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Working Paper Number	2018-012D
Revision Date	October 2019
Citable Link	https://doi.org/10.20955/wp.2018.012
Suggested Citation	Kozlowski, J., 2019; Long-Term Finance and Investment with Frictional Asset Markets, Federal Reserve Bank of St. Louis Working Paper 2018-012. URL https://doi.org/10.20955/wp.2018.012

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Long-Term Finance and Investment with Frictional Asset Markets

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September 26, 2019

Abstract

Trading frictions in financial markets affect more long- than short-term bonds generating an upward sloping yield curve. Long-term financing is more expensive in economies with higher trading frictions so firms choose to borrow and invest in shorter horizons and lower productivity projects. The theory guides a new identification of the slope of liquidity spread in the data. We measure and calibrate the model for the US, and counterfactual exercises suggest that variations in trading frictions can have significant effects on maturity choices and investment. A policy intervention improves liquidity, reduce long-term financial costs and promotes investment in longer-term projects.

JEL Classifications: E44, G30, O16.

Keywords: Debt maturity, Liquidity, Over-the-counter market, Secondary markets.

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*My special thanks and gratitude to my advisor Ricardo Lagos and committee Thomas Sargent, Laura Veldkamp and Venky Venkateswaran for their encouragement and support. I thank participants at seminars and conferences for helpful comments and Sebastian Di Tella and Konstantin Milbradt for insightful discussions. The views expressed are mine and do not necessarily reflect official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors. I thank Mahdi Ebsim for his valuable assistance with the data. Email: kozjuli@gmail.com.

1 Introduction

Firms in emerging economies tend to borrow and invest at shorter maturities compared to those in advanced countries.[1] This paper develops a theoretical and empirical framework to study the term-structure of liquidity spreads and the interaction with maturity and investment choices of firms. The theory builds on the following key feature of capital markets: Both in advanced and emerging economies, corporate debt markets exhibit trading frictions. The main result is that more severe frictions make long-term finance relatively more expensive, inducing firms to borrow and invest at shorter horizons. We use the insights of the model to measure the slope of liquidity spreads in the data and quantify the theory.

The central mechanism of this paper is that a long-maturity asset will trade in the secondary market more times than a short-maturity one. Hence, the lack of liquidity in secondary markets—a severe trading friction—affects long-maturity assets more than short ones, generating two important results for the yield curve. First, the liquidity spread increases with maturity. Second, economies with less-liquid secondary markets have a steeper yield curve, and firms invest at shorter horizons and in lower productivity projects. A calibrated model matches key features of the yield curve and gestation lags of projects in the US. Counterfactual exercises show that variations in trading frictions generate sizable variation in maturity choices and profitability. Finally, a policy intervention with subsidized financial intermediaries can improve liquidity, stimulate long-term finance, and induce investment at longer horizons, generating substantial welfare gains.

The modeling framework combines an over-the-counter (OTC) secondary market for debt as in Duffie et al. (2005) with a fairly standard production economy. To finance investment, firms borrow in the primary market and choose the maturity structure of their liabilities. On the real side, investors choose the gestation period of their project, taking into account that longer-term projects have higher productivity but are more expensive due to an upward sloping yield curve.

Liquidity in secondary markets shapes interest rates in two ways. First, the liquidity spread is increasing in maturity. Liquidity-need shocks hit debt holders, which cause them to become potential sellers. However, trading frictions prevent them from immediately selling the asset as they need to search for a counterpart and bargain over the terms of trade. Alternatively, the maturity of the asset also provides liquidity to debt holders. Hence, gains from trade in the secondary market increase with the maturity of the asset, which delivers an upward-sloping yield curve. Second, liquidity is more important for long-term assets in the sense that a reduction

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[1] For empirical evidence see Demirgüç-Kunt and Maksimovic (1998) and Levine (2005), among others. Some policy concerns are expressed in World Economic Forum (2011); European Commission (2013); OECD (2013); Group of Thirty (2013), and World Bank (2015).

in trading frictions not only reduces the liquidity spread for all maturities but also lowers the slope of the yield curve. Therefore, improvements in liquidity generate a flattening of the yield curve.

Decentralized asset markets affect investment costs, which propagate through the real economy. We use Compustat data to show that there exists a positive relation between maturity and profitability across sectors. Thus, the model assumes that long-maturity projects have high returns but need financing for a prolonged horizon. If the yield curve is upward sloping, short-term projects are relatively more attractive than longer ones. As a result, variations in trading frictions change the steepness of the yield curve and distort financial and investment choices.

Free entry to the secondary market determines the equilibrium liquidity. To evaluate variations in trading frictions—i.e., financial development—we consider changes in the matching efficiency of the secondary market. In more-efficient financial markets, more buyers are willing to enter so liquidity increases in equilibrium. Financial development generates more liquid markets in which liquidity spreads diminish for all maturities, but in particular for long-term debt. As a result, firms invest in more profitable and high-productivity, longer-term projects. This result suggests that the empirical evidence that firms in emerging economies borrow at shorter maturities can be the result of a substitution effect between maturity and liquidity of secondary markets.

The insights from the theory provide a novel identification of the slope of liquidity spreads by studying firms that issue two or more bonds of different maturity on the same day. If default spreads are constant across maturities, the difference on credit spreads across maturities within firm issuances on a given day uncover the slope of liquidity spreads. Several validation exercises using safe but illiquid assets and credit default swaps show that it is reasonable to assume that, for this sample, default is constant in maturity. Intuitively, the identification works because default is a property of the firm with similar effects for all bonds of different maturities issue on a given date. Liquidity, however, is a characteristic of each security and maturity is a key variable because assets of the same firm but of different maturities have different liquidity spreads. We apply the empirical strategy to the US and find that each additional year increases liquidity spreads by about five basis points. These estimates are useful to calibrate the model and perform counterfactual experiments.

We calibrate the model to the US, targeting the slope of the yield curve among other standard moments. Next, we validate the estimation with additional measures of liquidity. In particular, the model does a reasonable job on also matching the level of the liquidity spread, which is not a target of the calibration. Counterfactual experiments show that variations in trading frictions generate sizable effects on maturity choices and profitability. As an example, we apply the empirical strategy to measure the slope of liquidity spreads for Argentinean data and find that liquidity spreads increase more with maturity in Argentina than in the US. Then, we discipline a counterfactual search friction with the estimates for Argentina and find that variations in trading frictions account for 20% of the maturity differences between Argentina and the US, leading to a lower profitability for Argentina than the US. Although the model is stylized and tractable, the quantitative results suggest that the theory captures important features of corporate debt markets.

The main results are robust both quantitatively and qualitatively in several extensions. First, in the benchmark model, firms can borrow only at the beginning of the project, so the maturity of the project matches the maturity of financing by assumption. An extension of the model allows entrepreneurs to rollover short-term contracts to finance long-term projects with a fixed cost of issuance. Quantitatively, the effect of a change in liquidity on the choice of projects is similar both with and without rollover opportunities. Second, in the benchmark model, there is a single secondary market for assets of different maturities. This paper also considers a specification in which buyers direct themselves to markets segmented by maturity. The main takeaway is that even though market tightness (defined as the ratio of sellers-to-buyers) for short-term debt increases, tightness for long-term assets remains similar to the benchmark model with a single market. As a result, the yield curve is similar to the curve for the benchmark economy. Finally, we show that the main predictions of the model apply to bank lending instead of corporate bonds.

The presence of frictional asset markets suggests that a policy intervention that increases the liquidity of financial markets and improves credit conditions for the corporate sector can benefit the real economy. Based on existing policies like Government-Sponsored Enterprises (i.e., Fannie Mae and Freddie Mac) or large-scale asset purchases (i.e., quantitative easing), this paper evaluates one intervention that subsidizes financial intermediaries in the secondary market, named *Government-Sponsored Intermediaries* (GSIs). The government has four instruments: the size of the intervention, the prices at which the government-sponsored dealers buy and sell from private investors, and a distortionary tax rate to finance the costs of GSIs. Under the optimal policy, government intermediaries buy at higher prices than those in private meetings to provide more gains from trade to private sellers. On the other hand, government agents sell securities at a lower price than those in private meetings to stimulate the entry of potential private buyers. The optimal policy increases the liquidity, flattens the yield curve, and stimulates the use of long-term finance. The policy generates larger gains when there are more trading frictions.

Related literature Many papers have studied the term structure of interest rates through the lens of the consumption-based capital asset pricing model (see Gürkaynak and Wright, 2012, for a recent review). Those papers extend the expectation hypothesis framework, which posits that long-term interest rates are expectations of future average short-term rates. This paper is closer to a classic idea present in several discussions, such as empirical papers or textbooks (e.g., Mishkin, 2015), that attribute the shape of the yield curve to liquidity considerations. Geromichalos et al. (2016) propose a monetary-search model, with assets of two maturities, to rationalize the term-structure of liquidity spreads. The main contribution of this paper is to present a model with a continuum of maturities and study both theoretically and quantitatively the implications for maturity choices of the corporate sector, the effects on investment, and the role of policy interventions.[2]

More generally, the paper relates to the literature on OTC markets following the seminal work of Duffie et al. (2005). Some papers applied the theory to corporate bond markets (e.g., Chen et al., 2012; He and Milbradt, 2014; Chen et al., 2017), other studied the interaction between safety and liquidity (e.g., He and Milbradt, 2014; Geromichalos et al., 2018), while others consider the interaction between primary and secondary markets (Bruche and Segura, 2017; Arseneau et al., 2017; Bethune et al., 2017). In particular, the financial structure is a hybrid between He and Milbradt (2014) and Bruche and Segura (2017). On the one hand, He and Milbradt (2014) study the interaction between liquidity and default and abstract from the effects on maturity. They apply the model to the US corporate debt market in a framework with exogenous elements such as profitability, maturity, and liquidity (in the sense that meeting intensities are not an equilibrium outcome). In contrast, this paper studies the interaction between liquidity and maturity so it is key to have both maturity and liquidity as equilibrium outcomes. On the other hand, the paper extend and complement the analysis of Bruche and Segura (2017) (BF henceforth). The contribution of BS is normative, showing that search frictions in secondary markets induce firms to choose inefficiently low maturities. This paper, however, is on the positive side and studies how variations in search frictions affect both liquidity spreads and firms' choices. Moreover, BS is silent about the effect of trading frictions for the term-structure of liquidity spreads while this paper shows, both analytically and empirically, that liquidity spreads are increasing in maturity.[3]

This paper also contributes to the literature on maturity choice by proposing a novel channel

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[2] The version of the model with segmented markets provides a rationale for the model of Vayanos and Vila (2009) in which investors have preferences for specific maturities, so interest rates are influenced by demand and supply shocks local at that maturity.

[3] There are some modeling differences with BS as well. For example, BS assume exogenous and constant profits and simplify the maturity to Poisson arrivals rather than at a deterministic dates. Moreover, BS does not provide any empirical or quantitative results, while those are key contributions of this paper.

based on trading frictions in the secondary market, which generates an upward sloping yield curve. In the canonical models of Diamond (1991) and Leland and Toft (1996), frictions between lenders and borrowers shape maturity choices, while in this paper the friction is within lenders in financial markets. Milbradt and Oehmke (2015) also study the joint determination of financing terms and investment decisions in which limited commitment between borrowers and lenders leads the firm to adjust their investment toward shorter-term, second-best projects, while in this paper the friction is within lenders in financial markets.

More broadly, this paper provides a different perspective on how financial development can influence aggregate outcomes, the subject of a large body of work (e.g., Greenwood et al., 2010; Buera et al., 2011; Moll, 2014; Midrigan and Xu, 2014; Cole et al., 2016, among others). All these papers focus on contracting frictions between lenders and borrowers and interpret financial development as a reduction of that friction.[4] Instead, this paper considers trading frictions within lenders in financial markets, in which financial development increases the liquidity of the market and focuses on the choice of maturity, which is absent in previous analyses. Choudhary and Limodio (2018) explore a natural experiment in Pakistan and show that liquidity can affect long-term finance and aggregate outcomes.

The rest of the paper is organized as follows. Section 2 presents the model and Section 3 characterizes the equilibrium. Section 4 describes the empirical strategy and results, while Section 5 quantifies the model and performs counterfactual experiments. Next, Section 6 introduce rollover and performs policy analysis. Finally, Section 7 concludes. Proofs, additional results, and data descriptions are gathered in the Appendices.

2 Model

Time is continuous, starts at $t = 0$, and goes on forever. The economy is populated by agents in the financial and production sector. Corporate bonds trade in an OTC secondary market by members of the financial sector. In the production sector, firms choose investment projects from a menu of opportunities such that the return increases with the duration of investment. To finance the projects, firms borrow from the financial sector.

2.1 Financial Sector

Primary Market In the primary market, firms from the production sector issue corporate bonds and lenders from the financial sector buy those securities. Assume that there are no

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[4] While traditional models of maturity choice also consider frictions between borrowers and lenders, I'm not aware of an application to study the maturity choices of firms in emerging economies. One exception is Broner et al. (2013) which studies the maturity choices of sovereigns in emerging economies.

frictions in this market and that there is a large mass of potential lenders willing to buy the assets. Section 6.1 extends the model to add frictions in the primary market and Section B.4 shows that we can also interpret the model as bank lending instead of corporate bonds. Free entry into the primary market implies that the price of the bond is equal to the value of holding the asset for a lender,

$$P(\tau, \lambda) = D^H(\tau; \lambda), \tag{1}$$

where $D^H(\tau, \lambda)$ is the value of holding a bond with time-to-maturity τ when the liquidity of the secondary market is λ.

Secondary Market The financial sector trades corporate bonds in an OTC market as in Duffie et al. (2005). An agent of the financial sector can have either zero or one asset.[5] An agent without the asset can pay a search cost c, enter into the secondary assets market, and search for a counterpart. There is a large measure of potential entrants, which implies a free-entry condition to the market.

An agent can buy the asset in either the primary or secondary market, and always starts as a *high valuation* agent. However, the agent faces an idiosyncratic liquidity risk of becoming *low valuation*. With Poisson intensity η, a high valuation agent becomes low valuation and has to pay a holding cost h per unit of time.[6] This idiosyncratic risk generates differences in valuations, causing motives for trade in the secondary market. Note that asset holders are heterogeneous in two dimensions. First, they can be either high or low valuation. Second, they hold assets with time-to-maturity $y \in [0, \tau]$. Let $\mu^H(y)$ and $\mu^L(y)$ denote the measure of high- and low-valuation agents holding an asset of time-to-maturity y, respectively.

All the low-valuation agents are the sellers in the secondary market. There is random matching in this market, so assets of different time-to-maturity trade in the same market and the total mass of sellers is $\mu^S = \int_0^\tau \mu^L(y)dy$. Appendix B.3 extends the model to consider markets segmented by maturity and shows that the results are similar to the model with a single secondary market. On the other hand, a measure μ^B of buyers are agents without an asset searching in the secondary market. Assume a constant-returns-to-scale matching function between buyers and sellers, $M\left(\mu^S, \mu^B\right) = A\left(\mu^S\right)^\alpha \left(\mu^B\right)^{1-\alpha}$, and define the market tightness as the ratio of sellers-to-buyers, $\theta = \frac{\mu^S}{\mu^B}$.

A seller finds a counterpart at rate $\lambda = A\theta^{\alpha-1}$, and a buyer finds a counterpart at rate

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[5] This portfolio restriction is common in the literature because it simplifies the tractability of the model. See Lagos and Rocheteau (2009) for a model with unrestricted asset holdings.

[6] The modeling assumptions about high- and low-valuation agents are standard in the literature (e.g., Duffie et al., 2005; He and Milbradt, 2014). A low investor may have (i) high discounting, (ii) high financing costs, (iii) hedging reasons, (iv) tax disadvantage, or (v) lower personal use of the asset.

$\beta = A\theta^{\alpha}$. Upon a match, with probability $\frac{\mu^L(y)}{\mu^S}$ a buyer meets with a seller of an asset with time-to-maturity y. It is useful to define the liquidity of the secondary market as λ because it is the key object through which the secondary market feeds back into the primary market and affects the borrower's problem.[7]

Let $P^S(y;\lambda)$ be the price in the secondary market for an asset of time-to-maturity y and assume a Nash Bargaining protocol. Let γ be the bargaining power of the seller so that

$$P^S(y) = \arg\max_{P^S(y)} \left(P^S(y;\lambda) - D^L(y;\lambda)\right)^{\gamma} \left(D^H(y;\lambda) - P^S(y;\lambda)\right)^{(1-\gamma)}, \tag{2}$$

where $D^H(y;\lambda)$ is the value of holding an asset for a high-valuation agent—the buyer—and $D^L(y;\lambda)$ is the value of holding an asset for a low-valuation agent—the seller.

The value of search in the secondary market for a buyer, $D^S(\lambda)$, is

$$\rho D^S(\lambda) = -c + \beta \int_0^{\tau} \frac{\mu^L(y)}{\mu^S} \left(D^H(y;\lambda) - D^S(\lambda) - P^S(y;\lambda)\right) dy. \tag{3}$$

The discounted value of search is equal to the search cost and the expected gains from trade. With intensity $\beta\frac{\mu^L(y)}{\mu^S}$, a buyer matches with a seller of a bond with time-to-maturity y, and gains from trade are $D^H(y;\lambda) - D^S(\lambda) - P^S(y;\lambda)$. The buyer becomes a high-valuation agent, with value $D^H(y;\lambda)$, and pays the price $P^S(y;\lambda)$. Free entry into the secondary market implies that, in equilibrium, $D^S(\lambda) = 0$.

2.2 Production Sector

Every period, a measure μ^0 of identical entrepreneurs chooses a new project from a menu differentiated by the life-cycle of returns. The main idea of the production side is to capture, in an stylized fashion, an entrepreneur choosing across sectors to invest. We model sectors by their maturity τ and profitability rate π. To fix ideas, consider two type of projects. On the short-term side, the entrepreneur can construct a new building. This project needs financing for a short period (a low maturity τ) but also has a low profitability rate. On the long-term side, the entrepreneur can develop a transportation company (e.g., a new airline), but this project needs financing for an extended period and has a higher profitability rate. While there are several differences across sectors, the objective is to study how frictions on the capital market can generate variations in the type of projects that entrepreneurs take.[8] Section 4.2 shows

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[7] Note that $\beta = A\left(\frac{\lambda}{A}\right)^{\frac{\alpha}{\alpha-1}}$. It is equivalent to define functions depending on the market tightness θ, but it is easier to derive the intuition of the results thinking in the space of the selling intensity λ.

[8] Theories of R&D with financial frictions, for example, can endogenously generate this reduced-form relationship (e.g., Aghion et al., 2010). There are several empirical studies about the relationship between maturity,

that indeed in the data a firm in the construction division has a maturity of 3.2 years and a profitability of 7.5%, while a firm in the transportation division has maturity of about 8 years and a profitability rate above 12%. A very simple but tractable way is to directly assume a reduced-form relationship in which the profit rate increases with maturity.

To capture this fact in a stylized model, we make a stark assumption and divide the life-cycle of firms into two stages: (i) *investment* for $t \leq \tau$ and (ii) *production* for $t > \tau$.[9] An entrepreneur chooses τ, the age at which it starts production. Therefore, entrepreneurs have a menu of potential projects summarized by $\tau \geq 0$. With Poisson arrival rate δ, the firm is hit by an exit shock and the value of the project goes to zero.[10]

In the first stage, the entrepreneur pays κ per unit of time to invests in research and development such that at maturity τ the project has a profitability $\pi(\tau)$ that is increasing in maturity. At age τ, the firm stops investing and starts production. The net present value of a firm that spent τ periods doing R&D is $F(\tau) = \pi(\tau) \int_0^\infty e^{-(\rho+\delta)t} dt$, where ρ is the discount factor. All firms are identical, so in equilibrium they choose the same maturity τ and have the same profitability π.

Borrowing Firms do not have internal funds and need to borrow for investment. The benchmark model assumes that firms borrow only at the beginning of the project; i.e., they match the maturity of the project and the debt. This assumption helps us to obtain a sharp characterization of the equilibrium and captures the idea that it might be costly or risky to borrow short-term to finance long-term projects. In fact, Section 4.2 reviews the empirical evidence showing that firms tend to match the maturity of assets and liabilities. Section 6.1 relaxes this assumption and allows firms to issue short-term debt to finance long-term projects. In that extension we assume a fixed cost of issuance and find similar results as in the benchmark model.[11]

Corporate bonds are zero-coupon, have a default arrival rate δ, and a face value of one. The firm deposits the proceeds from the issuance in a bank account with risk-free rate ρ and withdraws κ per unit of time for investment. Hence, a firm needs initial funds equal to $I(\tau) =$

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investment, and the life-cycle of cash-flows. For example, R&D or product quality can affect future profitability (e.g., Branch, 1974; Phillips and Sertsios, 2013). There are industry studies such as Mendes (2019) about the production cycle of the wine sector in Portugal, or Gilje et al. (2019) about the oil and gas industry. Finally, there is also evidence on the role of financial frictions for long-term investment (e.g., Choudhary and Limodio, 2018; Garicano and Steinwender, 2016).

[9] We assume that a firm does not produce in the investment stage. Results are similar if we allow firms to start producing earlier and use internal funds to finance a fraction of investment, while κ is the external funds needed.

[10] To reduce notation, assume that this process has the same intensity for firms in the investment and production stages. However, it is simple to consider two different processes.

[11] Appendix B.4 argues that we would obtain similar results in a model with bank lending or equity finance instead of corporate bonds.

