M. Phil. (Master of Philosphy) in Mathematics

Faculty of Science



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1. Program Outcomes

- PO1. Mathematics knowledge: Knowledge of advanced level in pure and applied mathematics.
- PO2. **Problem analysis:** Develop analytical skills to identify, formulate, and analyze complex mathematical problems.
- PO3. **Modeling and solutions:** Design solutions for complex problems and evolve procedures for solutions.
- PO4. **Modern analytical tool usage:** Select, and apply appropriate techniques, resources, and modern analytical tools.
- PO5. **Environment and sustainability:** Understand the impact of physical processes in societal and environmental contexts, and demonstrate the knowledge, and need for sustainable development.
- PO6. Ethics: Apply ethical principles and commit to professional ethics and responsibilities.
- PO7. **Communication:** Communicate effectively on complex scientific activities with the science community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
- PO8. **Project management and Research:** Demonstrate knowledge and understanding of the Mathematical concepts and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
- PO9. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

2. Program Specific Outcomes

POS1: The students will be able to demonstrate understanding of basic knowledge in modern mathematical techniques..

POS2: The students would acquire basic knowledge of research and skills to design and conduct classes and interpret the results.

POS3: The students will be able to reinforce research skills and high end recent advances in mathematics.

Curriculum and Credit Distribution

	Course	Title	LTP	Credits	ES
	Code				
	20MAT701	Research Methodology	3 1 0	4	A
Semester I	20MAT702	Foundations of Modern Mathematics	3 1 0	4	В
	20AVP501	Amrita Values Programme	100	1	F
				9	
		Total			
		Elective	3 1 0	4	E
Semester	20MAT799	Dissertation (based on the chosen elective)		17	P
II		Total		21	
	·	Total Credits		30	

ELECTIVES (any one)

CODE	TITLE	LTP	Credits	ES
20MAT731	Advanced Graph Theory and its		4	Е
	Applications			
20MAT732	Advanced Operator Theory	3 1 0	4	Е
20MAT733	Coding Theory and Cryptography	3 1 0	4	Е
20MAT734	Homological Algebra	3 1 0	4	Е
20MAT735	Stochastic Process	3 1 0	4	E
20MAT736	Theory of Fluid Dynamics	3 1 0	4	E
20MAT737	Theory of Knots and Briads	3 1 0	4	Е
20MAT738	Introduction to Mathematical Modelling	3 1 0	4	Е
20MAT739	Semigroup Theory	3 1 0	4	Е
20MAT740	Introduction to methods of applied	3 1 0	4	Е
	mathematics			

3.EVALUATION SCHEME AND GRADING SYSTEM

All M.Phil. students must complete 12 credits of course work which includes 4 credits on Research Methodology; 8 credits in the domain of research; a one-credit course on Amrita Values Programme, followed by a Thesis and Viva-voce, carrying 17 credits.

The courses (excluding Thesis and Viva-voce) shall be graded as follows;

Letter Grade Grade Points Ratings

O	10.00	Outstanding
A+	09.50	Excellent
A	09.00	Very Good

B+	08.00	Good
В	07.00	Above Average
C	06.00	Average
P	05.00	Pass
F	00.00	Fail
FA		Failed due to Lack of Attendance
I etc)		Incomplete (awarded only for Lab. courses / Internship,
W		Withheld

Thesis Submission and Evaluation

After the thesis synopsis is approved, the M.Phil. scholar can submit thesis within two weeks from the date of Synopsis submission.

The thesis adviser shall nominate four experts to evaluate the thesis. The Head of the institution can choose anyone from the given list. The Thesis Advisor and Co-Advisor are also invited to provide a formal evaluation of the thesis. All examiners will be given three weeks to provide their evaluation. Each examiner can give one of three recommendations: (i) Accept, (ii) Accept with Modifications, or (iii) Reject. If neither of the external examiners recommends a Reject, the candidate is permitted to proceed to the Viva Voce. If both external examiners recommend a Reject, the thesis is rejected and the candidate is required to leave the M.Phil. Programme. If only one of the examiners recommends a Reject, the thesis is sent to a third external examiner whose evaluation decides whether or not the candidate is permitted to proceed with the Viva Voce. Prior to the Viva Voce, the candidate must submit a revised Thesis taking into account the comments and suggestions made by all examiners.

