Online Appendix for "A Flexible Finite-Horizon Alternative to Long-run Restrictions with an Application to Technology Shocks"

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In this Appendix we describe the two models used to generate data in the Monte Carlo experiments (Section A). We also provide further robustness analysis (Section B) and more details on the U.S. data analyzed in the paper (Section C).

A Models Used as DGPs in Monte Carlo Experiments

A.1 RBC Model

In this section, we outline the RBC model used to generate the data for the Monte Carlo experiments. Households choose consumption, C_t, labor, N_t, and investment, I_t, to maximize the expected present-discounted value of utility:

$$U(C_t, N_t) = \sum_{t=1}^{\infty} \beta^{t-1} \left[\ln(C_t) + \Phi \ln(1 - N_t) \right],$$

subject to a standard budget constraint:

$$C_t + I_t = (1 - \varsigma_{nt}) W_t N_t + (1 - \varsigma_{kt}) r_t K_t + \delta \varsigma_{kt} K_t - \Psi_t;$$

the equation characterizing the evolution of capital, K_t:

$$K_{t+1} = (1 - \delta) K_t + I_t;$$

an economy-wide resource constraint:

$$C_t + I_t + G_t \leq Y_t;$$

and a government spending constraint:

$$G_t = \varsigma_{nt} W_t N_t + \varsigma_{kt} (r_t - \delta) K_t + \Psi_t,$$

where r_t is the pre-tax return on capital, W_t is the real wage rate, δ is the depreciation rate, β is the discount factor, Ψ_t is a lump-sum tax, ς_{nt} is the tax on labor, and ς_{kt} is the tax on capital income. Consumers own the capital and rent it to firms. The government balances its budget each period and finances its spending through a combination of lump-sum taxes and distortionary labor and capital income taxes. Tax rates on capital and labor income are stochastically determined by $\tau_{it} = \rho_i \tau_{it-1} + \sigma_{\tau_i} \varepsilon_{\tau_i}$, for $i = k, n$ where $\tau_{it} = \ln(\varsigma_{it}) - \ln(\overline{\varsigma_i})$, and $\overline{\varsigma_i}$ are the steady-state values.[1] The steady-state deviation of government purchases, g_t, has a similar first-order autoregressive process. Finally, output is determined by a Cobb-Douglas production technology:

$$Y_t = (Z_t N_t)^{\alpha} K_t^{1-\alpha},$$

where Z_t is an exogenous process for labor-augmenting technological innovation, $z_t = \rho_z z_{t-1} + \sigma_z \varepsilon_{z_t}$ is the log of technology, and $\varepsilon_z \backsim i.i.d.N\left(0, \sigma_z^2\right)$. Table A.1 presents the sets of parameter values used to simulate the neoclassical growth model and the stick price model, respectively. Parameterizations for the neoclassical growth model are similar to those used by EGG (2005) in their benchmark model without capital utilization to match moments in U.S. data.

[1] The steady-state value for the ratio of government to output deserves special mention. The difference between private output and the sum of private consumption and investment is treated as exogenous government consumption [see Figure 8 and the accompanying text in Uhlig (2003)]. Under this simplifying assumption, the international sector and government investment are not explicitly modeled, although they may, in fact, be relevant in the transmission of technology.

The model is solved by first eliminating non-stationarities arising from technology by dividing Y_t, K_{t-1}, I_t, C_t, G_t, W_t, and Ψ_t by Z_t. Next, the necessary first-order and steady-state conditions are computed based on selected parameter values. The model is log-linearized around the steady-state growth path and the recursive equilibrium law of motion is solved using the method of undetermined coefficients.