$\kappa \frac{1-e^{-\rho\tau}}{\rho}$ to invest for τ periods. The firm takes the price of a bond with maturity τ and liquidity λ, $P(\tau, \lambda)$, as given and chooses maturity τ and issues debt B to maximize its value,

$$\max_{\tau,B} e^{-(\rho+\delta)\tau} \left(F(\tau) - B\right) \qquad \text{s.t. } BP(\tau, \lambda) = I(\tau). \tag{4}$$

2.3 Equilibrium

Definition 1 states the steady-state equilibrium.

Definition 1. *A steady-state equilibrium is characterized by the selling intensity λ, debt maturity τ, prices in the primary market $P(y; \lambda)$, prices in the secondary market $P^S(y; \lambda)$ and measures $\mu^H(y)$, and $\mu^L(y)$ such that:*

1. *Free entry into the primary market solves (1);*
2. *$P^S(y, \lambda)$ solves the Nash Bargaining problem (2);*
3. *$D^S(\lambda) = 0$ solves free entry into the secondary market (3); and*
4. *Firms in the corporate sector solve (4).*

3 Equilibrium Characterization

This section characterizes the solution of the model. First, we solve for the distribution of agents in the financial sector. Then, the main results show how the liquidity of the secondary market affects prices in the primary market, interest rates, and maturity choices. Next, a fixed point between the maturity choice τ and liquidity λ characterizes the equilibrium of the model. Finally, counterfactual exercises examine the effects of higher trading frictions.

3.1 Lenders

First, we solve for the distribution of high- and low-valuation agents holding assets of different maturities. Buyers from primary and secondary markets start as high valuation. Over time, some agents receive liquidity shocks while others trade in the secondary market. The laws of motion for the measure of high- and low-valuation agents are

$$-\dot{\mu}^H(y) = -(\eta + \delta)\, \mu^H(y) + \beta \frac{\mu^L(y)}{\mu^S} \mu^B, \tag{5}$$

$$-\dot{\mu}^L(y) = \eta \mu^H(y) - (\delta + \lambda)\, \mu^L(y). \tag{6}$$

Equations (5) and (6) show that as we move closer to maturity (lower y), a fraction η of high-valuation agent becomes low-valuation agents, and a fraction δ of both types of agents is holding an asset that is hit by a default shock. Moreover, a measure $\beta\frac{\mu^L(y)}{\mu^S}\mu^B$ of buyers finds a counterpart in the secondary market and becomes high-valuation agents. Finally, a measure λ of low-valuation agents is able to sell in the secondary market. At issuance τ, all agents buy in the primary market and are high valuation, so boundary conditions are $\mu^H(\tau) = \mu^0$ and $\mu^L(\tau) = 0$. Lemma 1 characterizes the steady-state distribution of financiers.

Lemma 1. *The distribution of financiers is given by*

$$\mu^H(y) = \frac{\mu^0\eta}{\eta+\lambda}\left(e^{\delta(y-\tau)}\frac{\lambda}{\eta} + e^{(\eta+\lambda+\delta)(y-\tau)}\right), \tag{7}$$

$$\mu^L(y) = \frac{\mu^0\eta}{\eta+\lambda}\left(e^{\delta(y-\tau)} - e^{(\eta+\lambda+\delta)(y-\tau)}\right). \tag{8}$$

When the secondary market is well-functioning—the selling intensity, λ, is relatively high—the mass of low-valuation agents $\mu^L(y)$ is small. When λ diminishes, the secondary market is more illiquid and the mass of low-valuation agents increases. The measure of low-valuation agents, $\mu^L(y)$, enters into the free-entry condition for the secondary market (3), so we use (8) to solve for the equilibrium liquidity.

Private valuations Next, we derive the value for high- and low-valuation agents to hold the asset,

$$\rho D^H(y;\lambda) = -\frac{\partial D^H(y;\lambda)}{\partial y} + \eta\left(D^L(y;\lambda) - D^H(y;\lambda)\right) + \delta\left(0 - D^H(y;\lambda)\right), \tag{9}$$

$$\rho D^L(y;\lambda) = -h - \frac{\partial D^L(y;\lambda)}{\partial y} + \lambda\left(P^S(y;\lambda) - D^L(y;\lambda)\right) + \delta\left(0 - D^L(y;\lambda)\right). \tag{10}$$

At maturity, both types of investors receive the face value of the asset, implying boundary conditions $D^H(0;\lambda) = D^L(0;\lambda) = 1$. Equation (9) defines the value of high-valuation agents. The left-hand side is the required return from holding the bond. The first term on the right-hand side represents the change in value due to it being closer to maturity. The second term captures the liquidity shocks that transform the investor into a low-valuation agent, which occurs at intensity η. The third term captures the risk of default of the bond. Equation (10) captures the value of low-valuation agents and follows a similar intuition as the previous equation. A low-valuation investor incurs a holding cost h, and with intensity λ the investor meets a counterpart and sells his bond at price $P^S(y;\lambda)$.

Price in secondary market The price in the secondary market, $P^S(y;\lambda)$, is the solution of the Nash Bargaining problem between the seller and the buyer in (2),

$$P^S(y;\lambda) = D^L(y;\lambda) + \gamma\left(D^H(y;\lambda) - D^L(y;\lambda)\right). \tag{11}$$

The gains from trade are $D^H(y;\lambda) - D^L(y;\lambda)$, and the seller gets a fraction γ of them.

3.2 Effects of Liquidity on the Primary Market

The central result of this paper is the characterization of how the liquidity of the secondary market feeds back into prices in the primary market. Proposition 1 solves Equations (9) and (10) using the equilibrium price in the secondary market (11) and characterizes the price in the primary market.

Proposition 1. *The price in the primary market is*

$$P(\tau,\lambda) = e^{-(\rho+\delta)\tau} - \mathcal{L}(\tau,\lambda), \tag{12}$$

where the illiquidity cost $\mathcal{L}(\tau,\lambda)$ is

$$\mathcal{L}(\tau,\lambda) = h \quad \frac{\eta}{\eta+\lambda\gamma} \quad \int_0^\tau e^{-(\rho+\delta)y}\left(1 - e^{-(\eta+\lambda\gamma)y}\right)dy. \tag{13}$$

The illiquidity cost satisfies the following properties:

1. *$\mathcal{L}(\tau,\lambda)$ is non-negative*
2. *Sensitivity with respect to liquidity shocks η:*
   (a) *If there are no liquidity shocks, $\eta = 0$, then $\mathcal{L}(\tau,\lambda) = 0$; and*
   (b) *If $\eta \to \infty$ (i.e., the holder always has to pay the cost h), then*

$$\lim_{\eta\to\infty} \mathcal{L}(\tau,\lambda) = h\frac{1 - e^{-(\rho+\delta)\tau}}{\rho+\delta}.$$

3. *Sensitivity with respect to maturity τ:*
   (a) *$\mathcal{L}(\tau,\lambda)$ is increasing in τ; and*
   (b) *$\mathcal{L}(\tau,\lambda)$ has a finite limit, $\lim_{\tau\to\infty}\mathcal{L}(\tau,\lambda) = h\frac{\eta}{(\rho+\delta)(\rho+\delta+\eta+\gamma\lambda)}$.*
   (c) *If $\lambda \geq \eta$ then $\mathcal{L}$ is concave in τ.*

4. *Sensitivity with respect to liquidity λ:*

    (a) *$\mathcal{L}(\tau, \lambda)$ is decreasing in λ;*

    (b) *If there are no secondary markets, $\lambda = 0$, the liquidity term only represents the expected holding costs; i.e.,*

    $$\mathcal{L}(\tau, 0) = h \int_0^\tau e^{-(\rho+\delta)y} \left(1 - e^{-\eta y}\right) dy;$$

    (c) *If secondary markets are totally liquid (i.e., $\lambda \to \infty$), then $\mathcal{L}(\tau, \lambda) = 0$.*

5. *Liquidity is more important for long-term assets:* $\frac{\partial^2 \mathcal{L}(\tau,\lambda)}{\partial \tau \partial \lambda} \leq 0$.

Proposition 1 shows that we can decompose the price in the primary market $P(\tau, \lambda)$ in two terms.[12] The first component represents the *frictionless* solution: The value of a promise to pay one unit in τ periods when the discount rate is ρ and the default intensity is δ. Note that absent the second term, the expectation hypothesis holds: Long-term interest rates are equivalent to the average of short-term rates. The second term, $\mathcal{L}(\tau, \lambda)$, represents the *illiquidity cost.* When this term is different from zero, the expectation hypothesis does not hold and borrowing at longer horizons becomes more expensive than the average of short-term rates.

The illiquidity cost captures the expected discounted time that the holder of the asset is low valuation and has to pay the holding cost. If there are no secondary markets, the illiquidity cost is equivalent to the holding cost h times the expected discounted length of time between the stopping time in which the agent receives the idiosyncratic shock—which occurs at intensity η—and maturity. On the other hand, if there are no frictions in the secondary market, upon a shock the agent can sell the asset instantaneously and recover the fundamental value, which implies that the illiquidity cost would be equal to zero. Hence, both the ease of selling in the secondary market, captured by λ, and the amount of gains from trade that the seller can retain, measured by γ, shape the illiquidity cost.

To derive some intuition about the illiquidity cost, let $s^H(y)$ and $s^L(y)$ be the adjusted probabilities that a security of age y is held by agents with high and low valuations, respectively.[13] This is an adjusted probability because the transition takes into account the bargaining

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[12] Note that the price can be negative when τ is large enough. A sufficient condition for having a positive price is that $\tau < \frac{1}{\rho+\delta} \log\left(\frac{(\rho+\delta)(\rho+\delta+\eta+\gamma\lambda)}{h\eta}\right)$. When we take the model to the data this condition implies maturity is below 30 years, which is well below the maturities consider in the quantitative model.

[13] Note that y is the age of the asset, not the time to maturity. Also, note that it is absent of default, as we include the default rate δ in the discount factor.

power

$$\dot{s}^H(y) = -\eta s^H(y) + \lambda\gamma s^L(y),$$
$$\dot{s}^L(y) = \eta s^H(y) - \lambda\gamma s^L(y),$$

with initial conditions $s^H(0) = 1$ and $s^L(0) = 0$. Then, the illiquidity cost is

$$\mathcal{L}(\tau, \lambda) = h \int_0^\tau e^{-(\rho+\delta)y} s^L(y)dy.$$

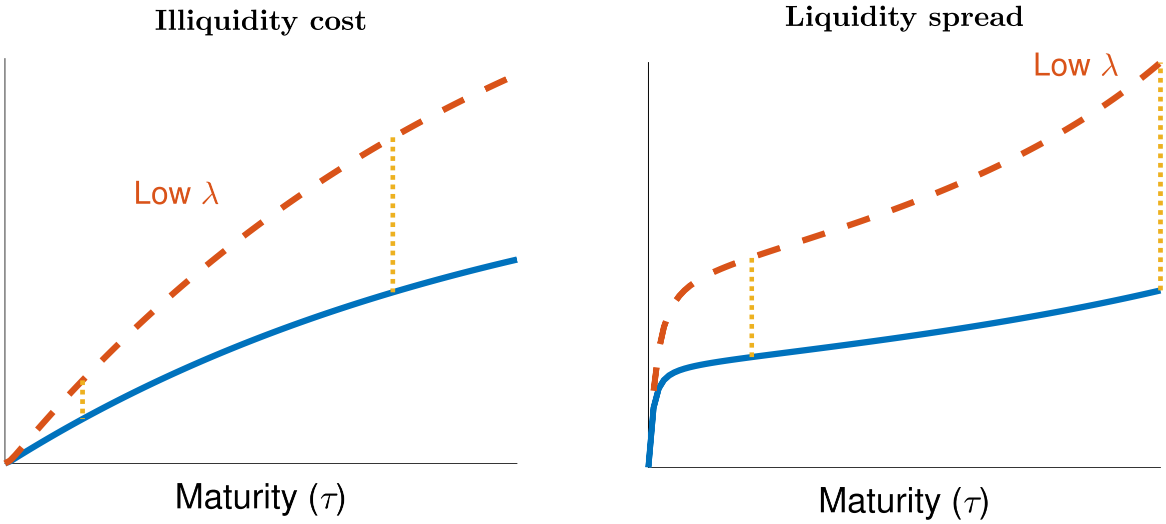
Therefore, the illiquidity cost is proportional to the expected discounted length of time that the holder of the asset has low valuation.

Proposition 1 establishes several results about the illiquidity cost. The first two properties show that $\mathcal{L}$ is non-negative, that if there are no idiosyncratic shocks $\mathcal{L}$ is equal to zero, and that if the agent always has to pay the holding cost then the illiquidity cost is equal to the net present value of paying h. More interestingly, the left panel of Figure 1 summarizes how maturity and liquidity affect the illiquidity cost. First, the illiquidity cost is increasing and concave in maturity. Longer securities spend more time in the market, which increases the expected length of time they are held by low-valuation agents. Second, the illiquidity cost is decreasing in liquidity λ. If the liquidity of the secondary market increases, the holders of that security spend less time paying the holding cost, which implies a lower illiquidity cost.

The central result is that the trading friction is more important for long-term assets; i.e., the cross-partial derivative of the illiquidity cost with respect to maturity and liquidity is negative: $\frac{\partial^2 \mathcal{L}(\tau,\lambda)}{\partial\tau\partial\lambda} \leq 0$. An investor that wants to exit a financial position can either sell in the secondary market or wait until maturity. Hence, the role of the secondary market is more important for an agent holding a longer-term asset. As a consequence, a reduction of the trading friction benefits more long-term than short-term securities. This result highlights the importance of decentralized asset trading to study long-term finance.

Inspection of equation (13) reveals that the product of λ times γ captures the feedback of secondary market liquidity to prices in primary markets, a standard result in the literature. The first term, λ, is the selling intensity in the secondary market—an equilibrium object. If λ increases, it becomes easier to sell in the secondary market. The second term, γ, is the bargaining power of the seller in the secondary market—a parameter. When γ increases, sellers keep a larger fraction of the gains from trade. Therefore, λ captures the feedback of the friction in the secondary market to prices in the primary market and investment choices.

Figure 1: **The Effect of Liquidity on Interest Rates**



Note: Illiquidity cost and liquidity spread are increasing in maturity and decreasing in liquidity λ*. Parameter values from calibration in Section 5.*

Yield curve For the empirical analysis in Section 4, it is useful to transform the price in the primary market into an interest rate schedule. Define $r(\tau, \lambda)$ as the compound interest rate that solves $P(\tau, \lambda) = e^{-r(\tau,\lambda)\tau}$, which is the value of the asset conditional on it being held to maturity without default or trading. The interest rate for a bond of maturity τ is

$$r(\tau, \lambda) = \rho + \delta + \frac{1}{\tau} \log \left(\frac{1}{1 - e^{(\rho+\delta)\tau} \mathcal{L}(\tau, \lambda)} \right). \tag{14}$$

The first term, ρ, is the risk-free rate, while the remaining two terms capture the credit spread. We can decompose the spread into a default and a liquidity component, following He and Milbradt (2014). Consider a marginal investor with no idiosyncratic liquidity risk. Such investor requires an interest rate equal to $\rho + \delta$. Therefore, the credit spread due to default is δ. Finally, define the credit spread due to liquidity by subtracting the default component. Hence, the third term corresponds to the liquidity spread,

$$cs^{liq}(\tau, \lambda) = \frac{1}{\tau} \log \left(\frac{1}{1 - e^{(\rho+\delta)\tau} \mathcal{L}(\tau, \lambda)} \right), \tag{15}$$

and $r(\tau, \lambda) = \rho + cs^{def} + cs^{liq}(\tau, \lambda)$. Note that in this model the term premium—the difference between long- and short-term rates—is equivalent to the credit spread due to liquidity. Hence, variations in interest rates across maturities are only explained by differences in the liquidity component of the security. All assets are traded in the same secondary market; however, the liquidity spread varies with maturity as the importance of the secondary market is different across assets. Section 4 exploits this observation to identify the liquidity component in the data. Lemma 2 describes the properties of the liquidity spread.

Lemma 2. *The liquidity spread* $cs^{liq}(\tau, \lambda)$ *is:*

1. *Increasing in maturity* $\frac{\partial cs^{liq}(\tau,\lambda)}{\partial \tau} \geq 0$*;*
2. *Decreasing in liquidity* $\frac{\partial cs^{liq}(\tau,\lambda)}{\partial \lambda} \leq 0$*; and*
3. *Increasing in the default intensity* $\frac{\partial cs^{liq}(\tau,\lambda)}{\partial \delta} \geq 0$*.*

Lemma 2 establishes three important properties about the liquidity spread. First, it is increasing in maturity and decreasing in liquidity. The right panel of Figure 1 shows the liquidity spread as a function of maturity for two levels of liquidity. Note that an increase in λ has a larger effect on the long-end of the yield curve. These results are also present in the illiquidity cost $\mathcal{L}(\tau, \lambda)$, and the yield curve preserves the properties.

Liquidity spreads are increasing in maturity because the measure of low-valuation agents is also increasing in maturity. The underlying intuition comes from Lemma 1. At issuance there is no low-valuation agents. However, as time evolves, a fraction of the high-valuation agents becomes low valuation, but not all of them are able to sell in the secondary market because of the search friction. As a result, the measure of low-valuation agents increases with maturity. This result implies that the prevalence of search frictions is more important for long-term rates, as shown in Lemma 2.

Lemma 2 also shows that there is a feedback-loop between default and liquidity; the liquidity spread is increasing in the default intensity δ. Note that δ has the same role as the discount factor ρ. An increase in the discount factor decreases the value of illiquidity at maturity, which increases the liquidity spread. Section B.2 extends the analysis and studies how changes in default risk interact with maturity choices.

The liquidity spread increases with maturity, while the default spread is constant. This result holds exactly in this model, but it is likely to hold in a large class of models for the following reasons. On the one hand, an aggregate default shock affects the value of the asset for all agents and is independent of the time-to-maturity. In particular, in this model the value is equal to zero. On the other hand, an idiosyncratic liquidity shock does not affect the value of the asset for other potential buyers or the value that the investor recovers at maturity. However, the closer the maturity of the asset is, the lower the cost associated with holding the security is, due to the frictions in the secondary market. Hence, in more sophisticated models this result might not hold exactly, but these forces indicate that the liquidity component will still affect the term structure, while the default component can generate either upward- or downward-sloping yield curves.[14]

Bid-ask spreads Define the proportional bid-ask spread as the gains from trade normalized by the mid-price,

$$BA\left(y;\lambda\right)=\frac{D^{H}\left(y;\lambda\right)-D^{L}\left(y;\lambda\right)}{1/2\left(D^{H}\left(y;\lambda\right)+D^{L}\left(y;\lambda\right)\right)}.$$

Lemma 3 shows that the proportional bid-ask spread is increasing in maturity. An asset of longer maturity has larger gains from trade, and as a result the bid-ask spread increases with maturity. Importantly, all the predictions in Lemmas 2 and 3 are consistent with the empirical evidence (see, for example, Edwards et al., 2007; Bao et al., 2011).

Lemma 3. *The proportional bid-ask spread is increasing in maturity.*

3.3 Entry to the Secondary Market

The free-entry condition to the secondary market characterizes liquidity λ as a function of the maturity at issuance of the bonds, τ. Replace the equilibrium price in the secondary market (11) in the free-entry condition (3) so that

$$c=\beta(1-\gamma)\int_{0}^{\tau}\frac{\mu^{L}(y)}{\mu^{S}}\left(D^{H}(y;\lambda)-D^{L}(y;\lambda)\right)dy. \tag{16}$$

Proposition 2 analyzes the free-entry condition (16). It describes how the selling intensity changes with the maturity at issuance, $\lambda(\tau)$.

Proposition 2. *$\lambda(\tau)$ is increasing in τ and $\lambda(\tau):\mathbb{R}_{+}\mapsto[0,\overline{\lambda}]$.*

When assets are of zero maturity there are no gains from trade, which implies no entry into secondary markets and a selling intensity equal to zero, $\lambda(0)=0$. Gains from trade are increasing in τ, which implies that there are more incentives to enter into the secondary market as τ increases. Hence, $\lambda(\tau)$ is increasing. When τ goes to infinity the gains from trade are bounded, which implies that λ converges to a finite number. Section 3.5 solves the equilibrium between lenders and borrowers in which $\lambda(\tau)$ represents the *lenders' curve.*

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[14] Indeed, in traditional models of corporate default (e.g., Merton, 1974; Duffie and Singleton, 1999) the change in the term premium with respect to a change in maturity can be either positive or negative.

3.4 Optimal Maturity

On the firm side, the solution of the firm's problem (4) is characterized by the following trade-off:

$$\frac{\partial F\left(\tau\right)}{\partial \tau}=\left(\rho+\delta\right)F\left(\tau\right)+e^{r(\tau,\lambda)\tau}\frac{\partial I\left(\tau\right)}{\partial \tau}+e^{r(\tau,\lambda)\tau}I\left(\tau\right)\mathrm{cs}^{\mathrm{liq}}(\tau,\lambda)\left(1+\epsilon_{\mathrm{cs}^{\mathrm{liq}}}(\tau,\lambda)\right), \tag{17}$$

where $\epsilon_{cs^{liq},\tau}\left(\tau,\lambda\right)$ is the elasticity of the liquidity spread with respect to maturity,

$$\epsilon_{cs^{liq},\tau}\left(\tau,\lambda\right)=\frac{\partial cs^{liq}\left(\tau,\lambda\right)}{\partial \tau}\frac{\tau}{cs^{liq}\left(\tau,\lambda\right)}.$$

Consider a marginal increase in τ. The left-hand side of Equation (17) represents the benefits of operating a firm with higher profitability, and the right-hand side captures the three associated costs. First, a project in which returns are more back-loaded requires more time to become profitable. This implies a higher time-discount on future profits. Second, a larger firm requires more investment. Note that even without financial frictions (i.e., constant interest rates, $r(\tau)=\rho+\delta$) we have an interior solution for τ, even if there is no default, $\delta=0$. Intuitively, there is an interior solution because as the firm chooses a larger τ, it is more costly and takes more time to complete.