Viva Voce

The M.Phil. Thesis Committee consists of the M.Phil. Committee and atleast one of the external experts who evaluated the thesis. The Convener of the M.Phil. Committee serves as the Convener of this Committee. If none of the external examiners can be present, the Head of the Institution can nominate a substitute examiner.

The Convener forwards the consolidated recommendation to the PGP Chair who forwards the same to the PGP Dean. The PGP Dean forwards the final recommendation to the Vice-Chancellor for his approval. After the Vice- Chancellor's approval, the PGP Dean issues the Provisional Certificate.

Objective

This course comprises of topics of research methodology in general, methodology of research in Mathematics, methodology of teaching Higher Mathematics, modern tools for teaching and research and ethics in research

Course Outcomes:

- CO 1: To understand the methods of scientific research and different research processes.
- CO 2: To familiarize various methods to collect the research materials for research.
- CO 3: To familiarize journals and conferences in different areas of mathematics.
- CO 4: To understand the challenges and opportunities in teaching of mathematics.
- CO 5: To familiarize different software tools for research.

1.1 Research

1.1.1. General introduction to research methodology

Meaning and objective of scientific research, Types and significance of research, Methods of scientific research, Research process and criteria for good research, Stages of research.

Reference: Research Methodology, by C.R. Kothari et al.

1.1.2. Methodology of Research in Mathematics

Identifying a broad area. Collecting materials for deep understanding of fundamentals as well as recent findings. Identifying an area for in-depth study. Collecting and reading as many documents as possible. Studying recent research findings and trying solutions independently. Fixing exact problem/concept or research.

1.1.3. Information Resources and Publication

This section deals with the sources of information. Classical sources, Modern sources, availability of online resources (free and subscribed), Accessibility of Journals and other print documents. Needs, ways and means of publication of research findings.

1.2 Teaching and learning

1.2.1. Modern and classical methods and techniques of teaching.

Teaching of Higher Mathematics. Challenges and opportunities. Difficulties in teaching and learning Mathematics. Traditional and Modern approaches. Teaching techniques, proof techniques-inductive reasoning, deductive reasoning, contrapositive, counter examples.

1.2.2. Innovative Methods of Teaching and Learning.

- * Graphing and computation (KMPlot, Geogebra, Scilab, SageMath, GGAP and R)
- * Document preparation (LaTeX)
- * Presentation (Beamer)

1.3 Ethics in research

- 1.3.1. Piracy and Plagiarism.
- 1.3.2. Morality and ethics.
- 1.3.3. Rules and regulations IPR.

L-T-P-Cr 3- 1- 0- 4

20MAT702 Foundations of Modern Mathematics

Objective

This course aims to provide a sound philosophical foundation of Modern Mathematics. It would also provide an understanding of the inter-relation between various branches of Mathematics. Applications of Mathematics in various fields including modern technology also to be discussed.

Course Outcomes:

- CO 1: To familiarize the history of Indian and other mathematicians.
- CO 2: To understand the fundamental concepts in Galois theory.
- CO 3: To Review the basic topics in topology and homotopy theory.
- CO 4: To Review the basic topics in mathematical analysis.
- CO 5: To Review the basic topics in measure theory.

2.1. History of Mathematics

Reference to contributions of Indian Mathematicians:

Aryabhata (476–550 AD), Varahamihira (505–587 AD), Yativṛṣabha (6 C-AD), Brahmagupta (598–670 AD), Bhaskara I (600–680 AD) Shridhara (650–850 AD), Mahavira (9 C-AD), PavuluriMallana (11 C-AD), Hemachandra (1087–1172 AD), Bhaskara II (1114–1185 AD), Narayana Pandit (1340–1400 AD), Sangamagrama Madhava (1340–1425 AD), Parameshvara, (1360–1455 AD), NilakanthaSomayaji, (1444–1545 AD), RaghunathaSiromani, (1475–1550

AD), Mahendra Suri (14 C-AD), ShankaraVariyar (c. 1530), Jyeshtadeva, (1500–1610), AchyutaPisharati (1550– 1621), Srinivasa Ramanujan, Harish Chandra, R.C.Bose, Srikhande, P.C.Mahalanobis.