To ensure the VAR representation exists under each parameterization, the model is written in its VARMA form. We can then verify that the MA portion is invertible. Here, we present the derivation of the VARMA representation. Given the recursive solution

$$\psi_t = p\psi_{t-1} + Q\xi_t, \tag{1}$$

$$y_t = W\psi_t + S\xi_t, \tag{2}$$

where ψ_t is a vector of endogenous state variables (in our case, capital, k, is the lone endogenous state variable), ξ_t is a vector of exogenous state variables (technology, A, government shock, g, and capital and labor tax shocks, τ_k and τ_n), and y_t is a vector of other endogenous variables (output, hours, consumption, and investment). The endogenous variables used to estimate the VAR are labor productivity (output-hours ratio), hours, consumption-output ratio, and investment-output ratio. Since these variables are basic transforms of the underlying variables, invertibility remains; of course we verified that this is indeed the case. The scalar p and the vectors Q, W, and S are determined by simulating the model, conditional on the parameter values from Table A.1.

Substituting (1) into (2) yields

$$y_t = pW\psi_{t-2} + WQ\xi_t + S\xi_t. \tag{3}$$

Realize that

$$W\psi_{t-1} = w_{t-1} - S\xi_{t-1}, \tag{4}$$

and substitute (4) into (3). Collecting terms yields:

$$y_t - py_{t-1} = S\xi_t + (WQ - pS)\xi_{t-1}. \tag{5}$$

We can rewrite this as:

$$S^{-1}y_t - (S^{-1}p)y_{t-1} = \xi_t + S^{-1}(WQ - pS)\xi_{t-1}, \tag{6}$$

Parameter	Description	Value
α	capital share	0.36
δ	quarterly depreciation rate	0.02
β	discount factor	1/1.03
Φ	preference parameter	1
ρ_z	autocorrelation of technology shock	1
ρ_k	autocorrelation of capital tax shock	0.98 (0.6)
ρ_n	autocorrelation of labor tax shock	0.98 (0.6)
ρ_g	autocorrelation of government spending shock	0.98 (0.6)
$\overline{g}/\overline{y}$	steady-state ratio of government to output	0.03
$\overline{n}$	steady-state labor	1/3
$\overline{\varsigma}_k$	steady-state capital tax rate	0.38
$\overline{\varsigma}_n$	steady-state labor tax rate	0.22
σ_z	technology shock standard deviation	0.0148
σ_{τ_k}	capital tax shock standard deviation	0.0148 (0.008)
σ_{τ_n}	labor tax shock standard deviation	0.0148 (0.052)
σ_g	government spending shock standard deviation	0.0148 (0.016)

Table A.1: Parameters for the RBC model augmented with preference shocks, capital, and labor income taxes. Numbers outside parentheses are for the benchmark model; and numbers in parentheses are for robustness analysis.

$$D(L)y_t = C(L)\xi_t.$$

Finally, given the parameterizations from Table A.1, we ensured that the roots of $C(L) = I + [S^{-1}(WQ - pS)]L$ lie outside the unit circle as required for invertibility.

A.2 New Keynesian Model

We use the DSGE model in EGG (2005) as an alternative DGP for Monte Carlo experiments.[2] The model is a standard medium-scale model built around a RBC core, with I(1) technology shocks. The model features real

[2] We are gratefule to Erceg, Guerrieri, and Gust for sharing their code with us.

"frictions" (habit formation in consumption, variable capital utilization, and investment adjustment costs) as well as nominal rigidities (sticky prices and wages). Other sources of fluctuations, besides technology shocks, are labor supply, labor tax rate, government spending, and monetary policy shocks. We summarize the log-linearized model, in terms of stationary variables. Table A.2 lists the model's variables; Table A.3 lists the parameters and their calibrated values.

