

## AMRITA AHEAD

### M Sc Mathematics (2021 admissions onwards) Syllabus

#### SEMESTER I

21MAT501A

Advanced Algebra

3 0 0 3

**Prerequisite: Abstract Algebra** (Groups, Rings, Vector Spaces and Field Extensions).

#### Unit 1

Conjugate Elements, Normalizer of an Element, Index of Normalizer, Center of a Group, Cauchy's Theorem on Prime Order, the Number of Conjugate Classes  $p(n)$  for a Permutation Group, Counting Principles, Cauchy Theorem,  $p$ - Sylow subgroups, Sylow's Theorems. (Sec. 2.11 and 2.12).

#### Unit 2

External and Internal Direct Products, Cyclic Groups, Abelian Groups, Invariants of a Group, Fundamental Theorem on Finite Abelian Groups (Sec. 2.13 and 2.14).

#### Unit 3

Polynomial Rings over the Rational Field, Primitive Polynomials, The Content of a Polynomial, Integer Monic Polynomial, Eisenstein Criterion, Polynomial Rings over Commutative Rings. (Sec. 3.10 to 3.11).

#### Unit 4

Euclidean Domains, Principal Ideal Domains, Unique Factorization Domains. (Sec. 8.1 to 8.3, 9.6 from Reference Book 1).

#### Unit 5

The Elements of Galois Theory, Group of Automorphisms and its fixed field, Galois Group, The Fundamental Theorem of Galois Theory. Solvable Groups, Solvability by Radicals. (Sec. 5.6 to 5.8).

#### TEXTBOOKS:

1. I. N. Herstein, 'Topics in Algebra', Second Edition, John Wiley and Sons, 2000.

#### REFERENCES

1. D.S. Dummit and R. M. Foote, 'Abstract Algebra', 2nd Ed., John Wiley, 2002.
2. M. Artin, 'Algebra', Prentice Hall inc 1994.
3. Joseph Rotman, 'Galois Theory', 2<sup>nd</sup> Ed., Springer, 2001.

**Note:** The Problems are to be referred from Reference Book 1.

21MAT502A

Advanced Real Analysis

3-0-0-3

**Prerequisite: Basic Real Analysis** (Limit, Continuity, Differentiability and Riemann Integration).

### **Unit 1**

Riemann-Stieltjes Integral: Definition and Existence of the Integral, Properties of the Integral, Integration and Differentiation, Integration of vector-valued functions.

(Chapter 6: 6.1 to 6.4)

### **Unit 2**

Sequences and Series of Functions: Sequence of functions and its point-wise limit, Discussion of main problems, Uniform convergence, Uniform convergence and continuity, Uniform convergence and Integration, Uniform convergence and Differentiation, Equicontinuous Families of Functions, The Stone-Weierstrass Theorem.

(Chapter 7: 7.1 to 7.7)

### **Unit 3**

Some Special Functions: Introduction to power series, The Exponential and Logarithmic Functions, The Trigonometric Functions.

(Chapter 8: 8.1 to 8.3)

### **Unit 4**

Some Special Functions and Functions of Several Variables: Fourier series, Gamma function and its properties.

(Chapter 8 & 9: 8.5 to 8.6)

### **Unit 5**

Functions of Several Variables: The Contraction principle, The inverse function theorem, The implicit function theorem.

(Chapter 9: 9.3 to 9.5)

### **TEXTBOOK:**

1. Rudin. W, "*Principles of Mathematical Analysis*", McGraw-Hill International Editions, Third Edition, 1976.

### **REFERENCE BOOKS:**

1. H.L. Royden and P.M. Fitzpatrick, "*Real Analysis*", Pearson Education Asia Limited, Fourth Edition, 2010.

2. Tom M. Apostol, "*Mathematical Analysis*", Narosa publishing house, New Delhi, Second Edition, 1989.

**21MAT503A**

**ORDINARY DIFFERENTIAL EQUATIONS**

**3 0 0 3**

**Prerequisite: Basics of ordinary differential equations.**

### **Unit 1**

Linear differential equations: Introduction, initial value problems, the wronskian and linear independence, reduction of order of a homogeneous equation, non-homogeneous equation.

**TB2 (3.1-3.6)(4 hours)**

**Existence - Uniqueness of Solutions to First Order Equations:** Equations with variable separated, Exact equations, the method of successive approximations, Lipschitz condition, Convergence of successive approximations.

**TB2 (5.2- 5.7)(8hours)**

### **Unit 2**

Systems of first order equations, Existence and uniqueness theorem, fundamental matrix, nonhomogenous linear systems, linear systems with constant coefficients.

**TB3 (4.2-4.7)(7 hours)**

### **Unit 3**

Complex n-dimensional space, Systems as vector equations, Existence and uniqueness of solutions to systems, Existence and Uniqueness of linear systems, Equations of order n.

**TB2 (6.4- 6.8) (5 hours)**

#### **Unit 4**

**Nonlinear equations:** Autonomous Systems, The Phase plane and its phenomena, Types of critical points. Stability, critical points and stability for linear systems.

**TB1 (11.58- 11.62) (6 hours)**

#### **Unit 5**

Stability by Liapunov's Direct method, stability by eigen values, Simple critical points of nonlinear systems.

**TB1 (11.58- 11.62) (4 hours)**

#### **TEXTBOOKS:**

1. George F. Simmons and John S Robertson, Differential equations with applications and historical notes, Tata McGraw Hill Education Private Limited, Second Edition, 2003.
2. E.A. Coddington, An introduction to ordinary differential equations, PHI learning, 1999.
3. S. G. Deo, V. Lakshmikantham and V Raghavendra, Text book of Ordinary differential equations, McGraw Hill Education Private Limited, second edition, 2013.

#### **REFERENCE:**

1. William E. Boyce and Richard C. DiPrima, Elementary differential equations and boundary value problems Wiley india, 9<sup>th</sup> edition, 2012.

**21MAT504A**

**Stochastic Processes**

**3 0 0 3**

**Prerequisite: Basic Probability Theory.**

**Unit – I** Introduction to Probability and Stochastic Processes:

Definition of Stochastic Processes, specification of Stochastic processes, Stationary processes– Markov Chains: definition and examples, higher transition probabilities, Generalization of Independent Bernoulli trials, classification of states and chains.

(Sections: 2.1, 2.2, 2.3, 3.1, 3.2, 3.3, 3.4)

**Unit – II** Markov Processes with Discrete State Space:

Poisson process, Poisson process related distributions, properties of Poisson process, Generalizations of Poisson Processes, Birth and death processes, continuous time Markov Chains.

(Sections: 4.1, 4.2, 4.3, 4.4, 4.5)

**Unit – III** Markov processes with continuous state space:

Brownian motion – Wiener Process - Differential equations for a Wiener process – Kolmogorov equations – first passage time distribution for Wiener process.

(Sections: 5.1 to 5.5)

**Unit – IV** Renewal processes and theory:

Renewal process – Renewal processes in continuous time – Renewal equation – stopping time – Wald's equation – Renewal theorems.