Ancient Indian systems of numbers - KATAPAYDI, BhutaSanghtya

Reference: Ancient Indian Mathematics: an overview, S.G. Dani, School of Mathematics, TIFR, Bombay

2.2. Algebra

Galois theory

References: Contemporary Abstract Algebra, Joseph A. Gallian, Fourth edition, Narosa

Publishing House, 2011.

Topics in Algebra, I. N. Herstein, Second edition, John Wiley and Sons.

2.3. Topology

Review of basic topology, Homotopy

References: Topology, James R Munkres, Prentice Hall (2000).

Lecture notes on elementary topology and geometry, I M Singer, J A Thorpe, New

York Springer 1967.

Elements of Algebraic Topology, James R. Munkres, Addison-Wesley Publishing

Company (1984)

2.4. Modern Analysis

Theory of distributions and Fourier Transform

Reference: Functional Analysis, Walter Rudin, McGraw-Hill Education (1973).

2.5. Measure theory

Review of basic measure theory, Radon-Nikodym theorem.

Reference: Real Analysis, Royden, Pearson, 3rd edition (1988).

ELECTIVES

L-T-P-Cr

20MAT731

Advanced Graph Theory and its Applications

3-1-0-4

Objective

The objective of the course is to explain basic concepts in graph theory and define how graphs serve as models for many standard problems. It provides an understanding of domination in graphs and its applications in networks and the concepts of product graphs.

Course Outcomes:

CO1: To understand and apply the fundamental concepts in graph theory

CO2: To understand the concepts of domination in graphs and its applications in networks

CO3: To understand the concepts of product graphs and their properties

Unit 1:

Introduction: Definition of graph-degree of vertex- Regular graphs - Connected graph, complete graphs - Bipartite graph - Euler graph necessary and sufficient conditions for Euler graph-Hamiltonian graph and its properties - Connectivity, vertex connectivity, edge connectivity. Trees - properties of Trees-spanning tree

Unit 2:

Domination theory - Definition of dominating sets in graphs- Domination number - bounds in terms of degree, diameter and girth- product graphs and Vizing's conjecture

Unit 3:

Changing and unchanging domination – changing vertex removal – changing edge removal – bondage number – unchanging vertex removal – unchanging edge removal

Unit 4:

Four Standard Graph Products - Cartesian Product - Strong Product - Direct Product - Lexicographic Product - distance formula

Unit 5:

Three Fundamental Products - Commutativity, Associativity- Projections and Layers

References:

- 1. Richard Hammack, Wilfried Imrich and Sandi Klavzar, Handbook Of Product Graphs, CRC Press, 2nd edition 2011.
- 2. Douglas B. West, Introduction to graph Theory, Second Edition, Pearson Publication, 2001.
- 3. Teresa W Hynes, Stephen T. Hedetniemi and Peter. J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker INC, New York.

L-T-P-Cr

20MAT732

Advanced Operator Theory

3-1-0-4

Objective

This course aims to provide an understanding of spectral theory and the interplay between the ideas and methods from operator theory and functional analysis with methods and ideas from function theory and commutative algebra.

Course Outcomes:

- 1. To understand the concepts of spectral theory for bounded operators.
- 2. To understand the concepts of Banach algebras, culminating in the Gelfand-Naimark theorem.
- 3. To understand the behavior of linear operators and C * -algebras of operators.
- 4. To understand the concept of Fredholm operators, Semi-Fredholm operators, index of a Fredholm operator
- 5. To Demonstrate the concept of Compact operators and operator equations; application to Sturm-Liouville system

Unit-1

Spectral Theory in Hilbert Spaces: Hermitian Symmetric Forms – Orthogonality – The Hilbert Space Adjoint – Self-adoint Bounded Linear Operators - Self-adjoint Compact Linear Operators – Positive Linear Operators - Orthogonal Projections – Functions of Self-adjoint Bounded Linear Operators – The Spectral Theorem.

Unit-2

Banach algebras, spectrum of a Banach algebra element, Holomorphic functional calculus, Gelfand theory of commutative Banach algebras.

Unit-3

Hilbert space operators, C * -algebras of operators, commutative C * -algebras.