The third term of Equation (17) captures the effect of the financial cost. First, as maturity increases, the firm has to pay the liquidity spread for a longer period. Second, Lemma 2 shows that the liquidity spread increases with maturity, which is captured by the elasticity $\epsilon_{cs^{liq},\tau}$. These two forces induce the firm to choose a shorter maturity than in the frictionless economy. Let $\tau(\lambda)$ be the optimal maturity when the selling intensity is equal to λ.

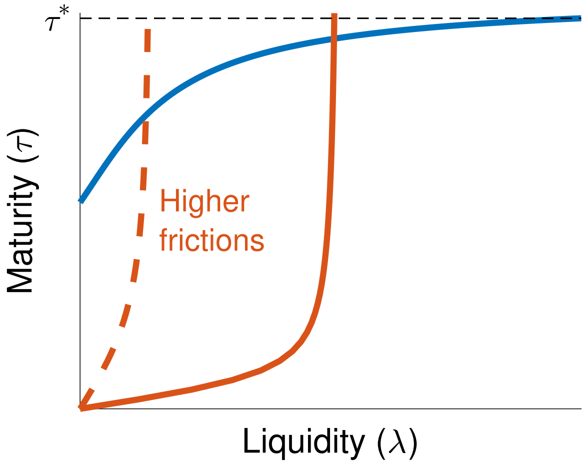
Proposition 3. *The optimal maturity is increasing in the liquidity of the secondary market and* $\tau(\lambda):[0,\overline{\lambda}]\mapsto[\underline{\tau},\overline{\tau}]$ *with* $0\leq\underline{\tau}\leq\overline{\tau}<\infty$.

The optimal maturity increases with the liquidity of the secondary market. By Lemma 2, the liquidity spread is lower when secondary markets are more liquid. This implies that it is cheaper to borrow, particularly at longer horizons. Hence, when λ increases, firms choose projects of longer maturity.

3.5 Equilibrium

The equilibrium is a fixed point between maturity and liquidity. On the one hand, firms take the liquidity of the secondary market as given and choose maturity—Equation (17)—which delivers a curve $\tau(\lambda)$. On the other hand, agents in the financial sector take the maturity of assets as

Figure 2: **Equilibrium characterization**



Note: Maturity and liquidity diminish when trading frictions in financial markets are higher. Parameter values from calibration in Section 5.

given, and the free-entry condition to the secondary market—Equation (16)—delivers $\lambda(\tau)$. An equilibrium is (τ^*, λ^*) such that $\tau(\lambda(\tau^*)) = \tau^*$. Proposition 4 states that an equilibrium exists. The proof follows directly from Propositions 2 and 3. Figure 2 shows the characterization of the equilibrium in the space of liquidity and maturity. The solid red and blue curves show the lenders' and borrowers' choices, respectively. The intersection of these curves characterizes the equilibrium of the economy.

Proposition 4. *A steady-state search and matching equilibrium always exists.*

3.6 Trading Frictions

We evaluate changes in trading frictions on the secondary market by increasing the matching efficiency A.[15] On the one hand, the optimal maturity, characterized by the curve $\tau(\lambda)$, depends on the equilibrium λ but not directly on A, which implies that the curve $\tau(\lambda)$ is independent of A. On the other hand, the curve $\lambda(\tau)$ depends directly on the matching efficiency. For a given maturity and market-tightness, a higher efficiency increase the selling intensity λ. Moreover, it also induces more potential buyers to enter the market, which reduces the market tightness and further increases λ. As a result, the curve moves to the right. The red-dashed curve in Figure 2 shows the new locus for the equilibrium condition under financial development: Both liquidity and maturity increase. This exercise shows that financial development—an increase in the efficiency of secondary markets—causes an increase of debt maturity. In Section 5 we

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[15] Reductions in the search cost c generate similar results.

evaluate this mechanism quantitatively and show that the liquidity of the secondary market can generate quantitatively large movements in maturity choices.

4 Empirical Analysis

This section provides empirical evidence that both liquidity spreads and profitability increase with maturity and provides quantitative guidance for the calibration of the model.

4.1 Maturity and Liquidity Spreads

We use the insights from the theory to measure the term-structure of liquidity spreads in the data. Previous research found that liquidity has stronger implications for long-term assets than shorter ones. For example, Longstaff et al. (2005) studied 68 firms during 2001-2002 and, using Credit Default Swaps (CDS) to control for default, found that there is a positive relation between the non-default component of credit spreads and the time-to-maturity of the bond. Edwards et al. (2007) and Bao et al. (2011) used trading data from secondary markets to estimate several measures of liquidity and found that bonds closer to maturity are more liquid. Gilchrist and Zakrajšek (2012) found that credit spreads increase in maturity after controlling for distance to default and bond-specific characteristics (amount outstanding, coupon rate, callable, industry fixed-effects, and credit rating fixed-effects). We contribute to the existing empirical literature by presenting a new empirical strategy and evidence on the slope of liquidity spreads with respect to maturity, which provide empirical guidelines for the quantification of the model.

We consider corporate debt issuances in the US on the Mergent Fixed Income Securities Database (FISD) and corporate CDS from Markit. We define, following Gilchrist and Zakrajšek (2012), credit spreads that are not subject to the "duration mismatch" by constructing a synthetic risk-free security that mimics exactly the cash flows of the corresponding corporate debt instrument. Appendix D.1 describes the data and the construction of credit spreads.

Empirical specification Let $s_{i,t,m}$ be the credit spread for a bond issued by issuer i, in period t, and of maturity m. As we explain later, a key step to identifying the slope of liquidity spreads consists of considering only firms that in a given period issue two or more bonds of different maturities. The empirical specification is

$$s_{i,t,m_j} - s_{i,t,m_1} = \beta(m_j - m_1) + \gamma \mathbf{X}_{i,t} + \epsilon_{i,t,m_1,m_j}. \tag{18}$$

We exploit the variation on credit spreads across maturities within issuances of a firm in a given date. The coefficient β measures the slope of credit spreads with respect to maturity. If $\beta > 0$, long-term spreads are larger than short-term spreads. The controls $\mathbf{X}_{i,t}$ include time, industry, and/or credit-rating fixed-effects. Appendix C.1 shows that the results are robust to adding non-linear effects which captures the concave implications of the theory.

We find a significant slope of credit spreads to maturity. The first three columns in panel A of Table 1 show different specifications with time and/or industry fixed-effects.[16] When we include time and industry fixed-effects (third column), the slope is about 5 basis points per year. We also exploit the variation within issuer-period assets. We define a "group" as all issuances of the same issuer in a given month and add a fixed-effect at the group level.[17] The fourth column shows that maturity differences within groups are significant and quantitatively similar to the other columns.

The previous estimate cannot distinguish between default and liquidity components. To uncover the slope of liquidity spreads, we propose and validate a novel identification scheme. Assume that for these issuances default spreads are constant in maturity, so the difference between the short- and long-term bonds identifies the slope of liquidity spreads. Theoretically, section 2 shows that if default arrives at a constant Poisson rate, then the credit spread due to default is constant in maturity. However, other theories of default (e.g., Merton, 1974; Duffie and Singleton, 1999) propose that default spreads can be either increasing or decreasing in maturity. More importantly, we perform several empirical exercises to validate the identification assumption. First, we use the information on credit ratings. Following Krishnamurthy and Vissing-Jorgensen (2012), we estimate Equation (18) only for the set of assets that are safe but illiquid and recover similar estimated coefficients. Second, we use corporate CDS to show that default spreads have a smaller estimated slope than the estimated liquidity component. Third, we focus on the set of assets in which we can match corporate bonds with their corresponding CDS to measure the non-default component. The three exercises validate the identification assumption showing that the slope in the benchmark specification can be attributed to liquidity.

Intuitively, focusing on the set of firms with multiple issuances in a given date, helps us to separate the credit spread due to default and liquidity. On the one hand, default is a characteristic of the firm and have similar effects on credit spreads of short- and long-term issuances in a given day. Hence, by focusing on variation within firm issuances on a given date control for the default spread.[18] On the other hand, liquidity is a characteristic of the asset

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[16] We define time fixed-effects at the month level, but results are robust to alternative length periods.

[17] Results are robust for different definitions of period (week instead of month) and if we restrict the sample so that issuers have more than three, four, or five issuances per issuer-period.

[18] In fact, after default, all bonds of different time-to-maturity enter together in the renegotiation process, and usually, holders of different bonds receive similar haircuts. Nevertheless, maturity may affect default spreads

and, as shown by the model, the time-to-maturity is a key variable to measure the liquidity spreads. Hence, looking at variation across maturities within firm issuances on a given date allow us to recover the slope of liquidity spreads.

Safe but illiquid bonds We now include information on credit ratings. First, we repeat the main estimation with the sample of rated bonds because not all bonds are rated, and the smaller and more illiquid bonds are probably not in the sample. The result is in the first column of panel B in Table 1. We find an estimated coefficient of about 4 basis points per year. The second column adds credit-rating fixed-effects and finds a significant and quantitatively similar coefficient for maturity differences.

Next we do the estimation only on the sample of issuances rated above A. These assets represent the High Quality Market (HQM) of corporate bonds and are safe but illiquid assets, so we can abstract from default considerations for this asset class. Expected credit losses and default rates are very small for these securities on average and were small during the 2008 financial crisis (see Appendix D.2). For example, the expected credit losses of a security rated A in 2008 were only 0.37%. Another concern is that long-term assets are more likely to be downgraded and then default. However, we also show in the appendix that five-year cumulative transition probabilities for these securities are quite small. Hence, we can abstract from default considerations for this group of securities and interpret the estimates as the liquidity component. The third column shows that the coefficient on maturity is significant and quantitatively similar to previous estimates. These results validate the identification assumption and suggest that the measured coefficients can be attributed to liquidity. The fourth column considers bonds only rated Aaa and also finds a significant and quantitatively similar coefficient, reinforcing the finding.

Corporate CDS The previous estimates rely either on the assumption that default spreads are constant with maturity or in the sample of safe but illiquid assets. On the empirical side, it is hard to get direct estimates of default intensities (e.g., Campbell et al., 2008). One common strategy is to use CDS to measure the credit spread due to default. However, it is worth noticing that CDS also trade in OTC markets, so CDS's prices also contain liquidity spreads and are not a pure measure of default. Nevertheless, for robustness we can look at corporate CDS and estimate how they change with maturity.

Let $cds_{i,t,m}$ be the implied yield for the CDS for issuer i, in month t, and of maturity m. We take as the short maturity the one-year CDS and compute the slope for CDS with maturities

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if default intensities depend on maturities. However, it is hard to get direct empirical estimates of default intensities (see Campbell et al., 2008), and there is no empirical evidence for the term structure of default intensities. Hence, we assume that it is constant on maturity.

Table 1: **Empirical evidence: Liquidity Spreads Increasing in Maturity**

A. Credit Spreads Differential

Maturity difference	7.876***	6.247***	4.877***	5.836***
	(0.234)	(0.359)	(0.299)	(0.325)
Observations	23,614	23,614	23,614	19,320
R-squared	0.046	0.104	0.173	0.858
FE	No	Time	Time, Industry	Firm-Time

B. Safe but Illiquid Bonds

Maturity difference	3.545***	3.684***	4.046***	3.620**
	(0.261)	(0.245)	(0.336)	(1.081)
Observations	15,471	15,471	11,956	867
R-squared	0.103	0.135	0.126	0.212
FE	Time	Time, Rating	Time	Time
Sample	All	All	Aaa-A	Aaa

C. Credit Default Swaps

Maturity difference	2.494***	2.412***	2.359***	2.073***	2.219***
	(0.046)	(0.230)	(0.224)	(0.201)	(0.042)
Spread short				-0.318***	
				(0.013)	
Observations	1,119,540	1,119,540	1,119,540	1,119,540	1,119,540
R-squared	0.003	0.023	0.027	0.624	0.860
FE	No	Time	Time, Industry	Time, Industry	Firm-Time

D. Direct Measure of Liquidity

Maturity difference	13.035**	9.215**	5.247***
	(3.272)	(2.672)	(0.885)
Observations	2,479	2,479	2,479
R-squared	0.180	0.361	0.903
FE	Time	Time, Industry	Firm-Time

*Note: The first panel shows the estimates of the benchmark specification. The second panel estimates the model for the set of safe but illiquid bonds. The third panel shows that the slope on CDS is smaller than in the benchmark specification. The last panel uses CDS to compute a direct measure of liquidity. See text for details. Standard errors in parentheses; *, **, and *** denote statistical significance at the 10, 5, and 1 percent level, respectively.*

equal to two, three, four, five, seven, ten, fifteen, twenty, and thirty years. The empirical specification replicates the estimates on total credit spreads (Equation (18)),

$$cds_{i,t,m_j} - cds_{i,t,m_1} = \beta(m_j - m_1) + \gamma \mathbf{X}_{i,t} + \epsilon_{i,t,m_1,m_j}. \tag{19}$$

As before, the coefficient β measures the slope of the default spread, and the controls $\mathbf{X}_{i,t}$

include time, industry, and/or firm-month fixed-effects.

Panel C of Table 1 shows that for the five different specifications the estimated coefficients are significant. Quantitatively, for each additional year, the spreads on CDS increase between 2 and 2.5 basis points. If we compare them with the slope of corporate yields (first panel), the slope of CDS represents between one-third and one-half of the total slope. Moreover, as corporate CDS also trade in OTC markets, this slope also contains liquidity spreads and is not a pure measure of default. We also estimated the slope on sovereign CDS, which trade in more liquid markets, and find that the slope on sovereign CDS is smaller than that on corporate CDS.

Non-default component Finally, we match corporate bond issuers with the CDS data, so we get a direct measure of the liquidity component (i.e., we match the corporate bond with the CDS for the same issuer at the same maturity). In the data, we don't have CDS with the exact same maturity, so we do a linear interpolation between the yields of the closest two maturities, but results are robust to alternative interpolation schemes. Define liquidity as $liq_{i,t,m} = s_{i,t,m} - cds_{i,t,m}$ and estimate

$$liq_{i,t,m_j} - liq_{i,t,m_1} = \beta(m_j - m_1) + \gamma \mathbf{X}_{i,t} + \epsilon_{i,t,m_1,m_j}. \tag{20}$$

Panel D of Table 1 shows that the estimated coefficients on the maturity difference are significant for alternative specifications. If we focus on the third column, which includes a firm-month fixed effect, the coefficient is about 5 basis points per year. Note that this coefficient is similar to those on the first panel in which we assume the default component is constant in maturity. We conclude that a significant fraction of the slope of credit spreads can be attributed to liquidity considerations.

Quantitative targets For the quantitative evaluation it is useful to summarize the empirical results with the effect at the median. We use estimates from the direct measure of liquidity; i.e., when we use CDS to control for the default component, Panel D of Table 1. The median firm has a maturity of 5 years, and it issues 3- and 7-year bonds. The estimated coefficients imply an increase on liquidity spreads of 21 bps (4 × 5.25).[19]

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[19] To gauge the quantitative magnitude of the effects we can compare with other papers measuring variation in credit spreads. For example, Gertler and Karadi (2015) show that the excess bond premium (as measured by Gilchrist and Zakrajšek (2012)) increases 8 basis points after a one standard deviation contractionary monetary policy surprise (in which the federal funds rate increases about 18 basis points). Nagel (2016) finds that for every percentage point increase in the fed funds rate, the liquidity premium rises roughly six basis points. Hence, the effects are around the same order of magnitude of what is consider a significant variation of liquidity in the literature.

One caveat about the mapping of the model to the data is that in the benchmark model there is only one bond issuance, while to recover the cost of liquidity in the data we consider the spread between two bond issuances. However, we argue that the empirical estimates recover the cost of financing at different maturities, which is the relevant measure for the model irrespective of the number of issuances on the same day. There are several extensions of the model that can capture the multiple issuances observed in the data, but it is not key for the main point of the paper. For example, a realistic assumption is that the life cycle of the project is more complicated than the one proposed in the model and that projects generate different cash flows at different points in time. In this case, it would be optimal to issue bonds of different maturities at period zero to match the cash-flow payments of the project.

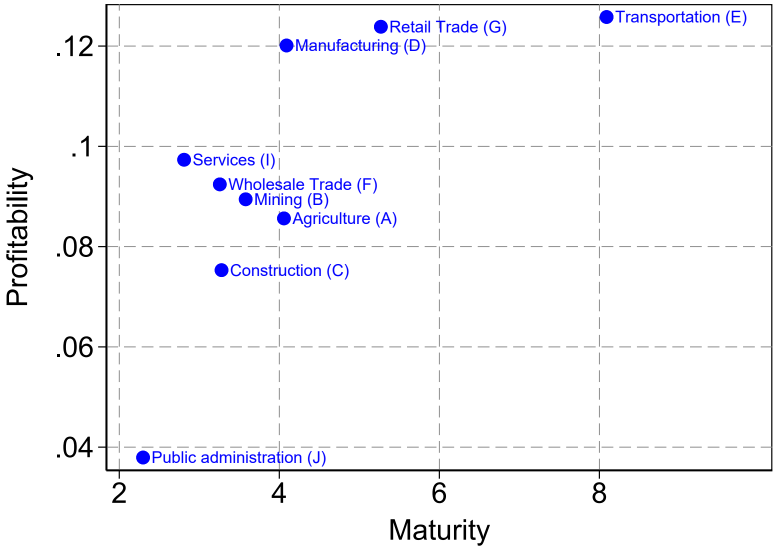
4.2 Maturity and Profitability

We provide empirical evidence that long-term projects have higher profitability. The intuition in the model is that entrepreneurs choose projects across sectors with different investment lags and profitability, which is supported by the following facts: (i) Firms tend to match the maturity of assets and liabilities; and (ii) There is a positive correlation between debt maturity and profitability at the sector level. These two observations validate the assumptions of the model and provide quantitative guidelines for the calibration.

Maturity Matching Empirical evidence shows that firms tend to match the maturity of assets and liabilities. For example, Guedes and Opler (1996) and Stohs and Mauer (1996) study firms in the US by measuring their asset maturity with depreciation rates and find that it matches their debt maturity. Graham and Harvey (2001) survey CFOs, and the most popular explanation of how firms choose between short- and long-term debt is that they match debt maturity with asset life, particularly for small and private firms. There is also international evidence of maturity matching, such as Schiantarelli and Sembenelli (1997) for the UK and Italy and Schiantarelli and Srivastava (1997) for India. Finally, Gopalan et al. (2016) take advantage of a natural experiment—the creation of debt recovery tribunals in India—and show that a reduction in enforcement constraints that increased the availability of funding for long-term projects indeed led to an increase in asset maturity among affected firms. This evidence validates the assumption that firms match the maturity of the bond with that of the project. The benchmark model has a stark assumption that firms fully match maturities, but Section 6.1 shows that the main results are robust when firms can rollover short-term debt to finance long-term projects.

Profitability Previous research has shown that there is a positive relation between debt maturity and profitability at the firm level (e.g., Yamarthy, 2016; Crouzet, 2017), while here we show that this relation also holds across sectors. Figure 3 shows the correlation of productivity and maturity across sectors. We use Compustat data; see Appendix D.3 for details.[20] On the short-term side, a firm in the construction division, for example a general building construction company, has a maturity of 3.2 years and a profitability of 7.5%. On the long-term side, a firm in the transportation division—for example, an airline company—has maturity of about 8 years and a profitability rate above 12%. Thus, sectors with higher maturity also have higher profitability.

Figure 3: **Profitability and Maturity Across Sectors**



Note: Positive correlation between profitability and maturity by industry.

We test the relation between profitability and maturity more formally by regressing profitability on alternative measures of long-term debt. At the firm level what we observe in the data are equilibrium outcomes and variation in maturity is not exogenous. To mitigate this problem we run the regressions at the sector level, exploiting the variation on average profitability and maturity across sectors. We define sector as the SIC division level, but results are similar for alternative industry classifications. First, we regress profitability on the ratio of long-term debt (defined as liabilities with outstanding maturity of more than one year) controlling for time and sector fixed effects. The first column of Table 2 shows a positive and

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[20] We define profitability as operating income before depreciation (oibdp) over assets (at), oibdp/at. Results are robust to alternative measures of profitability, such as oibdp to lagged assets (Gorodnichenko and Weber, 2016) or oibdp minus interest (xint) minus taxes (txt) over assets (Kahle and Stulz, 2017). We define sector as the SIC division level, but results are similar for alternative industry classifications.