(Sections: 6.1 to 6.5)

### Unit – V Branching Processes:

Introduction, properties of generating functions of Branching process, Distribution of the total number of progeny, Continuous-Time Markov Branching Process.

(Sections: 9.1, 9.2, 9.4)

#### Text Book:

1. J. Medhi, “Stochastic Processes”, 2<sup>nd</sup> Edition, New Age International Private limited, 2006.

#### Book for Reference:

1. Sheldon M. Ross, “Stochastic Processes”, 2<sup>nd</sup> Edition, Wiley, 1995.

2. J. Ravichandran, “Probability and Random Processes for Engineers”, 1<sup>st</sup> Edition, IK International, 2015.

21MAT581A

Mathematics Lab

0 0 2 1

- Introduction to a Mathematical software
- Explorations of various applications
- Implementation of Mathematical techniques.

### SEMESTER II

21MAT511A

ADVANCED COMPLEX ANALYSIS

3 0 0 3

**Prerequisite:** Basic Complex Analysis (Limit, Continuity, Analytic Functions, C-R Equations, Harmonic functions, Contour Integrals and Singularities).

#### Unit 1:

**Schwarz Reflection:** Schwarz Reflection by complex conjugation, Reflection along analytic Arcs, Application of Schwarz Reflection (Chapter 9)

#### Unit 2

**The Riemann Mapping Theorem:** Compact sets in Function Spaces, Statement and Proof of the Riemann Mapping Theorem, Behaviour at the Boundary (Chapter 10).

#### Unit 3

**Analytic Continuation:** Analytic Continuation along a curve, Monodromy Theorem, the Dilgrithm, Bloch-Wigner Function, Picard’s Theorem and its Application (Chapter 11)

#### Unit 4

**Entire and Meromorphic Functions:** Infinite Products, Absolute Convergence, Weierstrass Products, Functions of Finite Order, Canonical product, Minimum Modulus Theorem, Hadamard’s Theorem, Mittag-Leffler Theorem (Chapter 13) .

#### Unit 5

**Elliptic Functions:** Liouville Theorem, Fundamental Parallelogram, Elliptic Function, Weierstrass Function, Addition Theorem, Sigma and Zeta Functions (Chapter 14)

**TEXTBOOK**

Serge Lang, 'Complex Analysis' Springer, 4<sup>th</sup> Edition, First Indian Reprint 2005.

**REFERENCES**

1. S. Ponnusamy and H. Silverman, Complex Variables with Applications, Springer, 2006.
2. R. Roopkumar, Complex Analysis, Pearson Education, 2014, Chennai
3. Lars V. Ahlfors, Complex Analysis, 2<sup>nd</sup> Edition, McGrawHill, New York, 1966

**21MAT515A****Topology****3 0 0 3****Unit 1**

Infinite sets, Countable and Uncountable sets, the Axiom of Choice - continuum hypothesis, Well-ordered sets, The maximum principle.

Chapter 1: Sec 7 to 11(Text Book 2)

**Unit 2**

Metric spaces – Definition and examples - open balls and closed ball – Open Sets, Closed Sets and Convergence Sequences – Continuous Mappings between Metric Spaces – Examples – Complete Metric Spaces.

Chapter 9: Sec 9.1 to 9.4 (Text Book 1)

**Unit 3**

Compact spaces and their properties – Continuous functions on Compact spaces- Characterization of Compact Metric spaces -Separable Metric Spaces.

Chapter 9: Sec 9.5 and 9.6 (Text Book 1)

**Unit 4**

Three Fundamental Theorems: The Arzela-Ascoli Theorem – The Baire Category Theorem – The Banach Contraction Principle.

Chapter 10: Sec 10.1 to 10.3 (Text Book 1)

**Unit 5**

Topological spaces - Basis for a topology - The order topology - The product topology on  $X \times Y$  - The subspace topology-Closed sets and limit points.

Chapter 2: Sec 12 to 17(Text Book 2)

**TEXTBOOK:**

1. H.L. Royden and P.M.Fitzpatrick - "Real Analysis"-Pearson Education Asia Limited -2010 - Fourth Edition.
2. J.R. Munkers- "Topology" -Prentice Hall of India -2002- Second Edition.

**REFERENCE BOOKS :**

*I.J. Dugundji -" Topology" Allyn and Bacon, Boston-1966.*

**21MAT514A**

**PARTIAL DIFFERENTIAL EQUATIONS**

**3 0 0 3**

**Prerequisite:** The students must know the basic concepts on Calculus (both differential and integral), Differential Equations (ODE and PDE at UG Level), either metric space or topology to understand the words open set, closed set, compact, connected, region, continuous function, Vector Calculus in which the notion of curves, surfaces, tangent plane, normal, surface integral and volume integral and their evaluation, Fourier series and Fourier transforms.

**Unit 1**

Geometrical interpretation of a first-order pde, method of characteristics and general solutions, Monge cone, Lagrange's equations, canonical forms of first-order linear equations, method of separation of variables.

**Tb1:(2.4-2.8)**

**Unit 2**

Second-order equations in two independent variables, canonical forms, equations with constant coefficients, general solutions.

**Tb1: (4.1-4.6)**

**Unit 3**

The Cauchy problem, the Cauchy-Kowalewskaya theorem, homogeneous wave equations, the D'Alembert solution of wave equation, initial boundary-value problems.

**Tb1:(5.1-5.5)**

**Unit 4**

Basic concepts, types of boundary-value problems, maximum and minimum principles, uniqueness and continuity theorems. Dirichlet problem for a circle, Dirichlet problem for a circular annulus, Neumann problem for a circle.

**Tb1:(9.1-9.7)**

**Unit 5**

Derivation of the heat equation and solutions of the standard initial and boundary value problems, uniqueness and the maximum principle.

**TB2: (3.1-3.3) (10 hours)**

**TEXTBOOKS:**

1. Tyn Myint-U, Lokenath Debnath, Linear Partial Differential Equations for Scientists and Engineers, Birkhauser, Boston, Fourth Edition, 2007.
2. D. Bleeker, G. Csordas, Basic Partial Differential Equations, Van Nostrand Reinhold, New York, 1992.

**REFERENCES:**

1. L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol. 19, AMS, Providence, 1998.
2. I.N. Sneddon, Elements of partial differential equations, McGraw Hill, New York, 1986.
3. E. Zauderer, Partial Differential Equations of Applied Mathematics, John Wileys & Sons, New York, 2<sup>nd</sup> edition, 1989.

4. E. C. Zachmanoglou and D. W. Thoe, Introduction to Partial Differential Equations with Applications, Dover Publication, New York, 1986.

**21MAT512A**

**MEASURE THEORY**

**3 0 0 3**

**Prerequisite: Advanced Real Analysis**

**Unit 1** (Sections: 2.1 to 2.5 of [1])

Measure on the Real Line: Lebesgue Outer Measure - Measurable Sets – Regularity - Measurable Functions - Borel and Lebesgue Measurability

**Unit 2** (Sections: 3.1 to 3.4 of [1])

Integration of Functions of a Real Variable: Integration of Non-Negative Functions - The General Integral - Integration of Series - Riemann and Lebesgue Integrals.