Unit-4

Spectral Theory of Compact Operators and elements of Fredholm Theory. Fredholm operators, Semi-Fredholm operators, index of a Fredholm operator. Spectrum of compact operators

Unit 5

Compact operators and operator equations; Application to Sturm-Liouville system; Fredholm alternatives; Integral equations.

References

- 1. W. Arveson, "An invitation to C*-algebras", Graduate Texts in Mathematics, No. 39. Springer-Verlag, 1976.
- 2. N. Dunford and J. T. Schwartz, "Linear operators. Part II: Spectral theory. Self adjoint operators in Hilbert space", Interscience Publishers John Wiley I & Sons 1963.
- 3. R. V. Kadison and J. R. Ringrose, "Fundamentals of the theory of operator algebras. Vol. I. Elementary theory", Pure and Applied Mathematics, 100, Academic Press, Inc., 1983.
- 4. V. S. Sunder, "An invitation to von Neumann algebras", Universitext, Springer-Verlag, 1987.
- 5. J.B. Conway .A Course in Functional Analysis, GTM 96,Springer-1990
- 6. Douglas, R. G. Banach Algebra Techniques in Operator Theory (Academic Press 1972).

L-T-P-Cr

20MAT733 CODING THEORY AND CRYPTOGRAPHY

3-1-0-4

Objective

This course deals with the mathematical ideas underlying modern cryptography, including algebra, number theory and probability theory.

Course Outcomes:

CO1: To understand the concepts of linear codes and error correcting codes.

CO2: To Familiarize with Dual codes, Hamming codes

CO3: To understand the concepts of BCH codes.

CO4: Cryptanalysis of classical ciphers

CO5: To get an overview of encryption systems

Unit 1

Introduction to linear codes and error correcting codes. Encoding and decoding of a linear code,

Unit 2

Dual codes, Hamming codes and perfect codes.

Unit 3

Cyclic codes. Codes with Latin Squares, Introduction to BCH codes.

Unit 4

Classical ciphers: Cryptanalysis of classical ciphers, Probability theory, Perfect security. Block ciphers: DES, AES, Block cipher modes of operation.

Unit 5

Private-key encryption: Chosen plaintext attacks, Randomised encryption, Pseudo randomness, Chosen cyphertext attacks.

Text Books:

- 1. Raymond Hill, A first course in Coding Theory, Clarandon Press, Oxford (1986).
- 2. Jonathan Katz and Yehuda Lindell, Introduction to Modern Cryptography, CRC Press.

REFERENCES:

- 1. Katz and Lindell, Introduction to Modern Cryptography. Second Edition, Chapman & Hall/CRC Press, 2014.
- 2. Jonathan Katz and Yehuda Lindell, Introduction to Modern Cryptography, CRC Press.
- 3. Hans Delfs, Helmut Knebl, "Introduction to Cryptography, Principles and Applications", Springer Verlag.
- 4. Raymond Hill, A first course in Coding Theory, Clarandon Press, Oxford (1986).
- 5. J.H. Van Lint, Introduction to Coding Theory, Springer (1998).

L-T-P-Cr

20MAT734 HOMOLOGICAL ALGEBRA

3-1-0-4

Objective

This course will introduce the basic concepts and tools of homological algebra with examples in module theory and group theory. It deals with homological algebra for abelian categories in general, and modules over a ring in particular.

Course Outcomes:

- CO1: To Understand the basic concepts of homology groups
- CO2: To Understand the concept of restricted model of homology theory simplicial homology
- CO3: To familiarize the Axioms which characterize homology groups for CW complexes
- CO4: To understand the concept of Cohomology groups of spaces their basic properties
- CO5: To familiarize as an application the Jordan Curve Theorem, the Projective Spaces and Lens Spaces

Course Outcomes:

<u>Unit 1:</u> Simplical Complexes and Homology Groups

Definition and elementary properties-Siplices, Simplical complexes and Simplical maps, homology Groups, Zero-dimensional Homology, The Homology of a cone, Homomorphism induced by Simplicial Maps, Chain Complexes and Acyclic Carriers.

<u>Unit 2:</u> Relative Homology and Eilenberg-Steenrod Axioms

Relative Homology, the Exact Homology Sequence, the Zig-zag Lemma, Mayer-Vietoris Sequences, the Eilenberg-Steenrod Axioms, the Axioms for Simplicial Theory. Categories and functors.