Table 2: **Empirical evidence: Profitability Increasing in Maturity**

	(1)	(2)	(3)
Long-term ratio	0.427***		
	(0.031)		
Average maturity		0.030***	0.100***
		(0.003)	(0.011)
Average maturity-squared			-0.006***
			(0.001)
FE	Time Sector	Time Sector	Time Sector
Observations	369	369	369
R-squared	0.828	0.779	0.804

*Note: Standard errors in parentheses; *, **, and *** denote statistical significance at the 10, 5, and 1 percent level, respectively.*

significant coefficient on the ratio of long-term debt. Next, we compute the average maturity and regress profitability on maturity (column 2) and also include maturity squared (column 3), again controlling for time and sector fixed effects. The preferred specification is

$$\text{profitability}_{i,t} = \delta_t + \beta_1 \text{maturity}_{i,t} + \beta_2 \text{maturity}_{i,t}^2 + \varepsilon_{i,t}, \tag{21}$$

where $\text{profitability}_{i,t}$ and $\text{maturity}_{i,t}$ are the average profitability and maturity in period t, sector i. The second column shows that profitability is increasing in maturity and the third column suggest that this relation is concave. For the calibration we follow the estimates of the third column and assume that $\pi(\tau) = \pi_1\tau + \pi_2\tau^2$ with $\pi_1 = 0.100$ and $\pi_2 = -0.006$.

5 Quantitative Analysis

This section presents a quantitative evaluation of the theory presented in Section 2. We use the estimates from Section 4 to discipline the calibration of the model and exploit additional measurements as validation exercises. Next, we evaluate counterfactual scenarios to evaluate how trading frictions on financial markets affect the slope of the yield curve and the profitability of firms.

5.1 Calibration

We match moments from the corporate debt market and firm's profitability. Some parameters can be calibrated "externally," while others must be calibrated "internally" from the solution of the model. We proceed in five steps to calibrate each of the parameters of the model. Table 3 summarizes the parameter values and the target moments.

Externally calibrated One unit of time is equivalent to one year. The discount factor is set to $\rho = 0.02$ as it is standard in the literature (e.g., He and Milbradt, 2014), and the default rate is $\delta = 0.03$ to match the default rate of speculative-grade firms (Moody's, 2015). Normalize the measure of new firms by $\mu^0 = 1$. For the secondary market assume a constant-return-to-scale Cobb-Douglas matching function with elasticity $\alpha = 0.5$. Further, assume that the bargaining power of sellers is $\gamma = 0.5$.

Matching technology First, target an expected time to sell of two weeks (He and Milbradt, 2014) so that $\frac{1}{\lambda} = \frac{2}{52}$. Recall that $\lambda = A\theta^{\alpha-1}$ and normalize $\theta = 1$ as we do not have reliable information on the market tightness. Hence, the matching efficiency has to be equal to $A = 26$ to match the expected time to sell.

Turnover Second, target an annual turnover rate of 57% (He and Milbradt, 2014; Chen et al., 2017). Turnover is approximately equal to $(\eta^{-1} + \lambda^{-1})^{-1}$, so we can directly calibrate $\eta = 0.58$ to match this moment.

Liquidity Third, we target the slope of the liquidity spread from Table 1. The median maturity for the short and long issuances are 3 and 7 years, respectively. We target the increase in the liquidity spread between these two maturities to be 21 basis points; i.e., $cs^{liq}(7, \lambda) - cs^{liq}(3, \lambda) = 21$. Recall that this target controls for the slope attributed to liquidity using the implied slope on CDS. The only parameter missing to measure this moment is the holding cost h, which is set equal to 0.28 to match the target.

Profitability Fourth, we assume that the profit function is $\pi(\tau) = \pi_1\tau + \pi_2\tau^2$ with $\pi_1 = 0.100$ and $\pi_2 = -0.006$ from Table 2. We internally estimate κ to match a maturity choice of 5 years, which is in between the average maturity in FISD and Compustat.

Free-entry Finally, the free-entry condition delivers the value of the search cost c such that in equilibrium $\theta = 1$ and the free-entry condition holds. In particular, the entry cost has to be equal to 0.28.

Table 3: **Parameters and moments**

Parameter		**Value**	**Target/source**	**Model**	**Data**
Financial sector					
Matching efficiency	A	26.00	Expected time to sell	2.000	2.000
Intensity of liquidity shocks	η	0.58	Turnover rate	0.570	0.570
Holding cost	h	0.28	Slope liquidity	21	21
Search cost	c	0.26	Free entry		
Production sector					
Profits	π_1	0.10	Slope	0.100	0.100
Profits slope squared	π_2	-0.006	Slope squared	-0.006	-0.006
Cost	κ	0.33	Maturity	5.000	5.000
Matching					
Share of sellers	α	0.50	Normalization		
Bargaining power of sellers	γ	0.500	Normalization		
Others					
Discount factor	ρ	0.02	He Milbradt (2014)		
Default rate	δ	0.03	Moodys (2015)		

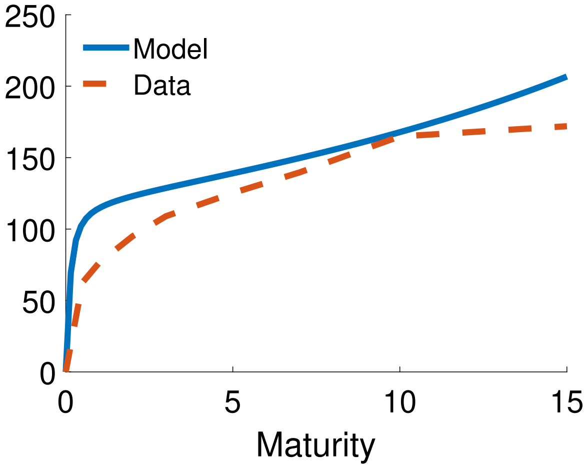
5.2 Validation and Empirical Evidence

The predictions of the model are consistent with several dimensions of the data not directly targeted in the calibration.

Yield curve The calibration directly targets the slope of liquidity spreads but not its level. While it is hard to get direct evidence of the level of liquidity spreads across maturities one option is to compare assets that both have small default risk but trade in markets with different liquidity. In particular, we focus on the positive spread between the yields on Treasuries and safe but illiquid corporate bonds, colloquially known as the *convenience yield* (e.g., Krishnamurthy and Vissing-Jorgensen, 2012). For the liquid asset we consider the zero-coupon yield curve for Treasuries. For the illiquid asset we use the zero coupon yield curve for the high-quality market of corporate debt. These securities only include bonds with ratings above A, such that, up to a first order, we can abstract from default risk (default credit losses for High-Quality Bonds are minimal, see Appendix D.2) but have a large illiquidity component. Note that we are not using this data to discipline the model but only as external validation. The red dotted line in Figure 4 shows that the spread between Treasuries and corporate bonds is positive and increases with maturity.

The calibration target is the slope of the liquidity spread at a specific point. Figure 4 shows that the model also predicts the level of the liquidity spread yield curve at different maturities, which was not a target of the calibration. For example, the model-implied liquidity spread at a

Figure 4: **Liquidity spread: Data versus Model**



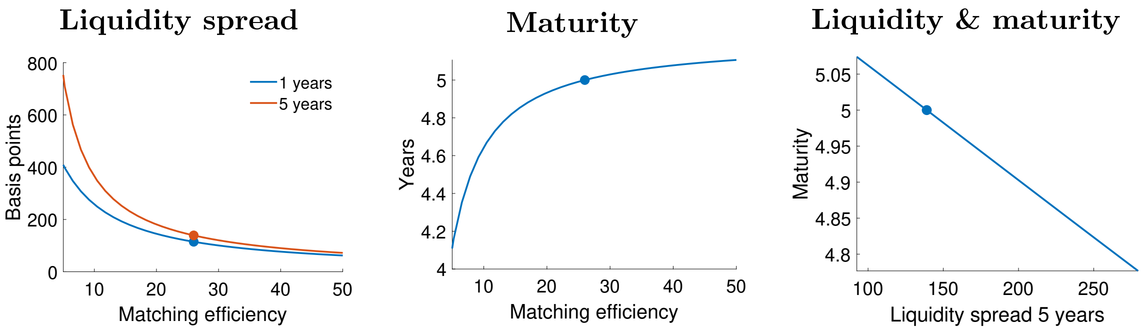
Note: Liquidity spread in the data is computed as the spread between treasury and corporate yield curves. See text for details.

maturity of 10 years is 167 basis points in the model and 164 in the data. Hence, the calibrated model is consistent with both the level and the slope of the liquidity-spread curve.

Liquidity and debt maturity Empirical evidence supports that when financial markets are more liquid, firms issue bonds of longer maturities. For example, Saretto and Tookes (2013) compare issuances of firms with and without CDS and argue that securities of companies with CDS trade in more-liquid financial markets. They find that firms with CDS increase maturity of their issuances between 0.68 and 1.79 years relative to those without CDS. Similarly, Cortina Lorente et al. (2016) study firms in emerging economies issuing bonds both in domestic and international markets, with domestic markets being more illiquid than international ones. The paper finds that maturity increases by 1.6 years for foreign issuances relative to domestic ones. The evidence corroborates that firms issue bonds of longer maturities when financial markets are more liquid.

Term premia, debt issuances, and investment The second mechanism of the paper argues that when term premia increase, firms choose to invest in shorter-term projects. Dew-Becker (2012) uses data from the US and concludes that the duration of investment decreases when the term premium increases. Yamarthy (2016) also finds that profitability and investment rates are higher when firms shift their long-term debt ratio to longer maturities. Foley-Fisher et al. (2016) use cross-sectional variation to show that companies with more dependence on long-term debt benefit more when the yield curve flattens. Garicano and Steinwender (2016) use firm-level data from Spanish firms and find that long-term investments fell more than

Figure 5: **Experiments: Matching efficiency**



Note: Effects of different matching efficiencies on liquidity spread and maturity choices.

shorter ones after the 2008 financial crisis, which can be interpreted through the lens of the model as a financial markets freeze. Therefore, the empirical evidence supports that variations in term premia can affect investment and profitability of the firm as predicted by the model.

5.3 Experiment: The Effect of Trading Frictions

We evaluate the importance of trading frictions for liquidity, maturity, and investment. We consider variations in the matching efficiency A while keeping the other parameters at the calibrated values, but variations in other parameters such us the search cost c yield similar results.

The equilibrium liquidity λ diminishes under a lower matching efficiency A because of two reinforcing effects. First, for a given market tightness, the selling intensity reduces. Second, the incentives to enter into the secondary market reduce, so the market tightness increases (i.e., there are more sellers per buyer). This second effect further diminishes the equilibrium liquidity. Figure 5 shows how changes in the matching efficiency affect the liquidity spreads and the maturity choices. The first panel considers the liquidity spread at maturities of 1 and 5 years. Note that the change in the liquidity spread due to more-severe trading frictions is more pronounced for the long-term asset as predicted by Proposition 1. For example, when the matching efficiency reduces from $A = 26$ to $A = 10$, the liquidity spread for a 5-year bond increases by 83 basis points more than the liquidity spread for a 1-year bond (the spread for the 5-year bond increases by 226 basis points, from 139 to 365, while for the 1-year bond increases by 143 basis points, from 114 to 257) showing that liquidity is more important for long-term than short-term interest rates.

An Example: Trading Frictions in Argentina As an example, in Appendix C.2 we repeat the estimates of the slope of liquidity spreads in Argentina to discipline the counterfactuals. We find that credit spreads are steeper in Argentina than in the US. We summarize the empirical results with the effect for the median firm. In Argentina the average maturity is 2.5 years, and when maturity increases from 1.5 to 3 years, credit spreads increase by 75 basis points, while CDS spreads increase by 14 basis points. We attribute the difference between corporate and sovereign CDS slope, 61 basis points, to the liquidity component.

To match the increase in liquidity spreads in the model, the matching efficiency reduces from 26 for the US to 7.8 for Argentina. Entry to the secondary market also adjusts; the market tightness—defined as the ratio of sellers-to-buyers—increases from 1.0 to 1.16. As a result, the liquidity spread increases and the yield curve becomes steeper.

Table 4 shows that firms in an economy like the US but with the financial system of Argentina would borrow at a maturity of 4.5 years. In the data, the average maturity for Argentina is 2.5 years. Therefore, trading frictions can explain about 20% of the maturity differences in the data. When long-term finance becomes more expensive, firms tilt their maturity choices toward the short-end. On the real side of the economy, this implies that entrepreneurs invest in shorter-term projects, which have lower profitability. Under the Argentinean financial system profitability $\pi(\tau)$ reduces by 6% due to investment in shorter-term projects. In terms of welfare, note that there is free entry of investors, so a natural candidate to measure changes in welfare is the value of firms, e.g., as in the firm's objective function $W = e^{-(\rho+\delta)\tau}(F(\tau) - B)$. The last row of Table 4 shows that under the Argentinean financial system welfare reduces by 7%.

What is *financial development*? There is a literal interpretation of it being related to the technology to execute trades. In developed markets, there are clearing houses such as *Euroclear* or *Clearstream*, while in emerging economies the time to execute a trade is delayed by technological constraints. For example, in Argentina investors liquidate securities in one place but make payments in a different institution.[21] A broader interpretation is to think about the participants in the market. In developed financial markets, there are large and active mutual funds, which are agents that trade more frequently than the rest of the market's participants. Because these funds are either small or nonexistent in emerging economies, this could also imply lower liquidity.

6 Extensions

In this section we show that the main results also hold when firms can rollover of short-term debt to finance long-term projects. We also consider a policy intervention aim to increase the

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[21] Recently in Argentina, BYMA—the local exchange market—is trying to unify these operations to increase the liquidity of the market.

Table 4: **Counterfactual: US and Argentina**

	US		**Argentina**	
	Data	Model	Data	Model
Matching efficiency		26.00		7.83
Market tightness		1.00		1.16
Liquidity (bps)				
Increase 7 - 3 years	21	21	162	127
Increase 3 - 1.5 years	8	9	61	61
Maturity (years)	5.0	5.0	2.5	4.5
Change in profitability				-0.06
Change in welfare				-0.07

liquidity of financial markets.[22]

6.1 Rollover

Changes in the liquidity of the secondary market have similar effects on the choice of investment projects even if borrowers can rollover short-term debt to finance long-term projects. Consider a firm that chooses the maturity of the project τ and issues zero coupon bonds to finance investment costs. A bond of maturity y has a fixed cost of issuance Φ and an interest rate $r(y)$.[23] Let J be the total number of issuances up to age τ. It is easy to show that a fixed cost of issuance implies a finite number of issuances.

The firm chooses the maturity of the project τ, the number of issuances J, the amount

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[22] Appendix B extends the model in other dimensions and shows that results do not hinge on many of the assumptions of the benchmark theory. For example, we show that the yield curve has the same shape regardless of whether buyers can direct themselves to markets segmented by maturity instead of having one market with random matching across maturities. We also argue that we can interpret the model as bank lending instead of corporate bonds.

[23] In the data, issuance costs include management fees, selling concessions, registration fees, underwriter fees, underwriter spread (the difference between the offering price and the guaranteed price to the issuer), underpricing (the difference between the market price and the offering price), and printing, legal and auditing costs. For the Eurobond market, Melnik and Nissim (2003) find that the total issuance cost is 37 basis points. Lee et al. (1996) find similar costs and reports evidence of economies of scale, reflecting that a significant fraction is a fixed cost. For the model, it is important to have fixed issuance cost, otherwise the firm would finance with overnight debt.

borrowed B_j, and the maturity structure of their liabilities y_j to solve

$$\max_{\tau,J,y_j,B_j} e^{-(\rho+\delta)\tau}\left(F(\tau)-B_j\right) \tag{22}$$
$$\text{s.t.} \quad B_j P(y_j,\lambda) = B_{j-1} + \Phi + I(y_j) \quad \text{for } j=1,\dots,J$$
$$B_0 = 0 \qquad \text{and} \quad \sum_{j=1}^{J} y_j = \tau.$$

Each issuance borrows to rollover existing debt B_{j-1}, cover the issuance cost Φ, and invest for the next y_j periods, $I(y_j)$.[24] Iterate on B_j to cast the firm's problem (22) as

$$\max_{\tau} e^{-(\rho+\delta)\tau} F(\tau) - \text{FIN}^{\text{COST}}(\tau),$$

in which the financial cost is

$$\text{FIN}^{\text{COST}}(\tau) = e^{-(\rho+\delta)\tau} \min_{J,\{y_j\}_{j=1}^{J}} \sum_{i=1}^{J} \left(\Phi + I(y_i)\right) e^{\sum_{s=i}^{J} r_s y_s}$$
$$\text{s.t.} \quad \sum_{j=1}^{J} y_j = \tau.$$

Financing cost Consider a project of a given maturity τ. The financial cost $\text{FIN}^{\text{COST}}(\tau)$ chooses the number of issuances J and the maturity structure y_j to minimize the net present value of issuance costs Φ and investment needs $I(y_j)$. Both the issuance costs and the liquidity spread affect financial decisions.

We evaluate the financial choices for different issuance costs Φ, given a project τ. The top panel of Figure 6 shows the number of issuances, the total financial cost, and the dispersion of maturities for different issuance costs Φ.[25] Naturally, as the issuance cost increases, the number of issuances decreases and the total financial cost increases. Note that if Φ is sufficiently large, the firm optimally chooses to issue only one time, matching the maturity of the project and the liabilities. In the benchmark model we focus on this particular case with $J = 1$ and derive a sharper analytical characterization of the effects of secondary-markets liquidity on the project's choice.

The bottom panel of Figure 6 evaluates how the liquidity of the secondary market affects

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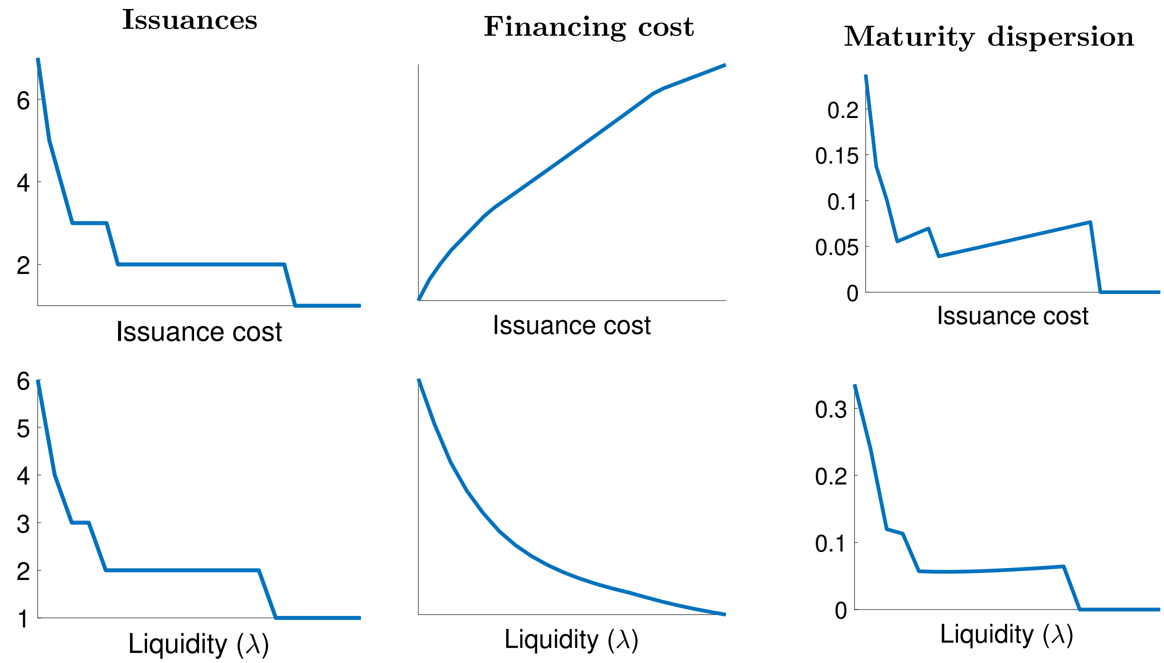
[24] A firm with positive cash holdings will always wait until it runs out of money to issue new debt. Hence, without loss of generality, we only have to consider the choices of the firm when outstanding debt matures, which coincides with the moment in which the firm runs out of money.

[25] Maturity dispersion is defined as the coefficient of variation of individual maturities y_j.

Figure 6: **Financial cost with rollover**



Note: Financial cost for a given maturity τ, different issuance costs Φ, and liquidity λ. Parameter values are discussed in Section 5. Maturity dispersion is defined as the coefficient of variation.

financial choices. In markets with lower trading frictions—higher λ—long-term finance is more attractive (Proposition 1), which induces firms to rollover less often and borrow at longer maturities, reducing the total financial cost. Hence, for a given project τ, the financial cost diminishes when λ increases. The next exercise shows that this effect induces firms to invest in projects of longer horizons.

Maturity structure The optimal maturity structure solves the trade-off between equalizing credit spreads across different issuances and decreasing future fixed issuance costs. In equilibrium, the maturity structure is decreasing—i.e., $y_1 \geq y_2 \geq \cdots \geq y_J$. On the one hand, Figure 6 shows that, conditional on the number of issuances, when the issuance cost increases firms choose to increase the maturity dispersion.[26] Intuitively, as Φ increases, firms want to postpone the fixed-cost payments of later issuances, and, as a result, they extend the maturity of the first issuances and decrease later ones. On the other hand, the bottom panel shows that for financial markets with larger trading frictions (lower λ), the dispersion of maturities diminishes to generate similar liquidity spreads across issuances.

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[26] Note that the *jumps* in the dispersion coincide when the firm decides to change the number of issuances. Hence, the dispersion is decreasing conditional on the number of issuances.