**Unit 3** (Section: 5.6 of [1])

Measure Spaces - Integration with Respect to a Measure.

**Unit 4** (Sections: 6.1 to 6.5 of [1])

Inequalities and the  $L^p$  Spaces: The  $L^p$  Spaces - Convex Functions - Jensen's Inequality - The Inequalities of Holder and Minkowski – Completeness of  $L^p(\mu)$ .

**Unit 5** (Sections: 8.1 to 8.3 of [1])

Signed Measures and their Derivatives: Signed Measures and the Decomposition - The Jordan Decomposition - The Radon-Nikodym Theorem.

**TEXTBOOK:**

1. Measure Theory and Integration by G.de Barra. First Edition. New Age International Publishers, Reprint 2000.

**Reference Book:**

1. Real Analysis by H.L. Royden and P.M. Fitzpatrick. Fourth Edition. Pearson Education Asia Limited, 2010.
2. Elias M. Stein & Rami Shakarchi, Real Analysis Measure Theory, Integration, and Hilbert Spaces (Princeton Lectures in Analysis), Princeton university press, 2007.

**21MAT513A**

**NUMERICAL ANALYSIS**

**3 0 0 3**

**Prerequisite: Calculus and Basic Real Analysis**

Unit I:

Review of errors and error propagation theorem;

(Roots of Transcendental and Polynomial Equations, Solution of equations in one variable: Rate of convergence for fixed point iteration method and Newton-Raphson method etc.;

System of nonlinear equations: Newton's Method, Steepest-Descent Method; (B1-10.2 and 10.4)

Solution of System of Linear Algebraic Equations: Decomposition method (LU), Ill- conditioned system, Iteration methods: Gauss-Jacobi method, Gauss- Seidel method; (B2-2.2, B2-2.4, B2-2.5)

Eigenvalues and Eigenvectors: Gershgorin theorem, Inverse power method. (B1-7.2, B3-3.6)

## **12 Hours**

Unit II:

Interpolation, Extrapolation and Approximation: Interpolating polynomials using finite differences, Hermite interpolation, Cubic-Spline interpolation, Richardson's Extrapolation. (B1-3.3, B1-3.4, B1-3.5, B1-4.2)

Numerical Differentiation: Numerical differentiation (Methods based on Interpolation, Finite difference operators, undetermined co-efficient); (B3-5.2)

Numerical integration: Trapezoidal, Simpson's 1/3rd, 3/8<sup>th</sup> rule, Gaussian Quadrature, Multiple integrals. (B1-4.3)

## **10 Hours**

Unit III:

Solutions of Ordinary Differential Equations: System of higher order differential equations, Stability, Stiff Differential equations; (B1-5.9, B1-5.10, B1-5.11)

Boundary value Problems of ODE: Shooting Method (B1-11.1, B1-11.2).

## **8 Hours**

Unit IV:

Solutions of Differential equations: Introduction to Finite element method: Mathematical Background, Finite Elements for ordinary differential equations, Finite Elements for ordinary differential equations, (B2-9.1, 9.2).

## **10 Hours**

Unit V:

Finite Elements for partial differential equations: Heat equations (Parabolic and Elliptic PDE) and Wave equations (Hyperbolic PDE) (B2- 9.3).

## **10 Hours**

TEXTBOOKS:

1. R.L. Burden, J. D. Faires, Numerical Analysis, Richard Stratton, 2011, 9th edition.
2. C. F. Gerald, P.O. Wheatley, Applied Numerical Analysis, Pearson Publishers, 2013, 7th edition.
3. M.K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical methods for scientific and Engineering computation, New Age International Publishers, 2007, 5th edition.

Reference Books:

4. E. Kreyszig, Advanced Engineering Mathematics, Wiley Publishers, 2015, 10th edition.
5. R.R. Bhat, S. Chakraverty, Numerical Analysis in Engineering, Narosa Publishing House, 2011.



- Finite Element Methods using MAT LAB or Finite element tools.

### SEMESTER III

21MAT601A

Advanced Graph Theory

3 0 0 3

**Prerequisite:** Basic Graph Theory (Graphs, degree and various definitions of graphs).

#### Unit 1

**Review of Graphs:** Graphs and Sub graphs, isomorphism, matrices associated with graphs, degrees, walks, connected graphs, shortest path algorithm.

**Trees:** Trees, cut-edges and cut-vertices, spanning trees, minimum spanning trees.

#### Unit 2

**Connectivity:** Graph connectivity, k-connected graphs and blocks.

**Euler and Hamilton Graphs:** Euler graphs, Euler's theorem. Fleury's algorithm for Eulerian trails. Necessary / sufficient conditions for the existence of Hamilton cycles.

#### Unit 3

**Matching:** Matchings, maximal matchings. Coverings and minimal coverings. Berge's theorem, Hall's theorem.

Coverings, Independent Sets and Cliques; Basic Relations.

#### Unit 4

**Colorings:** Vertex colorings, greedy algorithm and its consequences, Brooks' theorem.

#### Unit 5

**Planar graphs:** Euler formula. Dual graphs. Kuratowski's Characterization.

#### TEXTBOOKS

*J.A. Bondy and U.S.R. Murty, Graph Theory and Applications, Springer, 2008.*

#### REFERENCES BOOKS

1. D.B. West, *Introduction to Graph Theory*, P.H.I. 2010.

2. Frank Harary, *Graph Theory*, New York Academy of Sciences, 1979.

3. Russel Merris, *Graph Theory*, John Wiley, 2011.

21MAT603A

FUNCTIONAL ANALYSIS

3 0 0 3

**Prerequisite:** Basic Analysis and Linear Algebra

**Unit 1**(Sections: 3.1 to 3.5 of [1])

Normed Linear Spaces: Linear Spaces – Normed Linear Spaces – The Metric on a Normed Linear Space – Linear Subspaces – Bounded Linear Transformations.

**Unit 2**(Sections: 3.7 to 3.9 and 4.1 to 4.2 of [1])

Linear Homeomorphisms – An Elementary Integral – Regulated Mappings – Integration and Differentiation - Review of Compact Metric Spaces – Basic Results on Compact Subsets of a Metric Space – Separability of Compact Metric Spaces – Conditions Equivalent to Compactness - Borel – Lebesgue Theorem.

**Unit 3**(Sections: 4.3 to 4.6 of[1])

Compactness and Continuity – Dini’s Theorem - Finite Dimensional Normed Linear Spaces – Completeness.

**Unit 4**(Sections: 5.1 to 5.4 of [1])

Bounded Linear Functionals – Some Dual Spaces – The Hahn-Banach Theorem – The Existence of Bounded Linear Functionals – Reflexivity of the Banach Space  $\mathbb{R}^p$  - Annihilators.