Unit 3: Singular Homology

The Singular Homology Groups, the Axioms for Singular Theory, Excision in Singular Homology, Mayer-Vietoris Sequences, the Isomorphism between Simplical and Singular Homology, More on Quotient Spaces, CW Complexes and their Homology.

Unit 4: Cohomology and Homological Algebra

The Homfunctor, SimplicalCohomology Groups, Relative Cohomology, Cohomology theory, The Ext Functor, The Universal Coefficient Theorem for Cohomology, Torsion Products, The Universal Coefficient Theorem for Homology, Kunneth Theorems (for cohomology and homology proofs can be omitted)

Unit 5: Applications

Local Homology Groups and Manifolds, the Jordan Curve Theorem, Projective spaces and Lens Spaces, The Cohomology Ring of a Product Space

REFERENCES:

- 1. Algebraic Topology, Tammo tom Dieck, European Mathematical Society (2008)
- 2. Algebraic Topology, C.R.F.Maunder, Cambridge University Press, (1980)
- 3. Algebraic Topology, Edwin H. Spanier, Springer-Verlag, NewYork(1966), ISBN: 978-0-387-94426-5

L-T-P-Cr

20MAT735

STOCHASTIC PROCESSES

3- 1- 0- 4

OBJECTIVE:

This course aims at providing the necessary basic concepts in stochastic processes and its application. It introduces the topics Renewal process and Martingales.

Course outcomes:

CO1: Understand the fundamental concepts of random processes, particularly continuous-time Markov chains and related structures.

CO2: Demonstrate the specific applications to Poisson process. Understand some basic ideas about absorption probabilities and its basic properties, transient behaviour, the stationary distribution.

CO3: Demonstrate the application of Stochastic processes in Queueing Theory: arrival processes, service time distributions, Point Processes: Poisson process

CO4: Understand the concept of Renewal Processes: preliminaries, renewal function, renewal theory and applications, stationary and delayed renewal processes.

CO5: Understand a new concept Martingales: Super martingales and sub martingales with real life examples

Unit 1

Introduction to probability theory and random process

Unit 2

Markov Chains: Transistion Probability Matrices. Classification of States, Recurrence; Basic Limit Theorems. Discrete Renewal Equation, absorption Probabilities, Criteria for recurrence

Unit 3

Some Queuing Models (M/G/1, G/M/1), Random Walk: Continuous Time Markov Chains: Birth process, Birth & Death processes, Differential Equations of Birth & Death Processes, Absorption

Unit 4

Renewal Processes: Renewal Equations, Elementary renewal Theorem, Renewal Theorem and Applications. Generalizations and variations.

Unit 5

Martingales :Supermartingales and Submartingales, Optional Sampling Theorem, Martingale conveyance Theorems

REFERENCES

- 1. S. Karlin, H.M. Taylor, A first course in Stochastic Processes (Academic Press 1975)2ndEdn.
- 2. Fundamentals of Mathematical Statistics, S.C Gupta and V.K Kapoor, Sultan Chand Publications (2002)
- 3. Battacharya and Waymire: Stochastic Processes with Applications (John Wiley 1998)
- 4. Grimmett and Stirzaker: Probability & Random Processes (Clarender Press 1992) U.N.
- 5. Bhat, Gregory Miller: Applied Stochastic Processes (Wiley Inter 2002) 3rdEdn.

20MAT736

THEORY OF FLUID DYNAMICS

L-T-P-Cr 3- 1- 0- 4

OBJECTIVE:

This course aims at providing the necessary basic fluid kinematics and dynamics. Incompressible fluid flow and flow through pipes are discussed.

Course outcomes:

CO1: To Understand the difference between solids and fluids

CO2: To Understand the concept of Fluid Kinematics

CO3: To Discuss the similarity laws and models.

CO4: Getting familiarisation with Incompressible Fluid Flow

CO5: To Understand the concept of the Flow Through Pipes.

Unit 1 Basic Concepts and Properties

Fluid – definition, distinction between solid and fluid - Units and dimensions – Properties of fluids – density, specific weight, specific volume, specific gravity, temperature, viscosity, compressibility, vapour pressure, capillary and surface tension – Fluid statics: concept of fluid static pressure, absolute and gauge pressures – pressure measurements by manometers and pressure gauges.