Table 5: **Solution under alternative specifications**

Default	Secondary market	Issuance cost (p.p. spread)	Issuances	Maturity		Interest rate	Credit spread	
				Project	Bond		Default	Liquidity
				No rollover				
Yes	Centralized		1	5.2	5.2	5.0%	3.0%	0.0%
Yes	OTC		1	5.0	5.0	6.4%	3.0%	1.4%
Yes	Shut down		1	2.6	2.6	22.9%	3.0%	17.9%
No	Centralized		1	6.3	6.3	2.0%	0.0%	0.0%
No	OTC		1	6.0	6.0	3.3%	0.0%	1.3%
No	Shut down		1	2.8	2.8	20.8%	0.0%	18.8%
				Rollover with $\Phi = 0.015$				
Yes	Centralized	1.1%	3	5.4	1.8	5.0%	3.0%	0.0%
Yes	OTC	1.1%	4	5.3	1.3	6.2%	3.0%	1.2%
Yes	Shut down	1.2%	12	4.9	0.4	8.1%	3.0%	3.1%
No	Centralized	0.7%	1	6.3	6.3	2.0%	0.0%	0.0%
No	OTC	1.0%	3	6.1	2.0	3.2%	0.0%	1.2%
No	Shut down	1.2%	12	5.1	0.4	5.2%	0.0%	3.2%
				Rollover with $\Phi = 0.15$				
Yes	Centralized	6.4%	1	5.2	5.2	5.0%	3.0%	0.0%
Yes	OTC	6.3%	1	5.0	5.0	6.4%	3.0%	1.4%
Yes	Shut down	8.4%	4	4.2	1.1	12.5%	3.0%	7.5%
No	Centralized	6.2%	1	6.3	6.3	2.0%	0.0%	0.0%
No	OTC	6.0%	1	6.0	6.0	3.3%	0.0%	1.3%
No	Shut down	7.9%	4	4.8	1.2	10.3%	0.0%	8.3%

Note: The column "Secondary market" considers the cases of a centralized market (no trading frictions, $\lambda = \infty$), "OTC" (benchmark frictions, $\lambda = 26$), or "shut down" (no trading in secondary markets, $\lambda = 0$). On the second panel we choose the value of the issuance cost to match the direct costs of issuance. On the third panel we increase the issuance cost to capture the indirect effects.

Investment choice Proposition 3 and the quantitative exercises in Section 5 show that the liquidity of the secondary market is important for investment decisions when firms match the maturity of assets and liabilities. Moving from an OTC secondary market with liquidity as in the US economy ($\lambda = 26$), to a shut-down of the market ($\lambda = 0$) reduces the project's maturity by 2.4 years (from 5.0 to 2.6, first panel of Table 5, rows two and three).

If the firm can rollover short-term debt, the effects of trading frictions on the project's choice can be substantially weaker because as λ decreases, the firm can rollover more often, as suggested by Figure 6. We follow two calibration strategies to impose quantitative discipline on Φ. First, we set Φ to match the direct cost of issuance.[27] Altınkılıç and Hansen (2000) find that the average issuance cost are about 1.09%. The second panel of Table 5 shows that the model match this target by setting $\Phi = 0.015$. With an OTC secondary market, the firm chooses a project of 5.3 years of duration and issues bonds 4 times (i.e., bonds have a maturity of 1.3 years on average). However, when there is a shut-down of the secondary market, the firm chooses to rollover more often (12 times) and shortens the duration of investment to 4.9 years (bonds have a maturity of 0.4 years on average). Hence, the reduction in the duration of the project due to changes in trading frictions is 0.4 years, equivalent to the duration of one additional round of funding. The effects of liquidity are still large under this lower bound on the issuance cost. For example, removing default risk increase the maturity by 0.8 years while a shutdown of the secondary market reduce the maturity by 0.4 years, so a lower bound for the effects on liquidity are about one-half of the effects of default.

The assumption on issuance costs can more generally be interpreted as shorthand for other mechanisms that generate a preference for longer maturities, for instance, rollover risk (see, e.g., He and Xiong (2012)). For this interpretation we don't have a direct measurement of the issuance cost. One option is to increase the issuance cost Φ such that in the benchmark model the firm issue only one time and the issuance cost is as if the firm has to pay one extra year of interest (i.e., the issuance cost is 6.3% of the issuance which is equivalent to the annual interest rate). When the secondary market is shut-down, the firm decides to issue four times and the maturity of the project reduces by 0.8 years which is about the same effect than those generated by default. These exercises suggest that the results in Section 5 about how trading frictions affect investment decisions do not depend on the assumption of matching the maturity of the project and the bond.

One potential concern about these exercises is that the liquidity of the secondary market and rollover costs might be correlated. In the model, liquidity costs are endogenous while rollover

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[27] Issuance costs include management fees, selling concessions, registration fees, underwriter fees, underwriter spread (the difference between the offering price and the guaranteed price to the issuer), underpricing (the difference between the market price and the offering price), and printing, legal and auditing costs.

costs are exogenous and fixed, so they do not respond to changes in the liquidity of secondary markets. However, we expect that when the secondary market becomes more liquid, both issuance and rollover costs should diminish. Hence, it is conservative to assume fixed issuance costs. As in Table 5, when liquidity of secondary markets improves, the firm adopts a project of longer duration. Similarly, when issuance costs diminish, the firm chooses longer-term projects. If the two effects are present (an increase in liquidity and a reduction of issuance costs), the firm will extend the maturity of the project even more than with fixed issuance costs.

6.2 Policy Intervention

While it is outside the scope of this paper to fully characterize the inefficiencies or the optimal policy, we explore the effects of a government intervention designed to increase the liquidity of financial markets, namely *Government-sponsored intermediaries* (GSIs). The underlying inefficiencies are similar to those in Bruche and Segura (2017) in which the firms do not internalize the a longer maturity increases expected gains from trade in the secondary market, which attracts more buyers and, hence, facilitates the sale of debt issued by other firms. The intervention consists of government agents acting as intermediaries in the secondary market that buy and sell bonds at different prices than in private bilateral meetings, and to finance the policy the government charges a distortionary profit tax.[28] The government cannot avoid the search frictions or holding costs so that government agents face the same constraints as private agents. However, public agents can participate in secondary markets and take different actions (i.e., buying or selling prices) than they do in the private sector which increase the liquidity of the economy and so it does not generate a full crowding-out of private agents.[29]

We evaluate GSIs for economies with different trading frictions by considering alternative efficiencies of the matching function. We set the lower and higher value of A to match the calibration for Argentina and the US, respectively. The intervention has non-linear effects. Figure 7 compares economies with and without GSIs for different levels of trading frictions.

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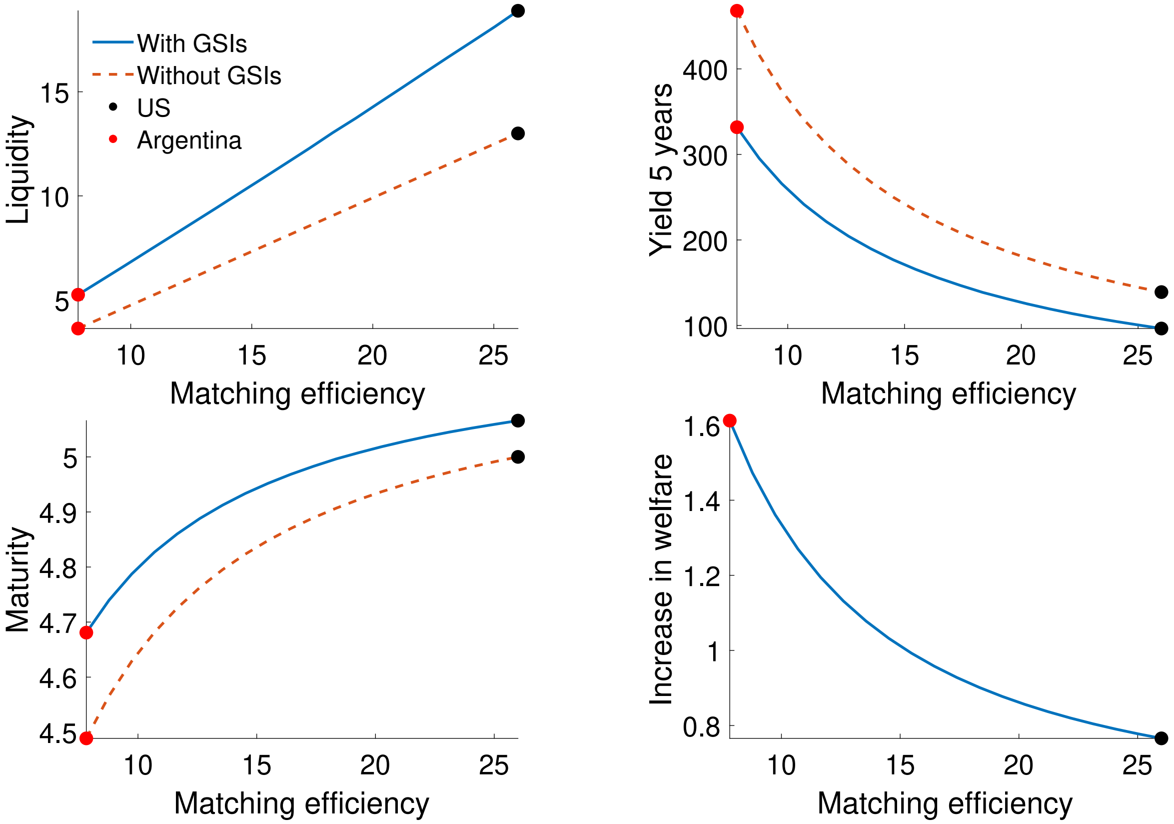
[28] The intervention has similarities with actual policies. For example, Government Sponsored Enterprises target the efficiency of household debt while GSIs focus on credit to the corporate sector. Moreover, during the 2008 financial crisis, the central bank performed large-scale asset purchases that were effective because of limits to arbitrage in private intermediation (Gertler et al., 2013; Ray, 2019). In a similar way, GSIs intermediate corporate debt to increase the liquidity. There are policies implemented or proposed in emerging economies that also have similarities with GSIs. In India, "priority-sector lending" requires banks to lend at least 40% of their net credit to the "priority sector"(Banerjee and Duflo, 2014), so banks trade to meet the specific targets, which increases the liquidity of the market (a similar motive for trade as in the Fed Funds Market in the US; see Afonso and Lagos, 2015). In Brazil, the private capital markets association (Anbima) propose the creation of a "Liquidity Improvement Fund," in which private agents manage public resources to act as market makers, similar to the proposed GSIs (Park, 2012).

[29] We present the main results here while Appendix B.1 describes and solve the extended model and present alternative interventions.

Figure 7: **GSIs: Effects for different trading frictions**



Note: Results with and without GSIs for different trading frictions.

The top-left panel shows that the policy is more efficient at improving liquidity of financial markets when search frictions are relatively low (i.e., higher matching efficiency). However, the right-top panel shows that the flattening of the yield curve due to the policy is more effective when there are larger frictions (i.e., lower matching efficiency). In an advanced financial market, the marginal effect of an increase in liquidity is smaller than in a less-developed financial system. Hence, even though the improvement in liquidity is lower in emerging markets the consequences might be larger.

The bottom-left panel shows that GSIs increase the equilibrium maturity of corporate debt by about 0.07 years in the US. Note that this effect is larger in less-developed financial markets. For example, in a system similar to Argentina, GSIs increase the maturity of corporate debt by 0.19 years. Finally, the bottom-right panel shows that the increase in welfare due to GSIs is 0.77% for the US and 1.61% for Argentina.

7 Conclusion

This paper studies the linkages between the maturity of corporate debt, the liquidity of financial markets, and the real economy. Long-term finance is particularly more expensive in economies with severe trading frictions that induce firms to invest at shorter horizons. A calibration of the model suggests that even though it is a stylized and tractable model, the theory reconciles data on maturities, credit spreads, and the real economy. Finally, an intervention like GSIs can improve the liquidity of financial markets, reduce long-term financial cost, and induce firms to borrow and invest at longer horizons. Several extensions suggest that the results of the paper do not hinge on particular modeling assumptions.

The framework and results developed in this paper transcend the particular application to corporate bonds and can be used to study other markets for long-term finance, such as households borrowing for real estate (mortgages) or education (student debt). Interestingly, Hicks (1969) argues that the products manufactured during the first decades of the Industrial Revolution had been invented much earlier. The critical innovation that ignited growth in England in the 18th century was capital market liquidity so that savers could easily sell their assets if needed, while at the same time the capital was committed for longer periods for investment (see Bencivenga et al., 1995).

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A Proofs

This section provides the proofs of the main results of the paper.

A.1 Lenders

A.1.1 Distribution of financiers

Proof of Lemma 1. Let $\mu(y) = \left[\mu^H(y), \mu^L(y)\right]$. Matching implies $\mu^B \beta \frac{\mu^L(y)}{\mu^S} = \lambda \mu^L(y)$. Then, (5)-(6) imply that $\dot{\mu}(y) = A\mu(y)$ with

$$A = \begin{bmatrix} \delta + \eta & -\lambda \\ -\eta & \delta + \lambda \end{bmatrix}.$$

The boundary condition is $\mu(\tau) = [\mu^0, 0]$. Note that A has two real and distinct eigenvalues. Let R be the vector with the eigenvalues and V be the matrix with eigenvectors of A. Define $B = (V)^{-1}\mu(\tau)$ so

$$V = \begin{bmatrix} -1 & \frac{\lambda}{\eta} \\ 1 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \eta + \lambda + \delta \\ \delta \end{bmatrix} \qquad B = \frac{\eta\mu^0}{\eta + \lambda}\begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

It is standard to show that

$$\mu^H(y) = \sum_{i=1}^{2} e^{R_i(y-\tau)} B_i V(1,i) \qquad \mu^L(y) = \sum_{i=1}^{2} e^{R_i(y-\tau)} B_i V(2,i).$$

Finally, a few lines of algebra deliver

$$\mu^H(y) = \frac{\mu^0\eta}{\eta+\lambda}\left(e^{\delta(y-\tau)}\frac{\lambda}{\eta} + e^{(\eta+\lambda+\delta)(y-\tau)}\right)$$
$$\mu^L(y) = \frac{\mu^0\eta}{\eta+\lambda}\left(e^{\delta(y-\tau)} - e^{(\eta+\lambda+\delta)(y-\tau)}\right).$$

□

A.1.2 Value functions

Proof of Proposition 1. Replace the price of the asset in the secondary market $P^S(y;\lambda)$ in (9)-(10) so

$$
\begin{aligned}
(\rho+\delta) D^H(y;\lambda) &= -\frac{\partial D^H(y;\lambda)}{\partial y} + \eta\left(D^L(y;\lambda) - D^H(y;\lambda)\right)\\
(\rho+\delta) D^L(y;\lambda) &= -h - \frac{\partial D^L(y;\lambda)}{\partial y} + \lambda\gamma\left(D^H(y;\lambda) - D^L(y;\lambda)\right).
\end{aligned}
$$

Let $H(y;\lambda) = D^H(y;\lambda) - D^L(y;\lambda)$, then

$$(\rho+\delta+\eta+\lambda\gamma) H(y;\lambda) = h - \frac{\partial H(y;\lambda)}{\partial y},$$

with $H(0;\lambda)=0$. It is straight forward to see that $H(y;\lambda) = h\frac{1-e^{-c_1 y}}{c_1}$ where $c_1 = \rho+\delta+\eta+\lambda\gamma$.

Next, solve for $D^H(y;\lambda)$ as

$$(\rho+\delta) D^H(y;\lambda) = -\frac{\partial D^H(y;\lambda)}{\partial y} - \eta h\frac{1-e^{-c_1 y}}{c_1},$$

with boundary $D^H(0;\lambda)=1$. The solution is $D^H(y;\lambda) = A + Be^{-(\rho+\delta)y} + Ce^{-c_1 t}$ with constants

$$A = -\frac{1}{\rho+\delta}\frac{\eta h}{c_1} \qquad C = -\frac{1}{\eta+\lambda\gamma}\frac{\eta h}{c_1} \qquad B = 1 + \frac{\eta h}{(\eta+\lambda\gamma)(\rho+\delta)}.$$

Finally, a few lines of algebra deliver

$$
\begin{aligned}
D^H(y;\lambda) =& e^{-(\rho+\delta)y} - \mathcal{L}(y,\lambda)\\
\mathcal{L}(y,\lambda) =& \frac{\eta h}{\eta+\lambda\gamma}\left(\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta} - \frac{1-e^{-(\rho+\delta+\eta+\lambda\gamma)y}}{\rho+\delta+\eta+\lambda\gamma}\right)\\
\mathcal{L}(\tau,\lambda) =& h \quad \frac{\eta}{\eta+\lambda\gamma} \quad \int_0^\tau e^{-(\rho+\delta)y}\left(1-e^{-(\eta+\lambda\gamma)y}\right) dy.
\end{aligned}
$$

The value of a low-valuation agent is $D^L(y;\lambda) = D^H(y;\lambda) - H(y;\lambda)$, that is

$$D^L(y,\lambda) = e^{-(\rho+\delta)y} - h\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta} + \frac{\lambda\gamma}{\eta}\mathcal{L}(y,\lambda).$$

Properties of the illiquidity cost:

1. **Positive:** $\mathcal{L}(\tau,\lambda)$ is positive as $\rho+\delta+\eta+\lambda\gamma \geq \rho+\delta$.

2. **Sensitivity with respect to maturity τ:**

(a) $\mathcal{L}(\tau,\lambda)$ is increasing in τ:

$$\frac{\partial \mathcal{L}(\tau,\lambda)}{\partial \tau} = h \quad \frac{\eta}{\eta+\lambda\gamma} \quad e^{-(\rho+\delta)\tau}\left(1-e^{-(\eta+\lambda\gamma)\tau}\right) \geq 0.$$

(b) The limit of $\mathcal{L}(\tau,\lambda)$ when τ goes to infinity is

$$\begin{aligned}\lim_{\tau\to\infty} \mathcal{L}(\tau,\lambda) &= h\frac{\eta}{\eta+\lambda\gamma}\left(\frac{1}{\rho+\delta} - \frac{1}{\rho+\delta+\eta+\lambda\gamma}\right)\\ &= h\frac{\eta}{(\rho+\delta)(\rho+\delta+\eta+\lambda\gamma)}.\end{aligned}$$

(c) $\mathcal{L}$ is concave in τ if $\lambda \geq \eta$: The second derivative of liquidity with respect to maturity is

$$\begin{aligned}\frac{\partial^2 \mathcal{L}}{\partial \tau^2} &= -(\rho+\lambda)\, h\frac{\eta}{\eta+\lambda\gamma} e^{-(\rho+\lambda)\tau}\left(1-e^{-(\eta+\lambda\gamma)\tau}\right)\\ &\quad + (\eta+\lambda\gamma)\, h\frac{\eta}{\eta+\lambda\gamma} e^{-(\rho+\lambda)\tau}\left(e^{-(\eta+\lambda\gamma)\tau}\right)\end{aligned}$$

that is

$$\frac{\partial^2 \mathcal{L}}{\partial \tau^2} = h\frac{\eta}{\eta+\lambda\gamma} e^{-(\rho+\lambda)\tau}\left(e^{-(\eta+\lambda\gamma)\tau}(\eta+\lambda\gamma-\rho-\lambda)-\rho-\lambda\right)$$

so, the curvature depends on the sign of

$$F(x,y,\tau) = e^{-x\tau}(x-y) - y$$

where $x = \eta+\lambda\gamma$ and $y = \rho+\lambda$. We want to find a sufficient condition for $\mathcal{L}$ to be concave, i.e. $F \leq 0$. First, note that F is decreasing in τ. So, the maximum value occurs be at $\tau = 0$. Then $F(x,y,0) = x - 2y$. That is

$$\begin{aligned}F(x,y,0) &= \eta+\lambda\gamma - 2(\rho+\lambda)\\ &= -2\rho - \lambda(2-\gamma) + \eta\end{aligned}$$

Recall $\gamma \in [0,1]$ so, for a sufficient condition we can consider the case of $\gamma = 1$ and

$$F(x,y,0) = -2\rho - \lambda + \eta$$

Finally, a sufficient condition is that $\lambda \geq \eta$. This condition means that it is faster to find a trading counterpart than receiving a preference shock, which is strongly supported by the data. Recall that this is a sufficient condition, but depending on other parameter values, such as γ, it is not a necessary condition. Moreover, note that as τ increases, the first term of $F(x, y, \tau)$ goes to zero and F becomes negative, independently of the parameter values.