Intermediate goods are produced by monopolistically competitive firms that set prices à la Calvo. Sluggish price adjustments result in a Phillips curve of the form

$$\pi_t = \beta \pi_{t+1} + \kappa_p \left[\zeta_t - (y_t - n_t) \right],$$

where

$$\kappa_p = \frac{1 - \psi_p}{\psi_p} \left(1 - \psi_p \beta \right).$$

Aggregate output is related to aggregate capital and labor with a Cobb-Douglas production function:

$$y_t = \left(1 - \tilde{\theta} \right) n_t + \tilde{\theta} \left(k_t + u_t - \sigma_z \epsilon_{zt} \right),$$

where

$$\tilde{\theta} = \left[\frac{1}{1 - \tau_K} \left(\frac{1 + \mu_z}{\beta} - 1 \right) + \delta \right] \frac{1 + \theta_p}{1 + \mu_z - (1 - \delta)} \frac{i}{y}.$$

Also, the ratio of factor prices is proportional to the aggregate capital-to-output ratio:

$$r_{Kt} - \zeta_t = n_t - (k_t + u_t - \sigma_z \epsilon_{zt})$$

Because of habit formation in consumption, the households' marginal utility of consumption depends on the current and lagged levels of consumption:

$$\lambda_{ct} = \frac{-1}{1 - \frac{\phi_c}{1 + \mu_z}} \left[c_t - \frac{\phi_c}{1 + \mu_z} \left(c_{t-1} - \sigma_z \epsilon_{zt} \right) \right]$$

The households' intertemporal Euler equation is given by

$$\lambda_{ct} = E_t \left[\lambda_{ct+1} + (f_t - \pi_{t+1}) - \sigma_z \epsilon_{zt+1} \right].$$

From the households' optimality conditions we also get an equation describing the replacement cost of capital, Tobin's q, as

$$q_t = \phi_i (1+\mu_z)(i_t - i_{t-1} + \sigma_z \epsilon_{zt}) - \phi_i \frac{(1+\mu_z)^2}{1+r}(i_{t+1} - i_t + \sigma_z \epsilon_{zt+1}).$$

The FONC for investment can be expressed in terms of q as

$$q_t = E_t \left[\frac{1-\delta}{1+r} q_{t+1} - (f_t - \pi_{t+1}) + \frac{r + \delta (1-\tau_K)}{1+r} r_{Kt+1} \right].$$

Households accumulate capital according to

$$\left(1 - \frac{1-\delta}{1+\mu_z}\right) i_t = k_{t+1} - \frac{1-\delta}{1+\mu_z}(k_t - \sigma_z \epsilon_{zt}).$$

The FONC with respect to capital utilization implies that the utilization rate is proportional to the rental rate on capital

$$\nu u_t = r_{Kt}.$$

Households supply monopolistically differentiated labor services to a competitive "labor market aggregator". The inertia in wage adjustment results in a wage Phillips curve of the form,

$$\pi_t^\omega = \beta E_t \pi_{t+1}^\omega + \kappa_w \left(\chi n_t + \chi_t - \lambda_{ct} - \zeta_t + \frac{1}{1-\tau_N} \tau_{Nt} \right),$$

where

$$\kappa_w = \frac{1-\psi_w}{\psi_w \left(1 + \chi \frac{1+\theta_w}{\theta_w}\right)} (1 - \psi_w \beta),$$

and τ_{Nt} and χ_t are a labor tax rate shock and a labor supply shock, respectively. Both labor market innovations are assumed to follow AR(1) processes,

$$\begin{aligned} \tau_{Nt} &= \rho_N \tau_{Nt-1} + \sigma_{\tau_N} \epsilon_{\tau_N t}, \\ \chi_t &= \rho_\chi \chi_{t-1} + \sigma_\chi \epsilon_{\chi t}. \end{aligned}$$

Changes in real wage, ζ, are related to nominal wage and price inflation as follows:

$$\Delta \zeta_t = \pi_t^\omega - \pi_t - \sigma_z \epsilon_{zt}.$$

The economy satisfies the following resource constraint

$$\left(1-\frac{g}{y}\right)y_t = \frac{c}{y}c_t + \frac{i}{y}i_t + g_t,$$

where $c/y = (1 - i/y - g/y)$ denotes the steady state consumption share of GDP; g_t is a government spending shock that follows an AR(1) process:

$$g_t = \rho_g g_{t-1} + \sigma_g \epsilon_{gt}.$$

Finally, we close the model with an interest rate rule describing monetary policy:

$$f_t = \gamma_i f_{t-1} + \gamma_\pi \pi_t^{(4)} + \gamma_y \Delta y_t^{(4)} + \sigma_f \epsilon_{ft},$$

where interest rate is set in response to year-on-year inflation and output growth:

$$\pi_t^{(4)} = \frac{1}{4}\sum_{j=0}^{3}\pi_{t-j}, \ \ \Delta y_t^{(4)} = \frac{1}{4}\sum_{j=0}^{3}\Delta y_{t-j},$$

and ϵ_{ft} is a monetary policy shock.