**Unit 5**(Sections: 5.5 to 5.7 of[1])

A Theorem on Convex Sets – The Riesz Representation Theorem.

**TEXTBOOKS:**

1. Elements of Functional Analysis by A.L. Brown and A. Page, Van Norstrand Reinhold Company, London,1970.

**References:**

1. Functional Analysis by Balmohan V Limaye, New Age International Publishers, Third Edition, Reprint 2014.
2. Introduction to Topology and Modern Analysis by G. F. Simmons, McGraw Hill Education, 2004.
- 3.Thamban Nair, Functional Analysis: A First Course, PHI, 2001.

**21MAT602A****Basics of Fluid Dynamics****3 0 0 3****Prerequisite: Basic Calculus and Differential Equations****Unit 1**

Kinematics of Fluids in motion – Lagrangian and Eulerian methods – Equation of continuity – Boundary conditions – Kinematic and physical – stream line, path line and streak line – velocity potential – vorticity - rotational and irrotational motion.

**Unit 2**

Equation of Motion of incompressible - General Properties – Equation of motion of inviscid fluid – Euler’s equation – impulsive force – physical meaning of velocity potential.

**Unit 3**

Lagrange’s hydrodynamical equations - Bernoulli’s equation and its applications - Motion in two-dimensions and sources and sinks – irrotational motion – complex potential - Milne-Thomson circle theorem – Blasius theorem.

**Unit 4**

General theory of irrotational motion – flow and circulation – Stoke’s theorem – Kelvin’s Circulation theorem – Permanence of irrotational motion - Kelvin’s minimum energy theorem.

**Unit 5**

Navier-Stokes Equations-Exact Solutions of Navier Stokes Equations – Small Reynold’s number flows – flow past a sphere – Stokes flow – Whitehead’s paradox - Flow past a circular cylinder – Stoke’s Paradox.

**TEXT BOOKS :**

- 1.MD Rasinghania, Fluid Dynamics with Complete Hydrodynamics and Boundary Layer Theory, Revised Edition, (2013), S. Chand & Company Pvt. Ltd., New Delhi.

**REFERENCES:**

1. G.K.Batchelor, “An Introduction to Fluid Dynamics”, Cambridge University Press, 1997.
2. L.M. Milne-Thompson, “Theoretical Hydrodynamics”, Dover Publications, 1968.
3. Victor L. Streeter and E.Benjamin Wylie, “Fluid Mechanics”, Mc Graw Hill, 1983.

**21MAT604A**

**Operator Theory**

**3 0 0 3**

**Prerequisite: Functional Analysis**

Unit 1: Compact operators on Hilbert Spaces.

Unit 2: Fredholm Theory and Index.

Unit 3:  $C^*$ - algebras - noncommutative states and representations.

Unit 4: Gelfand-Neumark representation theorem, Von-Neumann algebras.

Unit 5: Projections, double commutant theorem,  $L^\infty$  functional calculus, Toeplitz operators.

**TEXTBOOK:**

Elements of Functional Analysis by A.L. Brown and A. Page, Van Norstrand Reinhold Company, London, 1970.

Reference Books:

1. W. Arveson, "An invitation to  $C^*$ -algebras", Graduate Texts in Mathematics, No. 39. Springer-Verlag, 1976.
2. N. Dunford and J. T. Schwartz, "Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space", Interscience Publishers John Wiley i& Sons 1963.
3. R. V. Kadison and J. R. Ringrose, "Fundamentals of the theory of operator algebras. Vol. I. Elementary theory", Pure and Applied Mathematics, 100, Academic Press, Inc., 1983.
4. V. S. Sunder, "An invitation to von Neumann algebras", Universitext, Springer-Verlag, 1987.

## **SEMESTER IV**

**One Elective and Dissertation**

### **Electives**

**21MAT531A**

**ADVANCED BOUNDARY LAYER THEORY**

**3 0 0 3**

#### **Unit 1**

Introduction – limitations of ideal fluid dynamics – Importance of Prandtl's boundary layer theory - boundary layer equations in two dimensional flows – boundary layer flow over a flat plate – Blasius solution – Boundary layer over a wedge.

#### **Unit 2**

Energy integral equation for two-dimensional laminar boundary layers in incompressible flow – application of Von Karman's integral equations to boundary layer with pressure gradient.

#### **Unit 3**

Displacement, momentum, energy thickness – axially symmetric flows – momentum equation for laminar boundary layer by von Karman – Wall shear and drag force on a flat plate due to boundary layer – coefficient of drag. Boundary layer equations for a 2D viscous incompressible fluid over a plane wall – Similar solutions – Separation of boundary layer flow.

#### **Unit 4**

Hydromagnetic Boundary layers – Hartman Layer – MHD Blasius flow. Thermal boundary layers – thermal boundary layer equation in two dimensional flow – Thermal boundary layers with and without coupling of velocity and temperature field – forced convection in a laminar boundary on a flat plate.

### **Unit 5**

Polhausen's method of exact solution for the velocity and thermal boundary layers in free convection from a heated plate – thermal energy integral equation. Boundary layer control using suction and injection.

#### **TEXT BOOKS / REFERENCES:**

1. H.Schlichting and K.Gersten, "Boundary Layer Theory", Eighth Edition, Springer, 2000.
2. L. Rosenhead, "Laminar Boundary Layers", Dover, 1988.
3. G.K.Batchelor, "An Introduction to Fluid Dynamics", Cambridge University Press, 1993.
4. P.H.Roberts, "An Introduction to MHD", Longmans, 1967.

**21MAT532A**

**ALGEBRAIC GEOMETRY**

**3 0 0 3**

### **Unit 1 AFFINE AND PROJECTIVE VARIETIES**

Noetherian rings and modules; Emmy Noether's theorem and Hilbert's Basissatz; Hilbert's Nullstellensatz; Affine and Projective algebraic sets; Krull's Hauptidealsatz; topological irreducibility, Noetherian decomposition; local ring, function field, transcendence degree and dimension theory; Quasi-Compactness and Hausdorffness; Prime and maximal spectra; Example: linear varieties, hypersurfaces, curves.

### **Unit 2 MORPHISMS**

Morphisms in the category of commutative algebras over a commutative ring; behaviour under localization; morphisms of local rings; tensor products; Product varieties; standard embeddings like the segre- and the d-uple embedding.

### **Unit 3 RATIONAL MAPS**

Relevance to function fields and birational classification; Example: Classification of curves; blowing-up.

### **Unit 4 NONSINGULAR VARIETIES**

Nonsingularity; Jacobian Criterion; singular locus; Regular local rings; Normal rings; normal varieties; Normalization; concept of desingularisation and its relevance to Classification Problems; Jacobian Conjecture; relationships between a ring and its completion; nonsingular curves.

### **Unit 5 INTERSECTIONS IN PROJECTIVE SPACE**

Notions of multiplicity and intersection with examples.