3. **Sensitivity with respect to liquidity shocks η:**
   (a) If there are no liquidity shocks ($\eta = 0$), then $\mathcal{L}(\tau, \lambda) = 0$.
   (b) If $\eta \to \infty$ (i.e., always has to pay the cost h), then

$$\lim_{\eta \to \infty} \mathcal{L}(\tau, \lambda) = h \frac{1 - e^{-(\rho+\delta)\tau}}{\rho + \delta}.$$

4. **Sensitivity with respect to liquidity of the secondary market λ:**
   (a) $\mathcal{L}(\tau, \lambda)$ is decreasing in λ. Note that the illiquidity cost is

$$\begin{aligned}\mathcal{L}(\tau, \lambda) = {} & \eta h \left(\frac{1}{(\rho+\delta)(\rho+\delta+\eta+\lambda\gamma)} - \frac{e^{-(\rho+\delta)\tau}}{(\eta+\lambda\gamma)(\rho+\delta)} \right) \\ & + \eta h \frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta+\lambda\gamma)(\rho+\delta+\eta+\lambda\gamma)},\end{aligned}$$

so

$$\begin{aligned}\frac{\partial \mathcal{L}(\tau, \lambda)}{\partial \lambda} = {} & \eta h \left(-\frac{1}{(\rho+\delta)(\rho+\delta+\eta+\lambda\gamma)^2} + \frac{e^{-(\rho+\delta)\tau}}{(\rho+\delta)(\eta+\lambda\gamma)^2} \right) \\ & - \eta h \left(\frac{\tau e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta+\lambda\gamma)(\rho+\delta+\eta+\lambda\gamma)} + \frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta+\lambda\gamma)^2(\rho+\delta+\eta+\lambda\gamma)} \right) \\ & - \eta h \frac{e^{-(\rho+\delta+\eta+\lambda\gamma)\tau}}{(\eta+\lambda\gamma)(\rho+\lambda^D+\eta+\lambda\gamma)^2}.\end{aligned}$$

Let $a = \eta + \lambda\gamma$ and $b = \rho + \delta$ so

$$\frac{\partial \mathcal{L}(\tau, \lambda)}{\partial \lambda} = \eta h \left(-\frac{1}{b(a+b)^2} + \frac{e^{-b\tau}}{ba^2} \right) - \eta h \frac{e^{-(a+b)\tau}}{a(a+b)} \left(\tau + \frac{2a+b}{a(a+b)} \right).$$

We want to show that

$$\frac{e^{-b\tau}}{ba^2} \leq \frac{1}{b(a+b)^2} + \frac{e^{-(a+b)\tau}}{a(a+b)} \left(\tau + \frac{2a+b}{a(a+b)} \right). \tag{23}$$

Define $L(\tau)$ and $R(\tau)$ as the left- and right-hand-sides of (23), respectively. Note that $R(0) = L(0) = \frac{1}{ba^2}$. Hence, it is sufficient to show that the slope of $L(\tau)$ is lower than the slope of $R(\tau)$ for all τ. Note that

$$\frac{\partial L(\tau)}{\partial \tau} = -\frac{e^{-b\tau}}{a^2} \qquad \frac{\partial R(\tau)}{\partial \tau} = -\frac{e^{-(a+b)\tau}}{a}\left(\tau + \frac{1}{a}\right).$$

Hence, the slope of L is lower than the slope of R because $a\tau \geq \log(a\tau + 1)$.

(b) If there are no secondary markets; i.e., $\lambda = 0$, then the illiquidity cost represents the expected holding costs; i.e.,

$$\mathcal{L}(\tau, 0) = h \int_0^\tau e^{-\left(\rho+\lambda^D\right)y}\left(1 - e^{-\eta y}\right) dy.$$

(c) If secondary markets are totally liquid (i.e., $\lambda \to \infty$), then $\mathcal{L}(\tau, \lambda) = 0$.

5. **Liquidity is more important for long-term assets:** Recall that

$$\frac{\partial \mathcal{L}(\tau,\lambda)}{\partial \tau} = h\frac{\eta}{\eta + \lambda\gamma} e^{-(\rho+\delta)\tau}\left(1 - e^{-(\eta+\lambda\gamma)\tau}\right)$$
$$\frac{\partial \mathcal{L}(\tau,\lambda)}{\partial \tau} = \eta h e^{-(\rho+\delta)\tau}\int_0^\tau e^{-(\eta+\lambda\gamma)y} dy,$$

therefore,

$$\frac{\partial \mathcal{L}(\tau,\lambda)}{\partial \tau\lambda} = -\eta h e^{-(\rho+\delta)\tau}\int_0^\tau y e^{-(\eta+\lambda\gamma)y} dy \leq 0.$$

□

Finally, note that the price is positive if $e^{-(\rho+\delta)\tau} \geq \mathcal{L}(\tau)$. Recall that $\mathcal{L}(\tau) \leq h\frac{\eta}{(\rho+\delta)(\rho+\delta+\eta+\gamma\lambda)}$. Hence, a sufficient condition for having a positive price is

$$e^{-(\rho+\delta)\tau} \geq h\frac{\eta}{(\rho+\delta)(\rho+\delta+\eta+\gamma\lambda)},$$
$$\tau \leq \frac{1}{\rho+\delta}\log\left(\frac{(\rho+\delta)(\rho+\delta+\eta+\gamma\lambda)}{h\eta}\right).$$

A.1.3 Liquidity spread

Proof of Lemma 2. We show the following:

1. **The liquidity spread $cs^{liq}(\tau, \lambda)$ is increasing in maturity τ:**

$$\frac{\partial cs^{liq}\left(t, \lambda\right)}{\partial t} = \frac{1}{t^2} \log\left(1 - e^{(\rho+\delta)t} \mathcal{L}\left(t, \lambda\right)\right) + \frac{e^{(\rho+\delta)t}}{t} \frac{\left(\rho + \delta\right) \mathcal{L}\left(t, \lambda\right) + \frac{\partial \mathcal{L}(t,\lambda)}{\partial t}}{1 - e^{(\rho+\delta)t} \mathcal{L}\left(t, \lambda\right)}.$$

Recall that $\log\left(x\right) \geq \frac{x-1}{x}$. Hence

$$\log\left(1 - e^{(\rho+\delta)t} \mathcal{L}\left(t, \lambda\right)\right) \geq \frac{-e^{(\rho+\delta)t} \mathcal{L}\left(t, \lambda\right)}{1 - e^{(\rho+\delta)t} \mathcal{L}\left(t, \lambda\right)},$$

which implies that

$$\frac{\partial cs^{liq}\left(t, \lambda\right)}{\partial t} \geq \frac{1}{t^2} \frac{e^{(\rho+\delta)t} \mathcal{L}\left(t, \lambda\right)}{1 - e^{(\rho+\delta)t} \mathcal{L}\left(t, \lambda\right)} \left(t\left(\rho + \delta\right) + \frac{\partial \mathcal{L}\left(t, \lambda\right)}{\partial t} \frac{t}{\mathcal{L}\left(t, \lambda\right)} - 1\right).$$

Let $\varepsilon_{\mathcal{L},t} = \frac{\partial \mathcal{L}(t,\lambda)}{\partial t} \frac{t}{\mathcal{L}(t,\lambda)}$, and note that

$$\varepsilon_{\mathcal{L},t} = t\left[e^{-(\rho+\delta)\tau} - e^{-(\rho+\delta+\eta+\lambda\gamma)t}\right] \left[\frac{1 - e^{-(\rho+\delta)t}}{\rho + \delta} - \frac{1 - e^{-(\rho+\delta+\eta+\lambda\gamma)t}}{\rho + \delta + \eta + \lambda\gamma}\right]^{-1}.$$

A sufficient condition is $t\left(\rho + \delta\right) + \varepsilon_{\mathcal{L},t} - 1 \geq 0$. Let $a = \rho + \delta$ and $b = \eta + \lambda\gamma$, and define

$$E\left(t, a, b\right) = t\left(a + \left[e^{-at} - e^{-(a+b)t}\right] \left[\frac{1 - e^{-at}}{a} - \frac{1 - e^{-(a+b)t}}{a + b}\right]^{-1}\right) - 1.$$

It is easy to show numerically that $E\left(t, a, b\right) \geq 0$ for all $t, a, b \geq 0$. Hence, the liquidity spread is increasing in maturity. Finally, it is straightforward to see that the liquidity spread is decreasing in liquidity λ.

2. **The liquidity spread is increasing in the default intensity δ:** Note that

$$e^{(\rho+\delta)\tau} \mathcal{L}\left(\tau, \lambda\right) = \frac{\eta}{\eta + \lambda\gamma} \int_0^\tau e^{(\rho+\delta)(\tau-t)} \left(1 - e^{-(\eta+\lambda\gamma)t}\right) dt$$

$$\frac{\partial\left(e^{(\rho+\delta)\tau} \mathcal{L}\left(\tau, \lambda\right)\right)}{\partial \delta} = \frac{\eta}{\eta + \lambda\gamma} \int_0^\tau \left(\tau - t\right) e^{(\rho+\delta)(\tau-t)} \left(1 - e^{-(\eta+\lambda\gamma)t}\right) dt > 0.$$

□

Proof of Lemma 3. The mid-price is

$$\frac{1}{2}\left(D^H\left(y; \lambda\right) + D^L\left(y; \lambda\right)\right) = e^{-(\rho+\delta)y} - \frac{1}{2}\left(h \frac{1 - e^{-(\rho+\delta)y}}{\rho + \delta} + \left(\frac{\eta - \lambda\gamma}{\eta}\right) \mathcal{L}\left(y, \lambda\right)\right),$$

where

$$\left(\frac{\eta-\lambda\gamma}{\eta}\right)\mathcal{L}\left(y,\lambda\right)=h\frac{\eta-\lambda\gamma}{\lambda^{H}+\lambda\gamma}\left(\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}-\frac{1-e^{-\left(\rho+\delta+\lambda^{H}+\lambda\gamma\right)y}}{\rho+\delta+\eta+\lambda\gamma}\right).$$

The mid-price is

$$e^{-(\rho+\delta)y}-\frac{h}{\eta+\lambda\gamma}\left(\eta\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}-\frac{(\eta-\lambda\gamma)}{2}\frac{1-e^{-(\rho+\delta+\eta+\lambda\gamma)y}}{\rho+\delta+\eta+\lambda\gamma}\right).$$

Define the gains from trade as

$$GT\left(y\right)=h\frac{1-e^{-\left(\rho+\delta+\lambda^{H}+\lambda\gamma\right)y}}{\rho+\delta+\eta+\lambda\gamma},$$

so

$$BA\left(y\right)=GT\left(y\right)\left[e^{-\left(\rho+\lambda^{D}\right)y}-\frac{1}{\eta+\lambda\gamma}\left(h\eta\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}-\frac{(\eta-\lambda\gamma)}{2}GT\left(y\right)\right)\right]^{-1}$$

$$BA\left(y\right)=\left[\frac{e^{-(\rho+\delta)y}}{GT\left(y\right)}-h\frac{\eta}{\eta+\lambda\gamma}\frac{\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}}{GT\left(y\right)}+\frac{1}{2}\frac{\eta-\lambda\gamma}{\eta+\lambda\gamma}\right]^{-1}.$$

Note that $\frac{e^{-(\rho+\delta)y}}{GT(y)}$ is decreasing in y because of $e^{-(\rho+\delta)y}$ is decreasing and $GT\left(y\right)$ is increasing in y. Note that $\frac{\frac{1-e^{-(\rho+\delta)y}}{\rho+\delta}}{GT(y)}$ is increasing in y because the discount in GT is larger than in the numerator. Hence, with the negative sign it is decreasing. Therefore, everything in the square bracket is decreasing in y, and as it is to the power of -1, the $BA\left(y\right)$ is increasing in y. $\square$

A.1.4 Free entry

Proof of Proposition 2. Gains from trade are

$$D^{H}\left(y;\lambda\right)-D^{L}\left(y;\lambda\right)=h\frac{1-e^{-c_{1}y}}{c_{1}}\qquad c_{1}=\rho+\delta+\eta+\lambda\gamma.$$

The buyer gets $(1-\gamma)$ of the gains from trade. Hence, the free entry condition reads

$$c=(1-\gamma)\int_{0}^{\tau}\beta\frac{\mu^{L}(y)}{\mu^{S}}h\frac{1-e^{-c_{1}y}}{c_{1}}dy,$$

and $\theta = \frac{\mu^S}{\mu^B}$. Also, recall that $\mu^S = \int_0^\tau \mu^L(y)\,dy$ Hence, the free entry condition is

$$\begin{aligned} c &= \frac{(1-\gamma)\,h}{c_1} A\theta^\alpha \int_0^\tau \frac{\mu^L(y)}{\mu^S}\left(1 - e^{-c_1 y}\right) dy \\ c &= \frac{(1-\gamma)\,h}{c_1} A\theta^\alpha \left(1 - \frac{\int_0^\tau e^{-c_1 y}\mu^L(y)\,dy}{\int_0^\tau \mu^L(y)\,dy}\right). \end{aligned}$$

Define $c_2 = \eta + \delta + \lambda$ and note that

$$\int_0^\tau e^{-c_1 y}\mu^L(y)\,dy = \mu^0 \frac{\eta}{\eta+\lambda}\left(\frac{e^{-c_1\tau} - e^{-\delta\tau}}{\delta - c_1} - \frac{e^{-c_1\tau} - e^{-c_2\tau}}{c_2 - c_1}\right).$$

As a result, the ratio of integrals in the free-entry condition reads

$$\left(\frac{e^{-c_1\tau} - e^{-\lambda^D\tau}}{\delta - c_1} - \frac{e^{-c_1\tau} - e^{-c_2\tau}}{c_2 - c_1}\right)\left(\frac{1 - e^{-\tau\delta}}{\delta} - \frac{1 - e^{-(\eta+\delta+\lambda)\tau}}{\eta+\delta+\lambda}\right)^{-1}, \tag{24}$$

and the free-entry condition boils down to

$$c = \frac{(1-\gamma)\,h}{c_1} A\theta^\alpha \left(1 - \left(\frac{e^{-c_1\tau} - e^{-\delta\tau}}{\delta - c_1} - \frac{e^{-c_1\tau} - e^{-c_2\tau}}{c_2 - c_1}\right)\left(\frac{1 - e^{-\tau\delta}}{\delta} - \frac{1 - e^{-c_2\tau}}{c_2}\right)^{-1}\right).$$

First, note that it is easy to show that Equation (24) is increasing in τ. Next, consider $\tau = 0$. Note that the ratio of integrals in the free-entry condition is equal to 1 as

$$\begin{aligned} &\lim_{\tau\to 0}\left(\frac{e^{-c_1\tau} - e^{-\delta\tau}}{\lambda^D - c_1} - \frac{e^{-c_1\tau} - e^{-c_2\tau}}{c_2 - c_1}\right)\left(\frac{1 - e^{-\delta\tau}}{\delta} - \frac{1 - e^{-c_2\tau}}{c_2}\right)^{-1} \\ &= \lim_{\tau\to 0}\left(\frac{-c_1 e^{-c_1\tau} + \delta e^{-\delta\tau}}{\delta - c_1} - \frac{-c_1 e^{-c_1\tau} + c_2 e^{-c_2\tau}}{c_2 - c_1}\right)\left(\frac{\delta e^{-\tau\delta}}{\delta} - \frac{c_2 e^{-c_2\tau}}{c_2}\right)^{-1} \\ &= \lim_{\tau\to 0}\left(\frac{(c_1)^2 e^{-c_1\tau} - (\delta)^2 e^{-\delta\tau}}{\delta - c_1} - \frac{(c_1)^2 e^{-c_1\tau} - (c_2)^2 e^{-c_2\tau}}{c_2 - c_1}\right)\left(\frac{-(\delta)^2 e^{-\tau\delta}}{\delta} - \frac{-(c_2)^2 e^{-c_2\tau}}{c_2}\right)^{-1} \\ &= \left(\frac{(c_1)^2 - (\delta)^2}{\delta - c_1} - \frac{(c_1)^2 - (c_2)^2}{c_2 - c_1}\right)\left(\frac{-(\delta)^2}{\lambda^D} - \frac{-(c_2)^2}{c_2}\right)^{-1} \\ &= \left(\frac{(c_1+\delta)\left(c_1 - \lambda^D\right)}{\delta - c_1} - \frac{(c_1+c_2)(c_1-c_2)}{c_2 - c_1}\right)(c_2 - \delta)^{-1} \\ &= \left(-(c_1+\delta) + (c_1+c_2)\right)(c_2-\delta)^{-1} = \left(c_2 - \lambda^D\right)(c_2-\delta)^{-1} = 1, \end{aligned}$$

where we applied L'Hopital's rule in the second and third line. As a result, the free-entry condition is satisfied if and only if $\lim_{\tau\to 0}\theta = \infty$. Hence, $\lim_{\tau\to 0}\lambda = 0$. That is, $\lambda(0) = 0$.

Next, consider the case of $\tau \to \infty$. The ratio of integrals in the free-entry condition is equal to zero. Hence $c = \frac{h}{c_1}(1-\gamma) A\theta^{\alpha}$. Recall that $c_1 = \rho + \delta + \eta + \lambda\gamma$ and $\lambda = A\theta^{\alpha-1}$. Hence,

$$\rho + \delta + \eta + \gamma A\theta^{\alpha-1} = \frac{h(1-\gamma)}{c} A\theta^{\alpha}.$$

As $\alpha \in (0,1)$ the left-hand side is decreasing in θ and the right-hand side is increasing in θ. As a result, there exists a unique $\theta \in \mathbb{R}_+$. That is, $\lim_{\tau\to\infty} \lambda(\tau) = \overline{\lambda} \in \mathbb{R}_+$. $\square$

A.2 Borrowers

Proof of Proposition 3. Let $J(\tau,\lambda)$ be the value of the firm with maturity τ and liquidity λ, and let $Z = \frac{\zeta}{\rho+\delta}$. The first-order condition is

$$\begin{aligned} J_{\tau}(\tau,\lambda) =& e^{-(\rho+\delta)\tau} Z\left(1-(\rho+\delta)\tau\right) \\ &- e^{cs^{liq}(\lambda,\tau)\tau}\left[\frac{\partial I(\tau)}{\partial\tau} + (\Phi + I(\tau))\, cs^{liq}(\lambda,\tau)\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right)\right]. \end{aligned}$$

Note that τ is increasing in λ if the derivative of the first-order condition with respect to λ is positive

$$\begin{aligned} J_{\tau\lambda}(\tau,\lambda) = & -e^{cs^{liq}(\lambda,\tau)\tau}\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}\tau\left[\frac{\partial I(\tau)}{\partial\tau} + (\Phi+I(\tau))\, cs^{liq}(\lambda,\tau)\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right)\right] \\ & - e^{cs^{liq}(\lambda,\tau)\tau}\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}(\Phi+I(\tau))\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right) \\ & - e^{cs^{liq}(\lambda,\tau)\tau}(\Phi+I(\tau))\, cs^{liq}(\lambda,\tau)\frac{\partial\varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda}. \end{aligned}$$

Recall that $\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda} \leq 0$, so the first and second terms are positive. However, the last term involves $\frac{\partial\varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda}$ for which we do not know the sign. We can write $J_{\tau\lambda}(\tau,\lambda)$ as

$$\begin{aligned} J_{\tau\lambda}(\tau,\lambda) = & -e^{cs^{liq}(\lambda,\tau)\tau}\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}\frac{\partial I(\tau)}{\partial\tau}\tau - e^{cs^{liq}(\lambda,\tau)\tau}(\Phi+I(\tau)) \\ & \left[\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right)\left(\tau cs^{liq}(\lambda,\tau)+1\right) + cs^{liq}(\lambda,\tau)\frac{\partial\varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda}\right]. \end{aligned}$$

The first term is positive. A sufficient condition for $J_{\tau\lambda}(\tau,\lambda) \geq 0$ is that the second term is also positive. This implies

$$\frac{\partial cs^{liq}(\lambda,\tau)}{\partial\lambda}\left(1+\varepsilon_{cs^{liq},\tau}(\lambda,\tau)\right)\left(\tau cs^{liq}(\lambda,\tau)+1\right) \leq -cs^{liq}(\lambda,\tau)\frac{\partial\varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial\lambda}.$$

This expression depends only on $cs^{liq}(\lambda, \tau)$. By Lemma 2 we can approximate the liquidity spread as a linear function increasing in τ and decreasing in λ. Let $cs^{liq}(\lambda, \tau) = c_\tau \tau + c_\lambda \lambda$ with $c_\tau \geq 0$ and $c_\lambda \leq 0$. Then $\varepsilon_{cs^{liq},\tau}(\lambda, \tau) = \frac{c_\tau \tau}{c_\tau \tau + c_\lambda \lambda}$ and $\frac{\partial \varepsilon_{cs^{liq},\tau}(\lambda,\tau)}{\partial \lambda} = -\frac{c_\tau \tau c_\lambda}{(c_\tau \tau + c_\lambda \lambda)^2}$. The sufficient condition reads

$$\left(c_\tau \tau + cs^{liq}(\lambda, \tau)\right) cs^{liq}(\lambda, \tau)\, \tau + cs^{liq}(\lambda, \tau) \geq 0,$$

which is satisfied. Therefore, $J_{\tau\lambda}(\tau, \lambda) \geq 0$ and $\frac{\partial \tau(\lambda)}{\partial \lambda} \geq 0$. Finally it is straightforward to see that $\underline{\tau} = \tau(0) \leq \lim_{\lambda \to \infty} \tau(\lambda) = \tau^* < \infty$. $\square$

A.3 Existence of equilibrium

Proof of Proposition 4. First, Proposition 2 defines a schedule for the lenders $\tau^L(\lambda)$. Note that $\tau^L(0) = 0$, and there exists $\overline{\lambda}$ such that $\tau^L(\overline{\lambda}) = \infty$.

Second, Proposition 3 define $\tau^B(\lambda)$, and notice that $\tau^B(0) = \underline{\tau} > 0$ and $\tau^B(\lambda) \geq 0$ for all λ.

Finally, define $F(\lambda) = \tau^L(\lambda) - \tau^B(\lambda)$, and note that $F(0) = -\underline{\tau} < 0$ and $F(\overline{\lambda}) = \infty$. Hence, as F is continuous, Bolzano's theorem implies that there exits $\lambda^\star$ such that $F(\lambda^\star) = 0$, which defines the equilibrium. $\square$

B Extensions

This Appendix extends the model in several dimension and complements the main analysis.