B Robustness

B.1 Bias Over the Impulse Response Horizon

We might also be interested in how the bias changes over the response horizon. Figure B.1 shows the average absolute bias of the identified responses over the first 20 quarters for the baseline parameterizations of the RBC and NK models, respectively. For the RBC model, the productivity response identified by Max Share exhibit less bias on average than those identified by LR. For the other variables, Max Share exhibits less bias at short horizons—horizons typically used to distinguish the models. For the NK model, the bias advantage of Max Share is smaller but still apparent.[3]

B.2 Results with Less Persistent Non-Technology Shocks

Some recent studies [e.g., Francis and Ramey (2005) and Uhlig (2004)] argue that other shocks—capital tax shocks, for example—may contribute to the

[3] Because the NK responses are often close to zero, the reported percentage biases as functions of the true values are much larger than those for the RBC model.

Variable	Description
c_t	consumption
i_t	investment
y_t	output
k_t	capital
n_t	labor
u_t	utilization rate
q_t	Tobin's q
λ_{ct}	marg. util. of consumption
ζ_t	real wage
f_t	nominal interest rate
r_{Kt}	rental rate on capital
π_t	inflation rate
π_t^{ω}	wage inflation rate
$\pi_t^{(4)}$	y-on-y inflation rate
$\Delta y_t^{(4)}$	y-on-y output growth
ϵ_{zt}	technology shock
ϵ_{ft}	mon. policy shock
τ_{Nt}	labor tax rate shock
$\epsilon_{\tau_N t}$	labor tax rate innovation
g_t	govt. spending shock
ϵ_{gt}	govt. spending innovation
χ_t	labor supply shock
$\epsilon_{\chi t}$	labor supply innovation

Table A.2: Variables in the sticky price/wage model in EGG (2005)

Parameter	Description	Value
β	discount rate	$1.03^{-1/4}$
χ_0	s.s. normalization	1
$1/\chi$	Frisch elasticity of labor supply	1/1.5
δ	depreciation rate	0.02
θ	elasticity of output to capital	0.35
μ_z	deterministic trend	0.0037
g/y	govt. spending share of GDP	0.20
i/y	investment share of GDP	0.20
τ_N	avg. labor tax rate	0.22
τ_K	capital tax rate	0.38
ρ_g	autocorr. of govt. spending shocks	0.98 (0.6)
σ_g	std. dev. of govt. spending shocks	0.003
ρ_{τ_N}	autocorr. of labor tax shocks	0.98 (0.6)
σ_{τ_N}	std. dev. of labor tax shocks	0.0052
ρ_χ	autocorr. of preference shocks	0.95 (0.6)
σ_χ	std. dev. of preference shocks	0.0619
γ_i	int. rate smoothing	0.80
γ_π	int. rate response to inflation	0.61
γ_y	int. rate repsonse to GDP gr.	0.28
σ_f	std. dev. of mon. policy shocks	0.06/4
σ_z	std. dev. of technology shocks	0.0152
ϕ_c	habit persistence parameter	0.6
ν_0	s.s. normalization	s.t. $u = 1$
ν	elasticity of utilization cost	0.01
ϕ_i	inv. adj. cost parameter	2
$1 - \psi_p$	price reset probability	0.25
$1 - \psi_w$	wage reset probability	0.25
θ_p	price markup	0.05
θ_w	wage markup	0.2

Table A.3: Calibration of the sticky price/wage model in EGG (2005)