#### **TEXTBOOKS / REFERENCES BOOKS**

1. Robin Hartshorne, *Algebraic Geometry, Graduate Texts in Mathematics (GTM) 8th Printing, Springer, 1997.*
2. C. Musili, *Algebraic Geometry for Beginners, Texts and Readings in Mathematics 20, Hindustan Book Agency, 2001.*

**21MAT533A**

**ALGEBRAIC TOPOLOGY**

**3 0 0 3**

## **Unit 1**

Geometric Complexes and Polyhedra: Introduction. Examples. Geometric Complexes and Polyhedra; Orientation of geometric complexes.

Simplicial Homology Groups: Chains, cycles, Boundaries and homology groups, Examples of homology groups; The structure of homology groups.

## **Unit 2**

The Euler Poincare's Theorem; Pseudomanifolds and the homology groups of  $S_n$ . [Chapter 1 Sections 1.1 to 1.4 & Chapter 2 Sections 2.1 to 2.5 from the text].

## **Unit 3**

Simplicial Approximation: Introduction; Simplicial approximation; Induced homomorphisms on the Homology groups; The Brouwer fixed point theorem and related results;

## **Unit 4**

The Fundamental Group: Introduction; Homotopic Paths and the Fundamental Group; The Covering Homotopy Property for  $S^1$ ;

[Chapter 3 Sections 3.1 to 3.4; Chapter 4 Sections 4.1 to 4.3]

## **Unit 5**

Examples of Fundamental Groups; The Relation Between  $H_1(K)$  and  $\pi_1(K)$ ; Covering Spaces: The definition and some examples. Basic properties of covering spaces. Classification of covering spaces. Universal covering spaces. Applications.

[Chapter 4: Sections 4.4, 4.5; Chapter 5 Sections 5.1 to 5.5 from the text]

## **TEXT BOOK**

*Fred H. Croom: Basic Concepts of Algebraic Topology, UTM, Springer, NY, 1978.*

## **REFERENCES BOOKS:**

1. Eilenberg S and Steenrod N: *Foundations of Algebraic Topology, Princeton Univ. Press, 1952.*
2. S.T. Hu: *Homology Theory, Holden-Day, 1965.*
3. S.T. Hu: *Homology Theory, Academic Press, 1959.*

**21MAT534A**

**CODING THEORY**

**3 0 0 3**

**Unit 1** Introduction to linear codes and error correcting codes. Encoding and decoding of a linear code,

**Unit 2** Dual codes. Hamming codes and perfect codes.

**Unit 3** Cyclic codes. Codes with Latin Squares, Introduction to BCH codes.

**Unit 4** Weight enumerators and MDS codes.

**Unit 5** Linear coding theory problems and conclusions.

**TEXT BOOKS:**

1. Raymond Hill, *A first course in Coding Theory*, Clarendon Press, Oxford (1986).
2. J.H. Van Lint, *Introduction to Coding Theory*, Springer (1998).

**REFERENCES**

1. W. Cary Huffman and Versa Pless, *Fundamentals of Error Correcting Codes*, Cambridge University Press (2003).
2. W.W. Peterson, *Error Correcting Codes*, Cambridge, MA MIT Press (1961).
3. V. Pless, W.C. Huffman and R.A. Brualdi, *An Introduction to Algebraic Codes*, in *Hand book of coding theory*, Eds. Amsterdam Elsevier (1998).

**21MAT535A****COMMUTATIVE ALGEBRA****3 0 0 3**

**Unit 1** Rings and ideals, modules and operations on them (tensor product, Hom, direct sum and product).

**Unit 2** Rings and modules of Fractions, primary decomposition.

**Unit 3** Integral dependence and Valuations, Chain Conditions.

**Unit 4** Noetherian Rings and Artin Rings.

**Unit 5** Discrete valuation Rings and Dedekind Domains, Dimension theory.

**TEXT BOOKS / REFERENCES**

1. Atiyah-Macdonald, *Commutative Algebra*, Westview Press, 1994.
2. Zariski and Samuel, *Commutative Algebra I, II*, Springer, 1991.
3. Eisenbud, *Commutative Algebra with a View Towards Algebraic Geometry*, Springer, 1995.
4. Bourbaki, *Commutative Algebra*, Springer, 1989.

**21MAT536A****COMPUTATIONAL FLUID DYNAMICS****3 0 0 3**

**Unit 1** Review of Conservation equations for mass, momentum and energy; coordinate systems; Eulerian and Lagrangian approach, Conservative and non-conservative forms of the equations, rotating co-ordinates.

**Unit 2** Classification of system of PDEs: parabolic elliptic and hyperbolic; Boundary and initial conditions; Overview of numerical methods; Review of Finite Difference Method, Introduction to integral method, method of weighted residuals, finite elements finite volume method & least square method.

**Unit 3** Numerical Grid Generation: Basic ideas, transformation and mapping, unstructured grid generation, moving grids, unmatched meshes. Finite Volume Method: Basic methodology, finite volume discretization, approximation of surface and volume integrals, interpolation methods - central, upwind and hybrid formulations and comparison for convection-diffusion problem; Basic computational methods for compressible flows.

**Unit 4** Advanced Finite Volume methods: FV discretization in two and three dimensions, SIMPLE algorithm and flow field calculations, variants of SIMPLE, Turbulence and turbulence modelling, illustrative flow computations.

**Unit 5** Introduction to turbulence modelling, CFD methods for compressible flows.

**TEXT BOOKS / REFERENCE BOOKS:**

1. Anderson D A, Tannehill J C, and Pletcher R H, *Computational Fluid Mechanics and Heat Transfer*, 2nd edition, Taylor & Francis, 1997.
2. Ferziger, J. H. and Peric, M., *Computational Methods for Fluid Dynamics*, 3rd edition, Springer. 2003.

**21MAT537A**

**FINITE ELEMENT METHOD**

**3 0 0 3**

**Unit 1** Finite Element Method: Variational formulation - Rayleigh-Ritz minimization - weighted residuals - Galerkin method applied to boundary value problems.

**Unit 2** Global and local finite element models in one dimension - derivation of finite element equation.

**Unit 3** Finite element interpolation - polynomial elements in one dimension, two dimensional elements, natural coordinates, triangular elements, rectangular elements, Lagrangian and Hermite elements for rectangular elements - global interpolation functions.

**Unit 4** Local and global forms of finite element equations - boundary conditions - methods of solution for a steady state problem - Newton-Raphson continuation.

**Unit 5** One dimensional heat and wave equations.

**TEXT AND REFERENCE BOOKS**

1. J.N .Reddy, *An Introduction to the Finite Element Method*, McGraw Hill, NY.
2. Chung, *Finite Element Analysis in Fluid Dynamics*, McGraw Hill Inc.