B.1 Government-Sponsored Intermediaries

The government agency intermediates assets in the secondary market to improve the liquidity of financial markets. Government agents are subject to the same idiosyncratic risk of holding costs as private agents, so they act as both buyers and sellers in the secondary market.[30] One possible interpretation for these idiosyncratic shocks for public agents can be balance sheet requirements, such as the priority sector lending in India discussed before. However, the government can choose different prices than those charged in private meetings. If they buy (sell) at a high (low) price they will run a deficit, which is financed by distortionary taxes on the corporate sector.

Figure 8 shows a schematic representation of the model with GSIs. Private sellers can sell both to private and government agents and private buyers can buy in the secondary market

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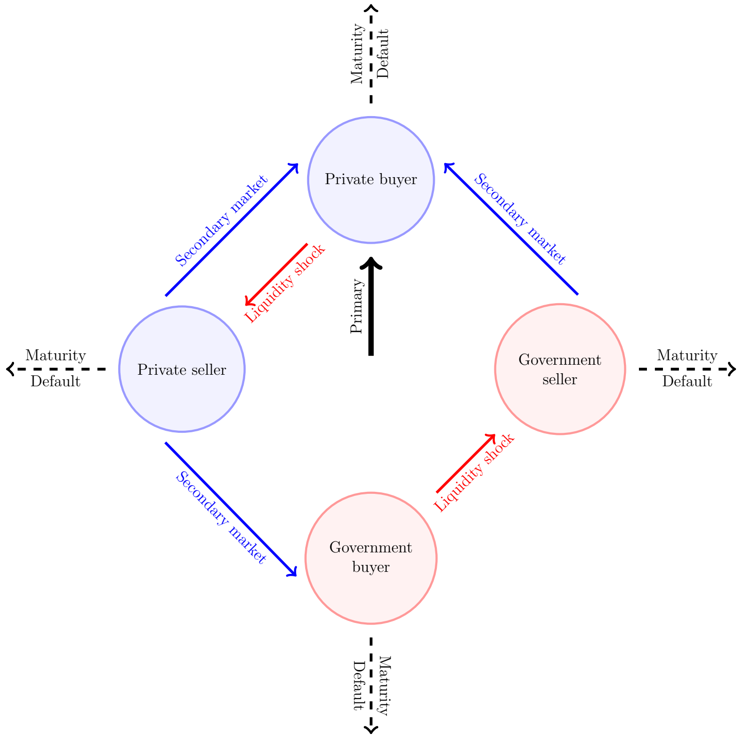
[30] The formulation of government agents is similar to Aiyagari et al. (1996).

Figure 8: **Schematic representation of GSIs**



from either private or government sellers. In this section, we describe the key features of the model with GSIs while Appendix B.1.3 contains additional details.

The government has four instruments: the size of GSIs, prices for buying and selling for their trading agents, and the corporate tax rate. The objective is to maximize aggregate steady-state welfare subject to running a balanced budget and equilibrium conditions. Recall that both primary and secondary financial markets are competitive—i.e., participants make zero profits in expectation. However, the production sector—i.e., the borrowers—have positive profits in equilibrium. Hence, we define the objective of the government as maximizing the value of the corporate sector.

Matching There is random matching between sellers and buyers. In the benchmark policy we assume that both government and private agents have the same efficiency to find a counterpart. For robustness exercises in Section B.1.2, we consider a general formulation in which government and private agents may differ in their efficiency to find a counterpart. Let $e^{i,j}$ be the efficiency for $i = p, g$ (private and government agents, respectively) of $j = b, s$ (buy and sell, respectively). In the benchmark model we assume that $e^{i,j} = 1$ for all i, j.

The total mass of sellers, μ^s, is composed of private and government agents. Private sellers are $\mu^{p,s} = \int_0^\tau e^{p,s}\mu^{p,l}(y)dy$, where $\mu^{p,l}(y)$ is the measure of low-valuation private agents holding an asset and willing to sell. Similarly, government sellers are $\mu^{g,s} = \int_0^{\tau} e^{g,s}\mu^{l,g}(y)dy$, where $\mu^{l,g}(y)$ is the measure of low-valuation government agents holding an asset and willing to sell. The total measure of buyers includes private buyers $\mu^{p,b}$, determined by a free-entry condition, and government buyers $\mu^{g,b}$, which is a policy instrument chosen by the government. Hence, $\mu^b = e^{p,b}\mu^{p,b} + e^{g,b}\mu^{g,b}$.

The market tightness is $\theta = \frac{\mu^s}{\mu^b}$, which affects the buying and selling intensities $\beta = A\theta^{\alpha}$ and $\lambda = A\theta^{\alpha-1}$, respectively. Let λ^{s-b} be the intensity at which a seller of type $s = p, g$ meets a buyer of type $b = p, g$. Similarly, let $\beta^{s-b}(y)$ be the intensity at which a buyer of type $b = p, g$ meets a seller of type $s = p, g$ with an asset of time-to-maturity y. The matching technology implies that

$$\lambda^{p-p} = \lambda e^{p,s} e^{p,b} \frac{\mu^{p,b}}{\mu^b} \qquad \lambda^{p-g} = \lambda e^{p,s} e^{g,b} \frac{\mu^{g,b}}{\mu^b} \qquad \lambda^{g-p} = \lambda e^{g,s} e^{p,b} \frac{\mu^{p,b}}{\mu^b}$$

$$\beta^{p-p}(y) = \beta e^{p,s} e^{p,b} \frac{\mu^{l,p}(y)}{\mu^s} \quad \beta^{g-p}(y) = \beta e^{g,s} e^{p,b} \frac{\mu^{l,g}(y)}{\mu^s} \quad \beta^{p-g}(y) = \beta e^{p,s} e^{g,b} \frac{\mu^{l,p}(y)}{\mu^s}.$$

Finally, we have to specify what happens after a meeting between a government buyer and a government seller. The idea is to interpret the government as a large player and private agents as atomistic. However, for tractability, we assume that all investors can hold either zero or one asset. To bypass this restriction, we assume that a government seller cannot trade with a government buyer; i.e., $\lambda^{g-g} = 0$. Note that this is a conservative assumption as the cost of the policy is smaller if intra-government trades can occur. In fact, Section B.1.2 solves the model with this type of trade and shows that there are larger effects.

Prices in secondary markets There are three types of meetings in secondary markets. Let $P^{S,s-b}(y)$ be the price when a seller of an asset with time-to-maturity y of type $s = p, g$ meets a buyer of type $b = p, g$. In a meeting between private agents, the price is determined by Nash Bargaining in which the seller has bargaining power γ so

$$P^{S,p-p}(y) = D^L(y) + \gamma(D^H(y) - D^L(y)).$$

The prices that involve either a government buyer or seller are determined by the government. In the quantitative solution we restrict prices to be in the following parametric family:

$$P^{S,g-p}(y) = D^L(y) + \gamma^{g,s}(D^H(y) - D^L(y)), \tag{25}$$

$$P^{S,p-g}(y) = D^L(y) + \gamma^{g,b}\left(D^H(y) - D^L(y)\right), \tag{26}$$

and let the government choose $\gamma^{g,s}$ and $\gamma^{g,b}$ in $[0, 1]$. Note that prices are similar to those in private meetings but that the government can choose a different bargaining power. As we will show latter, it is optimal to set $\gamma^{g,s} = 0$ and $\gamma^{g,b} = 1$. This implies that the government gives all the bargaining power to the private sector—i.e., the government buys at a *high* price and sells at a *low* price.

Of course, this is an important restriction on government prices, but it follows from the objective of finding a lower bound on the effects of the policy. For example, one can use the model with segmented markets presented in Appendix B.3 and allow the government to set different prices according to maturity. However, we will show that even without this flexibility, the effects of GSIs are quantitatively significant and the extension of targeting prices according to maturity is likely to improve the results from the lower bound identified in this exercise. Importantly, note that this alternative specification would work through the same channel as the mechanism described in the benchmark policy.

Private valuations The value of holding an asset for a high-valuation private agent is equivalent to the benchmark model, Equation (9). However, the value of a low-valuation private agent is different as now the agent can sell the asset to both private and government buyers. Under the government prices specified in (25), the price the government offers is equivalent to that offered in private meetings but in which the seller has a different bargaining power. Hence, the value for a low-valuation agent is equal to the benchmark model, Equation (10), with an augmented selling intensity: $\lambda = \lambda^{p-p}\gamma + \lambda^{p-g}\gamma^{g,b}$.

Let λ^{GSI} and λ^{EQ} be the equilibrium liquidity in the economy with and without GSIs, respectively. If $\lambda^{GSI} > \lambda^{EQ}$, Lemma 2 implies that the liquidity spread will be lower in an economy with GSIs. However, borrowers have to pay a distortionary tax to finance the intervention. Hence, ex-ante, we don't know which policies will increase welfare for borrowers.

Private buyers can meet with private and government sellers. The free entry condition is

$$c = \int_0^\tau \beta^{p-p}(y)\left(D^H(y) - P^{S,p-p}(y)\right)dy + \int_0^\tau \beta^{g-p}(y)\left(D^H(y) - P^{S,g-p}(y)\right)dy.$$

Cost of GSIs The government runs a balanced budget. The constraint is

$$\begin{aligned}&\mu^\pi(\tau)x^c f(\tau) + \left[\mu^{g,h}(0) + \mu^{g,l}(0)\right] + \lambda^{g-p}\int_0^\tau \mu^{g,l}(y)P^{S,g-p}(y)y \\ =&\mu^{g,b}c + \mu^{g,b}\int_0^\tau \beta^{p-g}(y)\,P^{S,p-g}(y)dy + h\int_0^\tau \mu^{g,l}(y)dy.\end{aligned} \tag{27}$$

The left-hand side of Equation (27) represents the government's income. First, the government charges a proportional corporate tax x^c to producing firms μ^f, where flow profits are $\pi(\tau)$.

Second, some of the securities held by government agents mature. Third, some low-valuation government agents sell the securities to the private sector.

The right-hand side of Equation (27) captures the expenditures. A measure $\mu^{g,b}$ of agents are searching in secondary markets, and some of them buy a bond. Moreover, some government agents are low-valuation and have to pay the holding cost h.

Optimal policy The objective of the government is to maximize steady-state profits of the production sector subject to the equilibrium conditions and the budget constraint (27)[31]

$$\max_{x^c,\mu^{g,b},\gamma^{g-b},\gamma^{g-s}} \mu^f(\tau)e^{-(\rho+\delta)\tau}\left((1-x^c)F(\tau)-I(\tau)e^{r(\tau)\tau}\right) \quad \text{s.t. (27) and equilibrium conditions.}$$

GSIs cause both a direct and an equilibrium effect. On the one hand, a larger intervention needs higher taxes, which lower welfare. On the other hand, if the policy increases the equilibrium liquidity, credit spreads for long-term borrowing decline, which benefits borrowers. Therefore, the optimal policy solves the trade-off between these two effects.

B.1.1 Optimal GSIs in the US

First, consider the optimal policy under the calibration for the US. The bargaining power when the government acts as a buyer, $\gamma^{g,b}$, directly affects the value of low- and high-valuation private agents. The optimal policy sets $\gamma^{g,b}=1$ so private sellers get more gains from trade when trading with the government. This generates a direct effect on increasing the value of private agents in the financial sector and reduces financial costs for the production sector.

The bargaining power when the government acts as a seller, $\gamma^{g,s}$, has a direct effect on the incentives of private agents to search in the secondary market. The optimal value is $\gamma^{g,s}=0$, i.e., the private buyer gets larger gains. Given the results in this section, the exercises set $\gamma^{g,b}=1$ and $\gamma^{g,s}=0$ and let the government choose $\mu^{g,b}$ and the tax rate.[32]

Finally, the measure of government agents searching in the secondary markets is optimally chosen to maximize the welfare gains. If $\mu^{g,b}=0$, the economy is equivalent to no intervention, while as $\mu^{g,b}$ increases, the tax rate also increases to balance the budget. Under the optimal policy, there is an increase in liquidity, which generates a drop on the five-year spread from 139 to 96 basis points. As a result, the optimal maturity increases from 5.00 to 5.07, and the welfare gains are about 0.77% (first and second row of Table 6).

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[31] We consider steady-state welfare because the transitions involve manipulating the boundary conditions of the distributions. However, note that this is a conservative assumption because during a transition, old generations holding a security issued before the intervention are better off because asset prices increase.

[32] In fact, in all exercises we verified that if the government can choose bargaining power then it chooses these values. However, this restriction simplifies the description of the results without adding additional intuition.

B.1.2 Robustness: Alternative Policies

The intervention considered so far should be thought of as a lower bound on the effects of GSIs. Table 6 explores alternative assumptions that can improve the effects of government interventions for two levels of financial frictions. The first panel considers a matching efficiency at the level calibrated for the US, while the second panel considers an economy with trading frictions similar to Argentina.

First, consider government agents that are more efficient at searching for counterparts. The third and fourth rows of each panel of Table 6 show the result of increasing the search efficiency of government agents by 10% and 50%, respectively (i.e., $e^{g,b} = e^{g,s} = 1.1$ and $e^{g,b} = e^{g,s} = 1.5$, respectively). Overall, the results show that as the efficiency of the government increases, the intervention becomes more effective in increasing the liquidity of the economy; the yield curve flattens even more; and firms issue at longer maturities.

Finally, recall that the benchmark policy assumes that $\lambda^{g-g} = 0$; i.e., a government seller cannot trade with a government buyer. For a given size $\mu^{g,b}$, the cost of GSIs decreases if the government can reallocate securities among its trading agents. The last row of Table 6 shows that if government agents can trade among themselves, GSIs are more efficient and the effects on credit spreads, maturity, and welfare improve.

There are legitimate reasons to imagine that government agents might have more flexibility than private agents. Hence, the results of the benchmark policy should be considered as a lower bound on the implications for GSIs. For example, Table 6 shows that the gains from government intervention can be larger if government agents are more efficient at finding counterparts or can trade among themselves.

B.1.3 Proofs

This appendix describes how to solve the distribution of financiers with GSIs. The total assets with time-to-maturity t are $\mu(t) = \mu^0 e^{-\delta t}$. These assets are held by four types of agents: $\mu(t) = \mu^{p,h}(t) + \mu^{p,l}(t) + \mu^{g,h}(t) + \mu^{g,l}(t)$. The laws of motions for the private sector are

$$
\begin{aligned}
-\dot{\mu}^{p,h}(t) &= -(\eta + \delta)\,\mu^{p,h}(t) + \left(\beta^{p-p}(t) + \beta^{g-p}(t)\right)\mu^{p,b} \\
-\dot{\mu}^{p,l}(t) &= \eta\mu^{p,h}(t) - \left(\delta + \lambda^{p-p} + \lambda^{p-g}\right)\mu^{p,l}(t),
\end{aligned}
$$

Table 6: **GSI: Alternative policies**

	Liquidity	**5-year spread**	**Maturity**	**Welfare gains**	**Profitability**
Low trading frictions (US)					
No GSIs	13.00	139	5.00		
Benchmark policy	18.89	96	5.07	0.77	0.55
Gov. 10% more efficient	19.18	95	5.07	0.83	0.57
Gov. 50% more efficient	20.04	91	5.08	1.03	0.63
Gov. transactions	21.26	86	5.08	1.21	0.70
High trading frictions (Argentina)					
No GSIs	3.63	468	4.49		
Benchmark policy	5.25	332	4.68	1.61	2.04
Gov. 10% more efficient	5.34	326	4.69	1.78	2.13
Gov. 50% more efficient	5.62	311	4.71	2.29	2.39
Gov. transactions	6.03	291	4.75	3.37	2.77

with boundary conditions $\mu^{p,h}(\tau) = \mu^0$ and $\mu^{p,l}(\tau) = 0$. The law of motions for government agents are

$$
\begin{aligned}
-\dot{\mu}^{g,h}(t) &= -(\eta+\delta)\,\mu^{g,h}(t) + \left(\beta^{p-g}(t) + \beta^{g-g}(t)\right)\mu^{g,b} \\
-\dot{\mu}^{g,l}(t) &= \eta\mu^{g,h}(t) - \left(\delta + \lambda^{g-p} + \lambda^{g-g}\right)\mu^{g,l}(t),
\end{aligned}
$$

with boundary conditions $\mu^{g,h}(\tau) = \mu^{g,l}(\tau) = 0$. Matching implies

$$
\begin{aligned}
\mu^{p,b}\beta^{p-p}(t) &= \mu^{p,l}(t)\,\lambda^{p-p} \\
\mu^{p,b}\beta^{g-p}(t) &= \mu^{g,l}(t)\,\lambda^{g-p} \\
\mu^{g,b}\beta^{p-g}(t) &= \mu^{p,l}(t)\,\lambda^{p-g} \\
\mu^{g,b}\beta^{g,b-g}(t) &= \mu^{g,l}(t)\,\lambda^{g-g}.
\end{aligned}
$$

Define $\mu(t) = \left[\mu^{p,h}(t), \mu^{p,l}(t), \mu^{g,h}(t), \mu^{g,l}(t)\right]$. The boundary condition is $\mu(\tau) = [\mu^0, 0, 0, 0]$ and the system is $\dot{\mu}(t) = A\mu(t)$ where

$$
A = \begin{bmatrix}
\eta+\delta & -\lambda^{p-p} & 0 & -\lambda^{g-p} \\
-\eta & \delta + \lambda^{p-p} + \lambda^{p-g} & 0 & 0 \\
0 & -\lambda^{p-g} & \eta+\delta & -\lambda^{g-g} \\
0 & 0 & -\eta & \delta + \lambda^{g-p} + \lambda^{g-g}
\end{bmatrix}.
$$

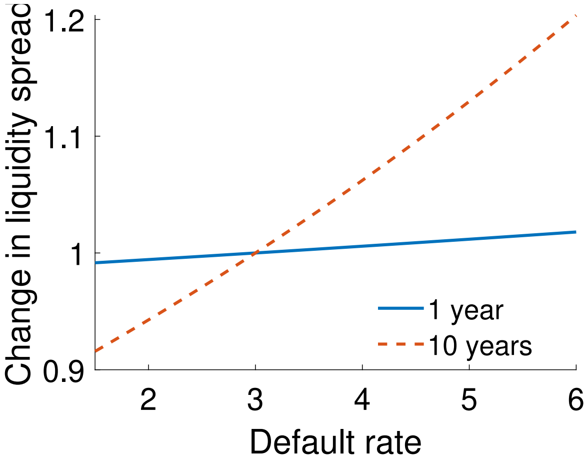
The solution of this system is standard. The only caveat is that we should pay attention to the real and complex eigenvalues of the matrix A.

B.2 Rollover and Default

Default affects credit spreads and investment decisions. Lemma 2 shows that the liquidity spread increases with the default intensity. Quantitatively, long-term rates react more than short-term rates to changes in default. Figure 9 shows how the default rate affects the liquidity spread at maturities of 1 and 10 years, relative to the benchmark of $\delta = 0.03$. The liquidity spread for long maturities reacts more to changes in the default rate. Hence, when δ increases, the yield curve shifts upward because of both default and liquidity. As a result, the firm chooses shorter-term projects (see rows four to six in Table 5).

Moreover, when there is no default risk and secondary markets are centralized, firms have no incentive to issue short-term debt regardless of the issuance cost. However, when default risk is positive, even if secondary markets are centralized, firms choose to rollover debt when the issuance cost is not too high. Hence, both default risk and trading frictions shape rollover choices.

Figure 9: **Liquidity-default interactions**



Note: The figure shows how the liquidity spread at 1 and 10 years changes with δ with respect to the benchmark of 0.03.

B.3 Segmented Markets

In the benchmark model, assets of different maturities are traded in a single secondary market. A potential concern could be that assets with short maturities, with small gains from trade, preclude the entry of buyers into the secondary market. To address this issue, we study an economy with secondary markets segmented by the time-to-maturity of assets. The main takeaway is that even though the market tightness for short-term bonds increases, the tightness for long-term assets remains similar to the case with only one market. Hence, the secondary market in the benchmark model is effectively a market for long-term assets. The intuition for this result is that, in equilibrium, the single market is not dominated by short-term assets, so there are always sufficient gains from trade.

Intuitively, in long-term markets, there are more gains from trade and therefore more entry of buyers. However, because there is Nash Bargaining over the gains from trade and buyers keep only a fraction $(1-\gamma)$ of the gains, the increase in the entry of buyers into long-term markets is not enough to compensate for the increase in the importance of the secondary markets for longer securities. As a result, the yield curve increases with maturity even with segmented markets. We describe the key features of the model and relegate to Appendix B.3.1 the full characterization of this extension.

Let τ be the initial maturity and consider the case in which secondary markets are segmented in N markets. Let $0=\tau_1<\cdots<\tau_{N+1}=\tau$, so each market $j=1,\ldots,N$ trades assets with time-to-maturity $t\in[\tau_j,\tau_{j+1}]$.

Matching and distribution of agents Let $\mu^j(y)=\left[\mu^{H,j}(y),\mu^{L,j}(y)\right]$ be the measure of high- and low-valuation agents holding an asset with time-to-maturity t in market j. We start with market N and solve for the distribution of agents backwards. The boundary condition is $\mu^N(\tau)=[\mu^0,0]$. Next, we iterate toward markets of shorter maturities with boundary conditions $\mu^j(\tau_{j+1})=\mu^{j+1}(\tau_{j+1})$ for $j=1,\ldots,N-1$. Lemma 4 characterizes the distribution of agents in each market.