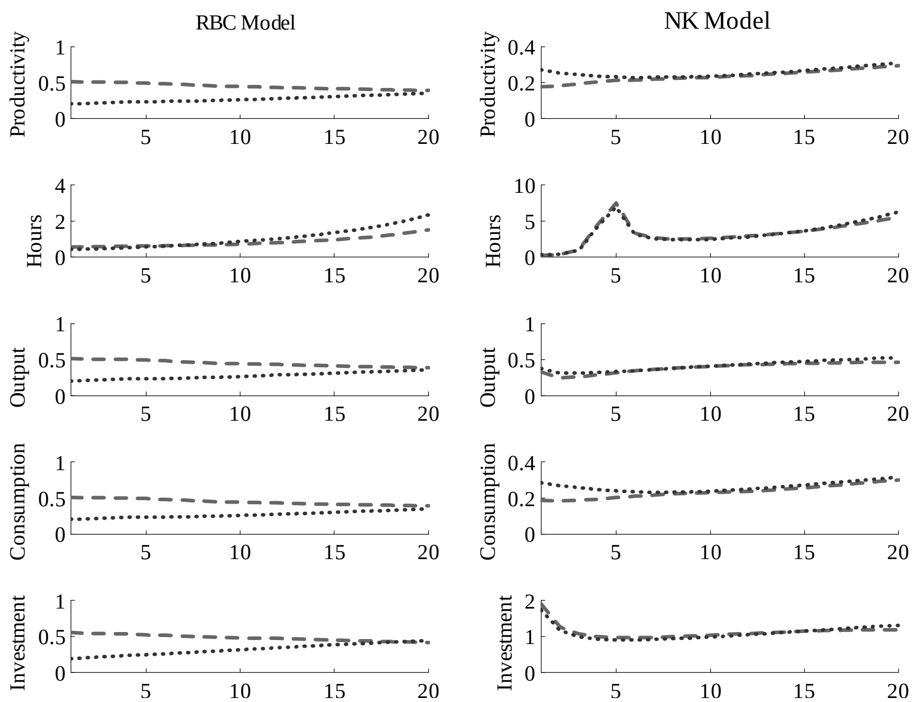

Figure B.1: Average absolute bias in LR (dashed) versus Max Share (dotted) across alternative maximization horizons for simulated data.

Bias is measured as the absolute difference between the median Max Share (or LR) and theoretical responses, averaged over the first four quarters. The underlying Max Share and LR responses are averages across 1,000 median estimates, each representing what an econometrician would estimate based on a sample with 247 observations.

Correlation Between Estimated and Model Technology Shocks					
RBC Model					
	Parameterization	Identification	16^{th} percentile	Median	84^{th} percentile
(1)	Baseline	LR	0.09	0.50	0.75
		Max Share	0.67	0.81	0.89
(2)	Less Persistent Non-technology Shocks	LR	0.10	0.55	0.81
		Max Share	0.85	0.90	0.93
NK Model					
	Parameterization	Identification	16^{th} percentile	Median	84^{th} percentile
(3)	Baseline	LR	0.39	0.68	0.85
		Max Share	0.68	0.81	0.89
(4)	Less Persistent Non-technology Shocks	LR	0.15	0.48	0.68
		Max Share	0.63	0.74	0.82

Table B.1: Correlations between the shocks estimated with LR/Max Share and the true shocks calculated for 1,000 Monte Carlo draws. The median, 16th, and 84th percentiles from the posterior distributions of the corrleations are reported.

variance of long-run labor productivity. In this section, we allow the non-technology shocks to play a greater role in determining labor productivity at long horizons. Specifically, the non-technology stochastic processes—e.g., government spending, capital, and/or labor taxes—are assumed to be highly persistent, with their innovation variances set equal to the variance of technology. Technology, however, remains the source of the unit root in productivity, consistent with (**??**). Increasing the persistence and variances of the non-technology processes allows them to have greater influence on labor productivity at horizons beyond the business cycle. This can be a source of possible contamination, making it more difficult for the either identification approach to isolate the technology process. Because (**??**) still holds, LR is still valid at the infinite horizon, potentially giving LR an advantage over Max Share, all else equal.