**21MAT538A**

**FIXED POINT THEORY**

**3 0 0 3**

**Unit 1** Contraction Principle, and its variants and applications;

**Unit 2** Fixed points of non-expansive maps and set valued maps, Brouwer-Schauder fixed point theorems,

**Unit 3** Ky Fan Best Approximation Theorem, Principle and Applications of KKM - maps, their variants and applications.

**Unit 4** Fixed Point Theorems in partially ordered spaces and other abstract spaces.

**Unit 5** Application of fixed point theory to Game theory and Mathematical Economics.

**TEXTBOOKS / REFERENCES BOOKS**

1. M.A. Khamsi and W.A. Kirk, *An Introduction to Metric Spaces and Fixed Point Theory*, Wiley - Inter Sci. (2001).
2. Sankatha Singh, Bruce Watson and Pramila Srivastava, *Fixed Point Theory and Best Approximation: The KKM - map Principle*, Kluwer Academic Publishers, 1997.

3. Kim C. Border, *Fixed Point Theorems with Applications to Economics and Game Theory*, Cambridge University Press, 1985.

**21MAT539A**

**FRACTALS**

**3 0 0 3**

**Unit 1** Classical Fractals, Self-similarity - Metric Spaces, Equivalent Spaces.

**Unit 2** The Space of Fractals, Transformation on Metric Spaces.

**Unit 3** Contraction Mapping and Construction of fractals from IFS.

**Unit 4** Fractal Dimension, Hausdorff measure and dimension, Fractal Interpolation Functions.

**Unit 5** Hidden Variable FIF, Fractal Splines, Fractal Surfaces, Measures on Fractals.

**TEXT BOOKS**

1. M.F. Barnsley, *Fractals Everywhere*, Academic Press, 1993.

2. P.R. Massopust, *Interpolation and Approximation with Splines and Fractals*, Oxford University Press, 2009.

3. K. Falconer, *Fractal Geometry (Mathematical Foundations and Applications)*, John Wiley & Sons, 2003.

**REFERENCES**

1. P.R. Massopust, *Fractal Functions, Fractal Surfaces and Wavelets*, Academic Press, 1994.

2. Heinz-Otto Peitgen and Peter Richter, *The Beauty of Fractals*, Springer, 1986.

3. Richard M. Crownover, *Introduction to Chaos and Fractals*, Jones and Bartlett Publishers, 1995.

4. Gerald A. Edgar, *Measure, Topology and Fractal Geometry*, Springer, 1990.

5. M.F. Barnsley, *Superfractals*, Academic Press, 2006.

6. B.B. Mandelbrot, *The Fractal Geometry of Nature*, Freeman, 1981.

**21MAT540A**

**HARMONIC ANALYSIS**

**3 0 0 3**

**Unit 1** Fourier series and integrals – Definitions and easy results – The Fourier transform – Convolution – Approximate identities – Fejer’s theorem – Unicity theorem – Parseval relation – Fourier Stieltjes Coefficients – The classical kernels.

**Unit 2** Summability – Metric theorems – Pointwise summability – Positive definite sequences – Herglotz’s theorem – The inequality of Hausdorff and Young.

**Unit 3** The Fourier integral – Kernels on  $\mathbb{R}$ . The Plancherel theorem – Another convergence theorem – Poisson summation formula – Bachner’s theorem – Continuity theorem.

**Unit 4** Characters of discrete groups and compact groups – Bochners’ theorem – Minkowski’s theorem.

**Unit 5** Hardy spaces - Invariant subspaces – Factoring  $F$  and  $M$ . Rieza theorem – Theorems of Szego and Beuoling.



**TEXT BOOK:**

*Content and Treatment as in Henry Helson, Harmonic Analysis, Hindustan Book Agency, Chapters 1.1 to 1.9, 2.1 to 3.5 and 4.1 to 4.3*

**21MAT541A****LIE ALGEBRA****3 0 0 3**

**Unit 1** Basic Concepts - Definition and Examples, Lie Algebra of Derivations, Adjoint Representation, Structure Constants, Direct Sums, Homomorphism and Isomorphisms, Ideals, Centre and Derived Algebra of a Lie Algebra, Simple Lie Algebras, The Normalizer of a Subalgebra and Centralizer of a Subset in Lie Algebras, Automorphism and Inner Automorphism of a Lie Algebra. (Book 1, Chapters 1 and 2).

**Unit 2** Descending Central Series of a Lie Algebra, Nilpotent Lie Algebras. Derived Series of a Lie Algebra, Radical of a Lie Algebra, Solvable Lie Algebras, Engel's Theorem. (Book 1, Chapter 3).

**Unit 3** Semisimple Lie Algebras - Theorems of Lie and Cartan, Jordan-Chevalley Decomposition, Cartan's Criterion. (Book 1, Chapter 4)

**Unit 4** Killing Form, Inner Derivations, Abstract Jordan Decomposition, Complete Reducibility of Lie algebras. (Book 1, Chapter 5)

**Unit 5** The Weyl Group, Root Systems. (Book 1, Chapter 10)

**TEXT BOOKS / REFERENCES BOOKS**

1. *Jacobson, Lie Algebras, Dover, 1979.*

2. *J.P. Serre, Lie Algebras and Lie Groups, Benjamin, 1965 (Translated from French).*

3. *J.E. Humphreys, Introduction to Lie Algebras and Representation Theory, Springer-Verlag, 1980.*

**21MAT542A****Linear Algebra and its Applications****3 0 0 3**

**Unit 1** Review: Vector Spaces.

Inner Products, Angle and Orthogonality in Inner Product Spaces, Length of a Vector, Schwarz Inequality, Orthogonal Vectors, Orthogonal Complement, Orthogonal Bases: Gram-Schmidt Process. **(Sec. 4.4)**

**Unit 2** The Algebra of Linear Transformations, Characteristic Roots, Invertible Linear transformations, Characteristic Roots, Characteristic Vector, Minimal Polynomial, Matrices, Matrix of a Linear Transformation. **(Sec. 6.1 to 6.3).**

**Unit 3** Canonical Forms: Triangular, Nilpotent Transformations, Jordan and Rational Canonical Form, invariant subspaces, cyclic subspaces. **(Sec. 6.4 to 6.6).**

**Unit 4** Trace and Transpose, Determinants, Symmetric and Skew Symmetric Matrices, Adjoint and Hermitian Adjoint of a Matrix, Hermitian, Unitary and Normal Transformations, Self Adjoint and Normal Transformations. **(Sec. 6.8 to 6.10)**

**Unit 5** Problems in Eigen Values and Eigen Vectors, Diagonalization, Orthogonal Diagonalization, Quadratic Forms, Diagonalizing Quadratic Forms, Conic Sections. (Sec. 7.1 to 7.3 and 9.5 to 9.6 from Reference Book 2)

**TEXT BOOK:**

1. I. N. Herstein, 'Topics in Algebra', Second Edition, John Wiley and Sons, 2000.

**REFERENCES:**

1. David C. Lay, *Linear Algebra and its Applications*, Pearson.
2. Gilbert Strang, 'Linear Algebra and its Applications, Fourth Edition, Cengage Learning, 2014.
3. Howard Anton and Chris Rorres, 'Elementary Linear Algebra', 9<sup>th</sup> Edition, Wiley, 2005.
4. Nabil Nassif, Jocelyne Erhel, Bernard Philippe, *Introduction to Computational Linear Algebra*, CRC press, 2015.