Lemma 4. *The measure of agents for markets $j=1,\ldots,N$ is given by the following backward recursion:*

$$\begin{bmatrix}\mu^{H,N+1}(\tau)\\ \mu^{L,N+1}(\tau)\end{bmatrix}=\begin{bmatrix}\mu^0\\ 0\end{bmatrix}$$

and

$$\begin{aligned}\mu^{H,j}(y)=&\frac{\eta}{\eta+\lambda^j}\Big[\frac{\lambda^j}{\eta}e^{\delta(y-\tau_{j+1})}\left(\mu^{H,j+1}(\tau_{j+1})+\mu^{L,j+1}(\tau_{j+1})\right)\\&-e^{\left(\eta+\lambda^j+\delta\right)(y-\tau_{j+1})}\left(-\mu^{H,j+1}(\tau_{j+1})+\lambda^j\mu^{L,j+1}(\tau_{j+1})\right)\Big]\\ \mu^{L,j}(t)=&\frac{\eta}{\eta+\lambda^j}\Big[e^{\delta(y-\tau_{j+1})}\left(\mu^{H,j+1}(\tau_{j+1})+\mu^{L,j+1}(\tau_{j+1})\right)\\&+e^{\left(\eta+\lambda^j+\delta\right)(y-\tau_{j+1})}\left(-\mu^{H,j+1}(\tau_{j+1})+\lambda^j\mu^{L,j+1}(\tau_{j+1})\right)\Big],\end{aligned}$$

where λ^j is the selling intensity in market $j=1,\ldots,N$.

Valuations Let $D^{j}(y) = \left[D^{H,j}(y), D^{L,j}(y)\right]$ be the values for high- and low-valuation agents of holding an asset with time-to-maturity y in market $j = 1, \ldots, N$. To solve for the value of holding the asset, start with the first market, in which the boundary condition is that at maturity the value is equal to one, and then iterate forward, toward longer-term markets. The boundary condition for market $j = 1$ is $D^{1}(\tau_1) = [1, 1]$. Value matching for market $j = 2, \ldots, N$ implies $D^{j}(\tau_j) = D^{j-1}(\tau_j)$, and the Hamilton-Jacobi-Bellman equations are the same as in the benchmark model, Equations (9) and (10).

Free entry Free entry in each market implies that

$$c = (1-\gamma) \int_{\tau_j}^{\tau_{j+1}} \beta^{j}(y) \left(D^{H,j}(y) - D^{L,j}(y)\right) dy,$$

where β^{j} is the intensity at which a buyer finds a seller in market $j = 1, \ldots, N$. Appendix B.3.1 provides analytical solutions for the value functions and the free-entry condition.

Results The first panel of Figure 10 shows the market tightness relative to the case of only one market when $N = 2$ and $N = 3$. With segmentation, markets for short-term assets are tighter (more sellers to buyers), as there are fewer gains from trade. However, for long-term bonds we find a tightness similar to the case of no segmentation. The second panel repeats the exercise under different degrees of segmentation ($N = 1, \ldots, 50$). Note that even with 50 different markets, the tightness for markets with maturity above four years is almost identical to the case of no segmentation.

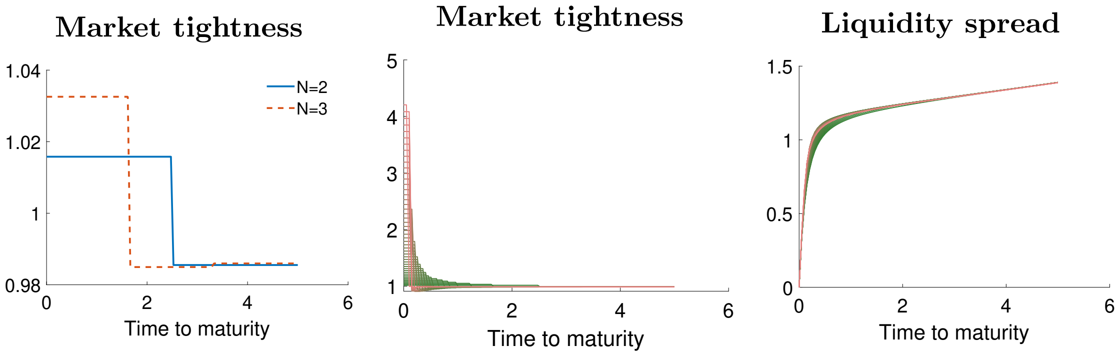
The third panel of Figure 10 shows the effects on the liquidity spread for different models with $N = 1$ to $N = 50$. As the market tightness after four years is identical in all these models, the implied liquidity spread is also the same. For short-term assets (maturities up to 4 years), there are some differences in the market tightness. However, they generate small variations in the yield curve. Therefore, we conclude that the secondary market in the benchmark model with $N = 1$ is effectively a market for long-term assets.

B.3.1 Proofs

Distributions of financiers

Proof of Lemma 4. Total assets are $\mu^{j}(t) = e^{(t-\tau)\delta}\mu^{0}$. The evolution for high- and low-valuation

Figure 10: **Segmented markets**


Market tightness

Market tightness

Liquidity spread

Note: The first (second) panel shows the market tightness relative to no segmentation for $N = 2, 3$ ($N = 2, \ldots, 50$). The third panel shows the liquidity spread for $N = 1, \ldots, 50$. In the second and third panels green lines are economies with larger N.

agents in market j are

$$-\dot{\mu}^{H,j}(t) = -(\eta + \delta)\,\mu^{H,j}(t) + \mu^{L,j}\beta^j(t)$$
$$-\dot{\mu}^{L,j}(t) = \eta\mu^{H,j}(t) - \left(\delta + \lambda^j\right)\mu^{L,j}(\tau_j).$$

Matching implies that $\mu^{B,j}\beta^j(t) = \mu^{L,j}(t)\,\lambda^j$. Hence, the system is

$$\dot{\mu}^j(t) = \begin{bmatrix} \eta + \delta & -\lambda^j \\ -\eta & \delta + \beta^j \end{bmatrix} \begin{bmatrix} \mu^{H,j}(t) \\ \mu^{L,j}(t) \end{bmatrix}$$

with eigenvalues δ and $\eta + \lambda^j + \delta$. Define V^j to be the matrix with the eigenvectors and R^j the diagonal matrix with the eigenvalues and $B^j = \left(V^j\right)^{-1}\mu^{j+1}(\tau_{j+1})$. Then

$$\mu^{H,j}(t) = \sum_{i=1}^{2} e^{R^j(i)(t-\tau_{j+1})}B^j(i)\,V^j(1,i)$$
$$\mu^{L,j}(t) = \sum_{i=1}^{2} e^{R^j(i)(t-\tau_{j+1})}B^j(i)\,V^j(2,i)\,.$$

For $j = 1, \dots, N-1$ we have that

$$\begin{aligned}
\mu^{H,j}(t) =& \frac{\eta}{\eta+\lambda^j}\left[\frac{\lambda^j}{\eta}e^{\delta(t-\tau_{j+1})}\left(\mu^{H,j+1}(\tau_{j+1})+\mu^{L,j+1}(\tau_{j+1})\right)\right] \\
&-\frac{\eta}{\eta+\lambda^j}\left[-e^{\left(\eta+\lambda^j+\delta\right)(t-\tau_{j+1})}\left(-\mu^{H,j+1}(\tau_{j+1})+\lambda^j\mu^{L,j+1}(\tau_{j+1})\right)\right] \\
\mu^{L,j}(t) =& \frac{\eta}{\eta+\lambda^j}\left[e^{\delta(t-\tau_{j+1})}\left(\mu^{H,j+1}(\tau_{j+1})+\mu^{L,j+1}(\tau_{j+1})\right)\right] \\
&+\frac{\eta}{\eta+\lambda^j}\left[+e^{\left(\eta+\lambda^j+\delta\right)(t-\tau_{j+1})}\left(-\mu^{H,j+1}(\tau_{j+1})+\lambda^j\mu^{L,j+1}(\tau_{j+1})\right)\right]
\end{aligned}$$

□

Value functions Let $Z^j(t) = D^{H,j}(t) - D^{L,j}(t)$ and $c_j = \rho + \delta + \eta + \gamma\lambda^j$, then $c_j Z^j(t) = h - \dot{Z}^j(t)$. The solution is $Z^j(t) = A^{Z,j}e^{-c_j t} + B^{Z,j}$ with $B^{Z,j} = \frac{h}{c_j}$, and the boundary condition pins down $A^{Z,j}$.

For $j = 1$ the boundary condition is $D^{H,1}(\tau_1) = D^{L,1}(\tau_1) = 1$ so $A^{Z,1} = -\frac{h}{c_1}$.

For $j = 2, \dots, N$ we have that $Z^j(\tau_j) = Z^{j-1}(\tau_j)$, which implies $A^{Z,j} = e^{c_j\tau_j}\left(A^{Z,j-1}e^{-c_{j-1}\tau_j} + \frac{h}{c_{j-1}} - \frac{h}{c_j}\right)$.

Next, we can solve for the value of high- and low-valuation agents using Z^j and the initial conditions. For high-valuation agents

$$(\rho+\delta)D^{H,j}(t) = -\dot{D}^{H,j}(t) - \eta\left(A^{Z,j}e^{-c_j t} + \frac{h}{c_j}\right).$$

The solution is $D^{H,j}(t) = A^{H,j} + B^{H,j}e^{-\left(\rho+\lambda^D\right)t} + C^{H,j}e^{-c_j t}$, with $A^{H,j} = -\frac{\eta h}{(\rho+\delta)c_j}$, $C^{H,j} = \frac{\eta A^{Z,j}}{\eta+\gamma\lambda^j}$, and the boundary condition pins down $B^{H,j}$.

For $j = 1$, we have that $D^{H,1}(\tau_1) = 1$ and $\tau_1 = 0$, so $B^{U,1} = 1 - A^{U,j} - C^{U,j}$. For $j = 2, \dots, N$, we have that $D^{H,j}(\tau_j) = D^{H,j-1}(\tau_j)$ so

$$B^{H,j} = e^{(\rho+\delta)\tau_j}\left(A^{H,j-1} - A^{H,j} + B^{H,j-1}e^{-(\rho+\delta)\tau_j} + C^{H,j-1}e^{-c_{j-1}\tau_j} - C^{H,j}e^{-c_j\tau_j}\right),$$

which defines a recursion in $B^{H,j}$.

Free entry The free-entry condition in each market is

$$c_j = (1-\gamma)\int_{\tau_j}^{\tau_{j+1}}\beta^j(t)\left(D^{H,j}(t) - D^{L,j}(t)\right)dt,$$

where $\beta^j(t) = A\left(\theta^j\right)^\alpha\frac{\mu^{C,j}(t)}{\mu^{C,j}}$, and both the measures and value functions are the sum of exponential functions. Hence, it is easy to solve for the integrals on the free-entry condition in each market.

B.4 Bank Loans and other securities

The benchmark model assumes that firms borrow from corporate bond markets, but similar trading frictions apply to other sources of external finance such as bank loans, venture capital, or private equity funds. The main reason to study corporate bonds is that these assets are already well studied in the literature about trading frictions and we can quantify the effects (e.g., He and Milbradt, 2014). However, for these alternative sources of finance there are also frictional secondary markets. For example, in 2006 the U.S. secondary loan market reached a volume of more than 200 billion (see Drucker and Puri, 2008; Altman et al., 2010).

We can interpret the financial sector of the model as bank lending in which the primary market represents the origination of the loan and the secondary market is a market for loans across banks. A moment commonly used to compare intermediation costs across countries is the bank's net interest margin, which measures the difference between interest income and payments to lenders using bank balance-sheet data from Bankscope (the data is available at The World Bank Global Financial Development Database). For example, Greenwood et al. (2013) attribute all of the spreads to the intermediation costs related to acquiring information about borrowers. Through the lens of this paper, however, the net interest margin can also reflect the illiquidity cost that banks will charge at origination of the loan, taking into account that they might need to later sell the loan in a frictional secondary market. In the data, net interest margins are about 300 basis points higher, while the maturity of loans is 3.5 years shorter in emerging countries than in advanced ones.[33] According to this data, trading frictions also seems to be higher in emerging economies, implying higher interest rates for longer-term loans and inducing firms to borrow and invest at shorter maturities.

A wider interpretation is that the life-cycle of cash-flows matters for liquidity spreads. For example, consider two stocks with different cash-flow structures. On the one-hand imagine a "growth" company that is not paying dividends in the short-run. On the other-hand, consider a "mature" firm that pays a smooth and roughly constant dividend stream to shareholders. The growth stock looks like a longer-term bond relative to the mature stock. By dividend payments the investor is getting out of the position without trading in the secondary market. There are two differences between the growth and mature stock when an investor receives a shock and becomes low valuation. First, due to previous dividend payments, the mature investor has less exposure to the security. Second, as future dividends arrive, the mature investor is getting out of the financial position without trading in the secondary market. Hence, the growth stock will have a higher liquidity spread than the mature stock. This example shows that the life-cycle of

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[33] While we cannot decompose the net interest margin into the which fraction correspond to agency frictions and which fractions to liquidity components, we conjecture that an important fraction of it can be attributed to liquidity considerations. We left open the decomposition for future research.

cash-flows matters for liquidity spreads and similar results would arise in a model with equity finance as long as there are trading frictions in the secondary market.

C Additional Empirical Results

C.1 Non-linear effects on credit spreads

In this appendix we extend the main empirical specification to allow for non-linear effects on maturity. We split the maturity difference in five groups (less than 3 years, 3 to 6 years, 6 to 9 years, 9 to 12 years, and more than 12 years) and estimate fixed effects at the group level as:

$$s_{i,t,m_j} - s_{i,t,m_1} = \gamma_t + \sum_{g=1}^{5} \beta_g \mathcal{I}\left((m_j - m_1) \in G_g\right) + \epsilon_{i,t,m_1,m_j}.$$

Table 7 shows the maturities for each group and the estimated coefficient. The last column shows that the marginal effect of an additional year of maturity is about 13 basis points for the first group but it reduces to about 4 basis points for the last group. This results are consistent with the concavity results of the theory shown in Proposition 1.

Table 7: Non-linear effects on credit spreads

	Change in maturity			**Effect on spreads**		
Group	Minimum	Maximum	Mean	Coefficient	Standard Error	Marginal effect
1	0	3	1.8	22.8	1.29	13
2	3	6	4.6	26.2	1.47	6
3	6	9	7.7	48.5	1.65	6
4	9	12	10.5	29.4	2.26	3
5	12	30	18.6	65.3	2.32	4

C.2 Argentina

As an example, we repeat the estimates of the slope of liquidity spreads in Argentina to discipline the counterfactuals. We find that credit spreads are steeper in Argentina than in the US, with a slope of 50 basis points per year in Argentina, relative to about 5 in the US (first column of Table 8).[34] To control for default, we look at sovereign CDS and estimate how they change with maturity. We consider sovereign instead of corporate CDS because we only have data on

---

[34] Of course, the Argentinean market is much smaller than the US market, so when we restrict to firms issuing two bonds on the same day we end with a much smaller sample than in the US. Nevertheless, for the year 2017 we have 70 issuance of 15 firms generating 35 observations for the difference on credit spreads and maturity.

sovereign CDS for Argentina. Another advantage is that sovereign CDS are more liquid than corporate CDS, so the bias due to liquidity should be smaller. The second column of Table 8 shows that the estimated coefficient for sovereign CDS is 10 bps, about one-fifth of the total slope for corporate spreads. Interestingly, if we estimate the slope of sovereign CDS for the US we also find that they are about one-fifth of the slope of corporate spreads. We conclude that credit spreads are steeper in Argentina than in the US and that a large fraction of this slope can be attributed to liquidity considerations. For the quantitative evaluation it is useful to summarize the empirical results with the effect at the median. In Argentina the average maturity is 2.5 years, and when maturity increases from 1.5 to 3 years, credit spreads increase by 75 basis points, while CDS spreads increase by 14 basis points. We attribute the difference between corporate and sovereign CDS slope, 61 basis points, to the liquidity component.

Table 8: **Slope of credit spreads in Argentina**

	Corporate	Sovereign CDS
Maturity difference	50.04***	9.529***
	(7.377)	(0.104)
Observations	35	99
Number of firms	15	
R-squared	0.930	0.577
FE	Time	Time

*Note: Standard errors in parentheses; *, **, and *** denote statistical significance at the 10, 5, and 1 percent level, respectively.*

D Data sources

D.1 Credit spreads

We consider corporate debt issuances in the US on the Mergent Fixed Income Securities Database (FISD). We keep corporate bonds of domestic borrowers in local currency (i.e., US dollars) and with a fixed interest rate. We follow Gilchrist and Zakrajšek (2012) to define credit spreads that are not subject to the "duration mismatch" by constructing a synthetic risk-free security that mimics exactly the cash flows of the corresponding corporate debt instrument. We use the US Treasury yield curve estimated by Gürkaynak et al. (2007). Empirical results are similar to an alternative definition of spreads. For example, we find similar results when we define credit spreads as the difference in coupon rates between corporate and sovereign bonds

Table 9: **Summary Statistics**

	Mean	Median	SD
Corporate Bonds			
# of Bond Issuances per Firm/Month	6.69	3.00	7.46
Maturity at Issue (years)	6.95	5.00	6.45
Coupon Rate (pct.)	3.28	3.70	2.70
Nominal Effective Yield (pct.)	3.34	3.74	4.10
Nominal Effective Treasury Yield (pct.)	2.73	2.50	1.60
Credit Spread (bps.)	60	59	369
Firms			
Profitability	0.10	0.10	0.04
Average Maturity	4.82	4.38	1.95
Long Share	0.78	0.80	0.13

Note: Issuances: 994 issuers; 35,513 bonds of which 23,182 bonds are rated. Firms: there are 20,163 firms and 150,477 firm-year observations.

of similar maturity. For CDS we use Markit for 2000-2017.

Table 9 describes the data in the final sample. Our sample considers the set of firms that in a given period issues two or more bonds of different maturities. In the benchmark specification we define the period as a day and perform robustness exercises for definitions at the week and month level. As the length of the period increases (from day to week to month), there are more issuances within each group allowing us to also include firm-period fixed-effects. The reason for this sample selection is important for identification and is discussed in the main text. To ensure that our results are not driven by a small number of extreme observations, we trimmed the data at the top and bottom 1 percent. Our sample period is January 2000 to December 2017. There are 994 issuers and 35,513 bond issuances; 23,182 bonds are rated and the median rating from Moody's is A2. On average, a firm that is issuing bonds in a given month makes 6.69 different issuances; however, there is a large variation across firms. The average maturity is 6.95 years with an average credit spread of 60 basis points. Again, note the large dispersion in maturity and credit spreads across issuances.

D.2 High-quality corporate bonds

The corporate yield curve corresponds to the high-quality market (bonds rated above A), and it is available at `https://www.treasury.gov/resource-center/economic-policy/corp-bond-yield/`. Define the corporate yield curve as the monthly average for the year 2017. Tables 10 and 11 show the default rates, default credit losses, and the transition probabilities of credit ratings for high-quality issuers.

Table 10: **Default Credit Losses**

Rating	**Default credit losses** **Average** **1982-2014**	 **Maximum** **2008**	**Default rates** **Average** **1920-2014**	 **Maximum** **2008**
Aaa	0.00%	0.00%	0.000%	0.00%
Aa	0.03%	0.48%	0.061%	0.724%
A	0.03%	0.37%	0.096%	0.547%

Source: Moody's 2015.

Table 11: **Five-year Transitions (cumulative)**

	Aaa-A	**Baa-B**	**Caa-C**	**Default**
Aaa-A	88.70%	10.62%	0.15%	0.52%

D.3 Maturity and Profitability

We use Compustat data for 1976 to 2014. We follow the same cleaning as in Crouzet (2017). Firm-year observations are kept in the sample if (1) their 2-digit sic code is not between 60 and 69 (financials) or equal to 49 (utilities); (2) debt in current liabilities (dlc) and debt in long-term liabilities (dltt) are not missing and weakly positive; (3) book assets (at) is not missing and weakly greater than 1m$; (4) book leverage, the ratio of (dlc+dltt) to at, is between 0 and 1; (5) the variables ddi, for i = 2, ..., 5 (which capture the portion of long-term debt due in 2,...,5 years) are all non-missing and weakly positive; (6) their sum is weakly smaller than 1.01× dltt; and (7) operating income before depreciation (oibdp) is non-missing. There are no direct measures of average time to maturity of outstanding debt, but a proxy can be obtained from the data as $\frac{1}{dlc+dltt}\left(dlc + \sum_{i=1}^{5} dd_i \times i + dvlt \times x\right)$, where dvlt represents long-term debt due in more than five years and is defined as $dvlt = dltt - \sum_{i=1}^{5} dd_i$, and we set $x = 15$ as Crouzet (2017). Finally, we define the year-industry variables as the mean across firm-year observations of each sector but results are robust to consider the median instead. Table 9 describes the data. There are 20,163 firms and 150,477 firm-year observations with an average profitability and maturity of 10% and 4.82 years, respectively.

D.4 Credit spreads in Argentina

Consider all the active corporate bonds in August 2017 in the domestic market (MAE) and keep issuances in local currency, with 100% amortization, and interest rates as a spread on the Badlar rate (which is the reference short-term rate in Argentina). These are floating interest rate bonds with a fixed spread, so the credit spread is just the spread on the Badlar rate because non-arbitrage implies that agents can swap the variable Badlar rate for a fixed rate.