Figure **??** shows the responses for this parameterization of the RBC model when technology has a unit root, all non-technology processes have AR(1) coefficients of 0.6, and all stochastic processes have equal variances. This parameterization may make it easier to differentiate between the unit root shock and the less persistent shocks. This may give LR a better chance to identify

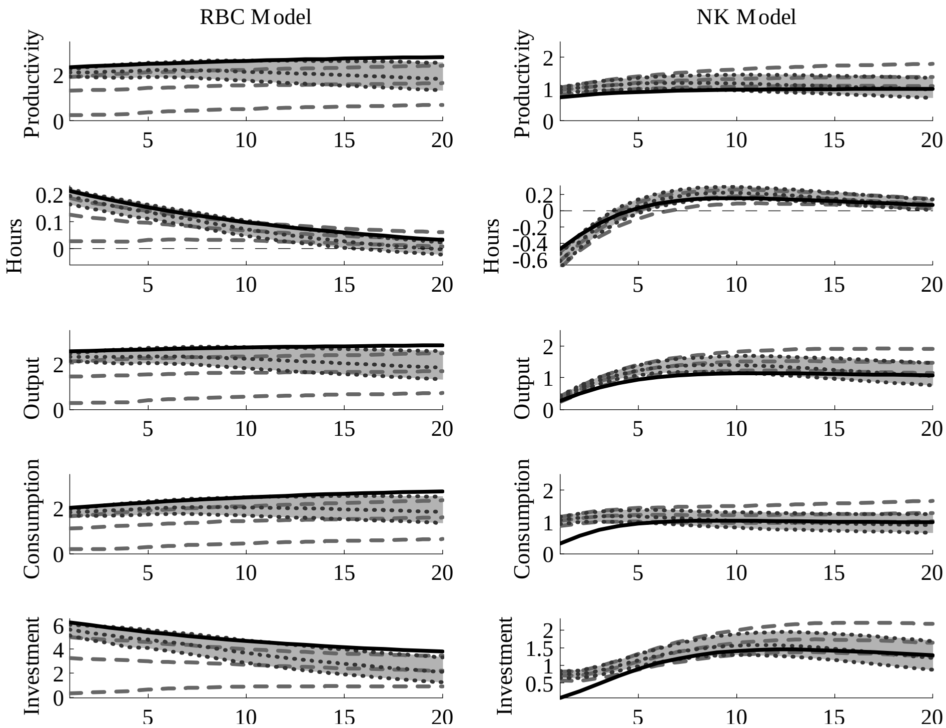

Figure B.2: Impulse responses to a technology shock: robustness analyses.

Theoretical responses [with AR(1) technology coefficient $\rho_z = 1.0$ and nontechnology AR(1) coefficients $\rho = 0.6$] are shown by thick solid lines. Median and 68-percent probability intervals for Max Share from Monte Carlo experiments are shown with dashed lines and shaded areas. LR median responses and 68-percent probability intervals are shown by dotted lines. Median estimates and error bands are averages across 1,000 estimates, each representing what an econometrician would estimate based on a sample with 202 observations and 1,000 draws from the posterior distributions for the impulse responses.

the technology shock. Figure **??** shows the responses for the NK model with less persistent nontechnology shocks (see Table A.3 in the appendix for details of the parameterization). Perhaps surprisingly, these results are similar to the previous parameterizations—the Max Share impulse responses demonstrate less bias than the LR responses. Decreasing the importance of nontechnology shocks assists both methods in identifying the unit root shock, but Max Share typically remains less biased overall. The second and fourth rows of Table B.1 shows that the Max Share-identified shocks are, on average, still more closely correlated with the model-generated shocks. Rows 1 and 3 are the results from the baseline model included for comparison. Therefore, even in the presence of less influential, non-technology components, the Max Share identification still outperforms the conventional identification approach.