**21MAT543A**

**MAGNETO-HYDRO DYNAMICS**

**3 0 0 3**

**Unit 1**

Electromagnetic field equations – Maxwell's equations - Electromagnetic effects and the magnetic Reynolds number – induction equation. Alfven's Theorem – Ferraro's Law of irrotation – Electromagnetic stresses.

**Unit 2**

Magnetohydrostatics and steady states – Hydromagnetic equilibria and Force free magnetic fields —Chandrasekhar's theorem – General solution of force free magnetic field is constant – Some examples of force free fields.

**Unit 3**

Steady laminar motion – Hartmann flow.

**Unit 4**

Magnetohydrodynamic waves - Alfven waves – Stability of hydromagnetic systems - Normal mode analysis . Instability of linear pinch – Flute instability – A general criterion for stability.

**Unit 5** Bernstein's method of small oscillations – Jeans Criterion for Gravitational stability – Chandrasekhar's generalization for MHD and rotating fluids.

**TEXT BOOKS / REFERENCES:**

1. Ferraro, V.C.A and Plumpton, C., "An Introduction to Magneto-Fluid Mechanics", Clarendon Press, Oxford, 1966.
2. M.R. Crammer, and Shi-I Pai, "Magneto-Fluid Dynamics for Engineers and Applied Physicists", Scripta Publishing Company, Washington, 1973.
3. P.H. Roberts, "An Introduction to Magnetohydrodynamics", Longmans, Green and Co, London, 1967.
- 4.S. Chandrasekhar, "Hydrodynamic and Hydromagnetic Stability", Dover Publications, 1981.

**Objective:** *This course intends to introduce applications of various mathematical techniques to problems of Theoretical Physics. Examples could be chosen from all 4 traditional divisions of Modern Fundamental Theoretical Physics – Classical Mechanics, Electrodynamics, Quantum Mechanics and Statistical Physics.*

**Unit 1**

Vector calculus and applications in electromagnetic theory and fluid mechanics.

**Unit 2**

Introduction to tensor calculus: review of basics, index notation, tensors in physics and geometry, Levi-Civita tensor, transformations of vectors, tensors and vector fields, covariance of laws of physics.

**Unit 3**

Calculus of variations and extremal problems, Lagrange multipliers to treat constraints, Introduction to the Lagrangian and Hamiltonian formulations of classical mechanics with applications.

**Unit 4**

Gamma and Beta functions, Dirac delta function, Special functions, Review of Legendre, Bessel functions and spherical harmonics (with applications to Quantum mechanics), series solutions, generating functions, orthogonality and completeness,

**Unit 5**

Applied linear algebra: Dirac notation, dual vectors, projection operators, symmetric hermitian, orthogonal and unitary matrices in physics, diagonalization, orthogonality and completeness of eigenvectors, spectral decomposition and representation, simultaneous diagonalization, normal matrices, applications to coupled vibrations, Schrodinger equation in matrix form.

**TEXT BOOKS:**

1. Arften and Weber, *Mathematical Methods for Physics*, Elsevier, 6<sup>th</sup> Ed., 2005.
2. Riley, Hobson and Bence, *Mathematical Methods for Physics and Engineering*, Cup, 3<sup>rd</sup> Edition, 2010.

Review of first order equations and characteristics.

**Unit 1** Weak solutions to hyperbolic equations - discontinuous solutions, shock formation, a formal approach to weak solutions, asymptotic behaviour of shocks.

**Unit 2** Diffusion Processes - Similarity methods, Fisher's equation, Burgers' equation, asymptotic solutions to Burgers' equations.

**Unit 3** Reaction diffusion equations - traveling wave solutions, existence of solutions, maximum principles and comparison theorem, asymptotic behaviour.

**Unit 4** Elliptic equations - Basic results for elliptic operators, eigenvalue problems, stability and bifurcation.

**Unit 5** Hyperbolic system.

**TEXT BOOK**

*J David Logan, An Introduction to Nonlinear Partial Differential Equations, John Wiley and Sons, Inc., 1994*

**21MAT546A            QUEUING THEORY AND INVENTORY CONTROL THEORY 3 0 0 3**

**Unit 1** Inventory concept – Components of Inventory model.

**Unit 2** Deterministic Continuous Review model - Deterministic Periodic Review model.

**Unit 3** The classical EOQ – Non zero lead time – EOQ with shortages allowed.

**Unit 4** Deterministic Multiechelon Inventory models for supply chain management.

**Unit 5**

A stochastic continuous review model – A stochastic single period model for perishable products.

**TEXT BOOKS**

1. *F S Hillier and Gerald J Lieberman, Introduction to Operations research, 8th edition, McGraw Hill.*

2. *Ravindran, Phillips and Solberg, Operations research Principles and Practice, 2<sup>nd</sup> Edition, John Wiley & Sons.*

**21MAT547A            STATISTICAL PATTERN CLASSIFICATIONS            3 0 0 3**

**Unit 1 Introduction and Bayesian Decision Theory**

Introduction – Pattern recognition systems – the design cycle – learning and adaptation – Bayesian decision theory – continuous features – Minimum error rate classification – discriminant functions and decision surfaces – the normal density based discriminant functions.

**Unit 2 Maximum-likelihood and Bayesian Parameter Estimation**

Maximum likelihood estimation – Bayesian estimation - Bayesian parameter estimation – Gaussian case and general theory – problems of dimensionality – components analysis and discriminants – hidden Markov models.

**Unit 3 Nonparametric Techniques and Linear Discriminant Functions**

Nonparametric techniques – density estimation – Parzen windows – nearest neighborhood estimation – rules and metrics – linear discriminant functions and

decision surfaces – generalized linear discriminant functions – two-category linearly separable case – minimizing the perception criterion function.

#### **Unit 4 Nonmetric methods and Algorithm-independent Machine Learning**

Nonmetric methods – decision trees – CART methods – algorithm-independent machine learning – lack of inherent superiority of any classifier – bias and variance for regression and classification – resampling or estimating statistics – estimating and comparing classifiers.

#### **Unit 5 Unsupervised Learning and Clustering**

Unsupervised learning and clustering – mixture densities and identifiability – maximum likelihood estimates – application to normal mixtures – unsupervised Bayesian learning – data description and clustering – criterion functions for clustering – hierarchical clustering – component analysis – low-dimensional representations and multi-dimensional scaling.

#### **TEXT AND REFERENCE BOOKS:**

1. *Richard O. Duda, Peter E. Hart and David G. Stork, Pattern Classification, Second Edition, 2003, John Wiley & Sons.*
2. *Earl Gose, Richard Johnson baugh and Steve Jost, Pattern Recognition and Image Analysis, 2002, Prentice Hall of India.*

#### **21MAT548A STATISTICAL QUALITY CONTROL AND SIX SIGMA QUALITY ANALYSIS 3 0 0 3**

**Unit 1** Introduction to Quality Management – Japanese System of Total Quality Management.

**Unit 2** Quality Circles - 7 Quality Control tools - 7 New Quality Control tools.

**Unit 3** ISO 9000 Quality system Standards - Project Planning, Process and measurement system capability analysis - Area properties of Normal distribution.

