithm is to solve for equilibrium using the associated Euler (accounting for occasionally binding irreversibility constraints) equations. Period zero capital stocks and price for land are given by the pre shock steady state. Individuals learn about a full new sequence of mortgage interest rates and credit conditions in period 1. We assume the new steady sate (associated to the last period values for the sequence of interest rates and credit conditions) is reached at period T (for large T typically longer than 80 years). In the long-run, our numerical simulations approach steady state in a smooth fashion several periods ahead of period T. We use a nonlinear solver and obtain very low values (typically lower than 10^{-7}) for the Euler equation residuals (the values of the Euler equations evaluated at the proposed solution).

This approach yields high accuracy, but as with any numerical nonlinear solver, requires a good initial guess for it to work. We follow the homotopy approach: We divide the distance between the target sequence and the period zero (constant) sequence of interest and credit conditions into N steps. We solve the system of equations step by step (so that the first step is very close to the original steady state) and use the solution of step n as initial guess for step n+1, iterating until we reach the targeted sequence. The algorithm is as follows:

1. Given parameters (before any shocks) compute a steady state.
2. Given step n's period T values for the mortgage interest rate and credit conditions, solve for the final steady state.
3. Given step n sequence of values for credit conditions and the interest rate on mortgages, given initial conditions (as given by the steady state computed in 1), imposing that capital stocks in period T+2 correspond to the steady state in 2, and using as initial condition the solution to the system of step n, solve the system of equations (the program uses two different methods, and iterates over them, using the solution of one as initial guess for the solution of the other several times, this was the way we found to achieve high accuracy levels we aimed for).
4. Store the new solution (step n+1), go back to step 2 unless the target sequence of interest rates and credit conditions has been reached.

5.2 Shocks to expectations

The shocks to expectations results can be generated by using the code "unexpectedshocks.f"The code is in Fortran and can be compiled and run using any Fortran90 compiler. It requires using Absoft's IMSL Fortran 7.0 numerical routines(for the nonlinear solvers, the user should be able to use alternative algorithms is IMSL is not available, but we have not tested alternatives).

The idea of the algorithm is essentially the same as in the perfect foresight experiments. What changes is that in this new case individuals learn a new sequence of interest rates and credit conditions that is constant through time at every period. Say we are in period 1, taking the stocks of capital chosen in period 0, and the price of land that prevailed in period 0 (as that determines the value of the mortgage). Individuals learn about new values for interest rates and credit conditions, and they expect these new values will prevail into the indefinite future. The program computes a whole transition path from the current capital stocks. To generate the time series of the shocks to expectations the user must only keep the capitals and land prices that prevail one period after such shock, and use those as initial conditions for next period, and keep iterating forward (during the boom years). To generate the bust simulation the relevant initial conditions determined in the last year of the boom can be employed, and the homotopy approach (and program) of the perfect foresight algorithm above can be used to slowly modify expectations from constant forever, to the sequence of falling and eventually increasing rates considered by the paper.