C U.S. Data

For the analysis of U.S. data, we use data from the St. Louis Fed's FRED and updated hours data from Francis and Ramey (2009).[4] The sample spans 1948:Q2 throughout 2009:Q4. The VARs include productivity (in log-level or growth rate), log hours, log consumption- and investment-to-output ratios:

$$
\begin{aligned}
y_t^{LR} &= \left[\Delta \log\left(\frac{Y_t^R}{H_t^{Tot}}\right), \log\left(H_t\right), \log\left(\frac{C_t^N}{Y_t^N}\right), \log\left(\frac{I_t^N}{Y_t^N}\right)\right]', \\
y_t^{MS} &= \left[\log\left(\frac{Y_t^R}{H_t^{Tot}}\right), \log\left(H_t\right), \log\left(\frac{C_t^N}{Y_t^N}\right), \log\left(\frac{I_t^N}{Y_t^N}\right)\right]'.
\end{aligned}
$$

[4] See `http://research.stlouisfed.org/fred2/` and `http://weber.ucsd.edu/~vramey/research.html`.

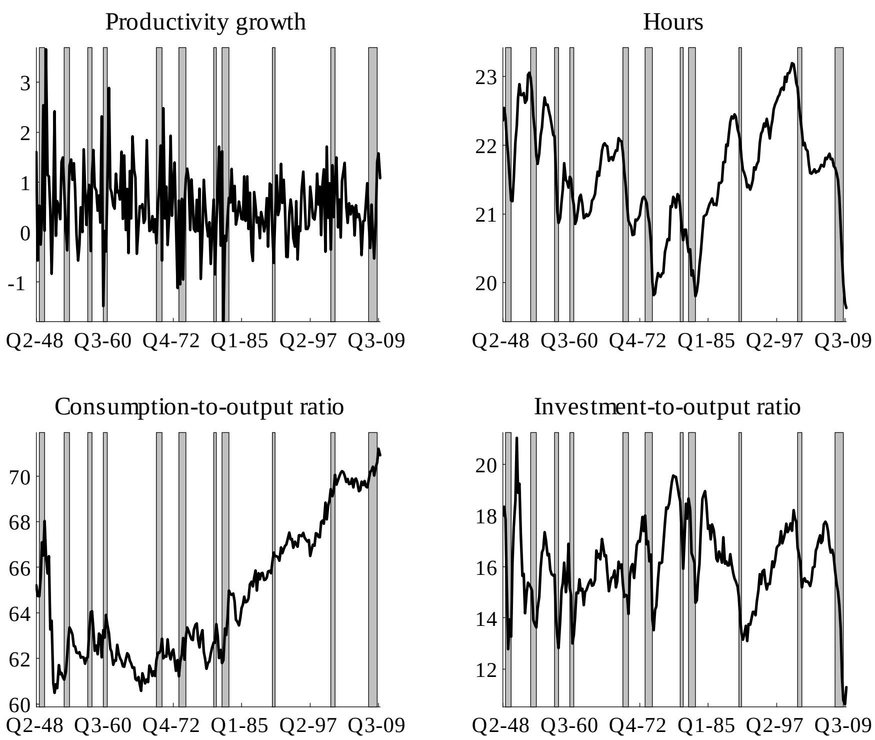

Figure C.1: US data.

Variable	**Description**	**Units**	**Source**
Y^N	Gross Domestic Product	Billions of dollars	FRED (GDP)
Y^R	Real GDP	Billions of chained 2005 dollars	FRED (GDPC1)
C^N	Personal Consumption Expenditures	Billions of dollars	FRED (PCEC)
I^N	Gross Private Domestic Investment	Billions dollars	FRED (GPDI)
H^{Tot}	Total hours worked	Billions of hours	Francis and Ramey (2009)
H	Average weekly hours	Hours per week	Francis and Ramey (2009)

Table C.1: U.S. data; sample 1948:Q2 to 2009:Q4.

References

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