**Unit 4** Metrics of Six sigma, The DMAIC cycle - Design for Six Sigma - Lean Sigma – Statistical tools for Six Sigma.

**Unit 5** Taguchi methods. Loss functions and orthogonal arrays and experiments.

#### **TEXT AND REFERENCE BOOKS**

1. *Ravichandran. J, Probability and Statistics for Engineers, 1<sup>st</sup> Edition 2012 (Reprint), Wiley India.*
2. *Montgomery Douglas C., Introduction to Statistical Quality Control, Sixth Edition. John Wiley & Sons, (2008).*
3. *Ishikawa K., Guide to Quality Control, 2<sup>nd</sup> Edition: Asian Productivity Organization, Tokyo (1983).*
4. *Taguchi G, Introduction to Quality Engineering: Designing Quality into Products and Processes Second Edition. (1991).*
5. *Harry, M and Schroeder R., Six Sigma: The Breakthrough Management Strategy. Currency Publishers, USA. (2000).*

**Unit 1**

Definition of Manifolds, Differentiable and Analytic Manifolds, Examples of Manifolds, Product of Manifolds, Mappings between Manifolds, Submanifolds, Tangent Vectors.

**Unit 2**

Differentials, The Differential of a Function, Infinitesimal Transformation, Tangent Space, Tangent Vector.

**Unit 3**

Cotangent Space, Vector Fields, Smooth Curve in a Manifold. Differential Forms– k-forms, Exterior Differential, its Existence and Uniqueness.

**Unit 4**

Exact Differential Forms. De Rham Cohomology Group, Betti Number, Poincare's Lemma, Inverse Function Theorem, Implicit Function Theorem and its Applications, Integral Curve of a Smooth Vector Field.

**Unit 5**

Orientable Manifolds– Definition and Examples. Smooth Partition of Unity– Definition and Existence. Riemannian Manifolds– Definition and Examples.

**TEXTBOOKS / REFERENCES:**

1. P.M.Cohn, "*Lie Groups*", Cambridge University Press, 1965.
2. Claude Chevalley, "*Theory of Lie Groups*", Fifteenth Reprint, Princeton University Press, 1999.

**21MAT551A THEORY OF SAMPLING AND DESIGNS OF EXPERIMENTS 3 0 0 3****Unit 1**

Stratified random sampling, estimation of the population mean, total and proportion, properties of estimators, various methods of allocation of a sample, comparison of the precisions of estimators under proportional allocation, optimum allocation and srs. Systematic sampling. Comparison of systematic sampling - srs and stratified random sampling for a population with a linear trend.

**Unit 2**

Unbiased ratio type estimators - Hartly-Ross estimator, regression method of estimation. Cluster sampling, single stage cluster sampling with equal and unequal cluster sizes, estimation of the population mean and its standard error. Two-stage cluster sampling with equal and unequal cluster sizes, estimation of the population mean and its standard error.

**Unit 3**

Unequal probability sampling, PPS sampling with and without replacement, cumulative total method, Lahiris method, Midzuno-Zen method, estimation of the population total and its estimated variance under PPS wr sampling, ordered and unordered estimators of the population total under PPS wor, Horwitz – Thomson estimator.

#### **Unit 4**

Elementary concepts (one and 2 way classified data) Review of elementary design (CRD, RBD, LSD) Missing plot technique in RBD and LSD with one and two missing values, Gauss-Markov theorem, BIBD: Elementary parametric relations, Analysis, PBIBD.

#### **Unit 5**

General factorial experiments, factorial effects, best estimates and testing the significance of factorial effects, study of  $2^3$  and  $2^4$  factorial experiments.

#### **TEXT AND REFERENCE BOOKS**

1. Cochran, W.C. *Sampling Techniques, Third Edition, Wiley Eastern, (1977).*
2. Des Raj, *Sampling Theory, Tata McGraw Hill, New Delhi, (1976).*
3. Murthy, M.N., *Sampling Theory, Tata McGraw Hill, New Delhi, (1967).*

#### **21MAT552A**

#### **TIME SERIES ANALYSIS**

**3 0 0 3**

**Unit 1** Time series, components of time series, additive and multiplicative models, determination of trend, analysis of seasonal fluctuations.

**Unit 2** Test for trend and seasonality, exponential and moving average smoothing, holt-winter smoothing, forecasting based on smoothing.

**Unit 3** Time series as a discrete parameter stochastic process, auto covariance and auto correlation functions and their properties, stationary processes, test for stationarity, unit root test, stationary processes in the frequency domain, spectral analysis of time series.

**Unit 4** Detailed study of the stationary processes: moving average (MA), autoregressive (AR), autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models.

**Unit 5** Estimation of ARMA models, maximum likelihood method (the likelihood function for a Gaussian AR(1) and a Gaussian MA(1)) and Least squares, Yule-Walker estimation for AR Processes, choice of AR and MA periods, forecasting, residual analysis and diagnostic checking.

#### **TEXT BOOKS**

1. Anderson, T.W. *The Statistical Analysis of Time Series, John Wiley, New York, 1971.*
2. Box, G.E.P. and Jenkins, G.M. *Time Series Analysis- Forecasting and Control, Holden-day, San Francisco, 1976.*
3. Kendall, Sir Maurice and Ord, J.K., *Time Series, Edward Arnold, London, 1990.*

#### **21MAT553A**

#### **WAVELETS ANALYSIS**

**3 0 0 3**

**Unit 1** Basic Properties of the Discrete Fourier Transform, Translation - Invariant Linear Transformations. The Fast Fourier Transform.

**Unit 2** Construction of Wavelets on  $\mathbb{Z}_N$ , The First Stage Construction of Wavelets on  $\mathbb{Z}_N$ , The Iteration Step's. Examples and Applications,  $\ell_2(\mathbb{Z})$

**Unit 3** Complete Orthonormal Sets in Hilbert Spaces,  $L_2([-\pi, \pi])$  and Fourier Series, The Fourier Transform and Convolution on  $L_2(\mathbb{Z})$ , First-Stage Wavelets on  $\mathbb{Z}$ , The Iteration Step for Wavelets on  $\mathbb{Z}$ , Implementation and Examples.

**Unit 4**  $L_2(\mathbb{R})$  and Approximate Identities, The Fourier Transform on  $\mathbb{R}$ , Multiresolution Analysis and Wavelets,

**Unit 5** Construction of Multiresolution Analyses, Wavelets with Compact Support and Their Computation.

**TEXT BOOK:**

*Michael W. Frazier, An Introduction to Wavelets Through Linear Algebra, Springer, 1999.*

**REFERENCES:**

1. Daubechis, *Ten Lectures on Wavelets*, SIAM, 1992.
2. S. Mallat, *A Wavelet Tour of Signal Processing*, Elsevier, 2008.