Unit 2 Fluid Kinematics

Fluid Kinematics - Flow visualization - lines of flow - types of flow - velocity field and acceleration - continuity equation (one and three dimensional differential forms)- Equation of streamline - stream function - velocity potential function - circulation - flow net .

Unit 3 Fluid Dynamics

Fluid dynamics - equations of motion - Euler's equation along a streamline - Bernoulli's equation - applications - Venturi meter, Orifice meter, Pitot tube - dimensional analysis - Buckingham's theorem - applications - similarity laws and models.

Unit 4 Incompressible Fluid Flow

Viscous flow - Navier - Stoke's equation (Statement only) - Shear stress, pressure gradient relationship - laminar flow between parallel plates - Laminar flow through circular tubes (Hagen poiseulle's).

<u>Unit 5</u> Flow Through Pipes

Hydraulic and energy gradient - flow through pipes - Darcy-weisback's equation - pipe roughness - friction factor - Moody's diagram - minor losses - flow through pipes in series and in parallel - power transmission - Boundary layer flows, boundary layer thickness, boundary layer separation - drag and lift coefficients.

REFERENCES:

- 1. White, F.M., Fluid Mechanics, Tata McGraw-Hill, 5th Edition, New Delhi, 2003.
- 2. An introduction to fluid dynamics, G K Bachelor, Cambridge University Press, 2000.
- 3. Streeter, V.L., and Wylie, E.B., Fluid Mechanics, McGraw-Hill, 1983.
- 4. Kumar, K.L., Engineering Fluid Mechanics, Eurasia Publishing House (P) Ltd., New Delhi (7th Edition), 1995

L-T-P-Cr

20MAT737

THEORY OF KNOTS AND BRAIDS

3- 1- 0- 4

OBJECTIVE:

This course aims at understanding the concept of Knot theory and theory of braids. It helps to familiarize different approaches for constructing invariant polynomials from link diagrams.

Course outcomes:

CO1 : To Understand the concept of Knot theory its properties, planar diagrams and the semigroup structure on knot, series of knots and link

CO2: To Understand the concept of fundamental group, Knot Quandle and Conway's algebra and to construct various knot invariants

CO3: To familiarize Different approaches for constructing invariant polynomials from link diagrams

CO4: To Understand the concept of theory of Braids its properties, braids in different spaces, Algorithms for constructing a braid by a link

CO5: To understand the basic notions of Vassiliev invariant theory, algebraic structure that arises on the set of Vassiliev knot invariants.

<u>Unit 1:</u> Knots, Knot arithmetic and Invariants:

Knots- Definition, Simple examples, Elementary properties, Polygonal links and Reidemeister moves Knot arithmetics and Seifert surfaces; Knots in Surfaces- Torus Knots, The linking coefficient, The Arf invariant and The colouring invariant.

<u>Unit 2:</u> Fundamental group, Quandle and Conway's algebra

Examples of unknotting, Fundamental group, Calculating knot groups, Quandle-Geometric description of the quandle, Algebraic description of the quandle, Completeness of the quandle, Special realisations, Conway algebra and polynomials, Realisations of the Conway algebra, Matrix representation.

Unit 3: Jones polynomial and Khovanov's polynomial

Kauffman's bracket, Jones' polynomial and skein relations, Kauffman's two-variable polynomial, Jones' polynomial. Khovanov's complex, Simplest properties, Tait's first conjecture and Kauffman-Murasugi's theorem, Classification of alternating links, The third Tait conjecture, A knot table, Khovanov's polynomial, The two phenomenological conjectures.

Unit 4:Braids, links and representations

Definitions of the braid group- Geometrical definition, Topological definition, Algebrogeometrical definition and Algebraic definition; Equivalence of the four definitions, The stable braid group, Pure raids, Links as braid closures, Braids and the Jones polynomial;

Representations of the braid groups- The Burau representation, The Krammer–Bigelow representation, Krammer's explicit formulae, Bigelow's construction, Alexander's theorem, Spherical and cylindrical braids; Vogel's algorithm.

<u>Unit 5:</u> Vassiliev's invariants and the Chord diagram algebra

Vassiliev's invariants-Definition and Basic notions, Singular knots, Invariants of orders zero and one, Examples of higher—order invariants, Conway polynomial coefficients, Other

polynomials and Vassiliev's invariants, An example of an infinite-order invariant, The Chord diagram algebra - Chord diagram algebra- Basic structures, Bi algebra structure, The four colour theorem.

REFERENCES:

- 1. Knot Theory, V. O. Manturov, CRC Press; 1st edition (2004).
- 2. Knot theory and its applications, K. Murasugi, Springer.
- 3, Braid and Knot theory in dimension four, S. Kamada, American Mathematical Society.

L-T-P-Cr

20MAT738

Introduction to Mathematical Modelling

3-1-0-4

Unit 1:

Mathematical Modelling; Need, Classification and simple illustrations, techniques of mathematical modelling, Some characteristics of mathematical models, Mathematical models through geometry, Mathematical models through algebra, Mathematical models through Trigonometry, Mathematical models through calculus, Limitations of Mathematical models.

Unit 2:

Random Variable and Distributions: Probability, random variable, discrete and continuous distribution functions: Binomial, Uniform, Normal, Poisson, Gamma distributions, expected values and standard deviations, Structure of a hypothesis test: the t-test, chi-square test, Testing regression parameters, Linear Regression, Multiple Linear Regressions, Non-Linear Regression.

Unit 3:

Mathematical modelling through Ordinary Differential Equations of first order, second order, Linear and Non-Linear Growth Decay Model, Predator-Prey Interaction, Lotka-Volterra Model

Unit 4:

Mathematical modelling through system of ordinary differential equations of first order, Population dynamics, Epidemics

Unit 5:

Mathematical modelling through partial differential equations, Limitations of ordinary differential equations, Well Posedness, Dirichlet and Neumann Conditions, Symmetry and Dimensionality, Example, Replacing Derivatives with Finite Differences, Formulating an Algorithm

Textbooks & References:

- 1. Mathematical Modelling J.N. Kapur, New Age International Publishers
- 2. Mathematical Modelling and simulation Introduction for scientists and Engineers, Kai Velten, Wiley-ECH
- 3. Principles of Mathematical Modelling, Clive L Dym, Elseiver Academic Press

20MAT739

Semigroup Theory

L-T-P-Cr 3-1-0-4

Unit I:

Basic Definitions- Monogenic Semigroups- Ordered Sets, Semi lattices and lattices- Binary relations; equivalences- Congruences- Free semigroups- Ideals and Rees Congruences

Unit II:

Greens Relations- Structure of D- classes- Regular D- classes- Regular Semigroups- The sandwich Set

<u>Unit III</u>: Simple and 0-simple semigroups- principal factors, Rees Theorem- Completely simple semigroups

Unit IV:

Varieties-Bands- Free Bands- Varieties of Bands (Chapter IV Section 4.3 - 4.6)

<u>Unit V</u>: - Inverse semigroups- Preliminaries- The Natural partial order relation on an inverse semigroup

Textbooks & References:

- 1. Fundamentals of Semigroup theory, J. M. Howie, Clarendon Press, Oxford ISBN0-19-851194-9
- 2. The Algebraic Theory of Semigroups- A. H. Clifford and G. B. Preston, American Mathematical Society 1961
- 3. Semigroups: An Introduction to the Structure Theory- P. A. Grillet, Marcel Decker INC. 1995
- 4. Techniques of Semigroup Theory- Peter M. Higgins, Clarendon press

20MAT740 Introduction to Methods of Applied Mathematics

Unit-I

Applications of Green Function to solve some ODE, Introductions of Integral equations, classification

Unit-II

Applications of Integral equation to solve various Problems Laplace Transforms and its properties, Application of Laplace Transforms to solve various Problems

Unit-III

Fourier Transforms and its properties, Application of Fourier Transforms to solve various Problems

<u>Unit-IV</u> Orthogonal expansions, orthogonal Polynomials and their Properties Application of Orthogonal Polynomials

<u>Unit-V</u> Introduction to Wavelets, Applications of Wavelets to solve various ODE/PDE

Textbooks & References:

- 1. Introduction to Applied Mathematics Gilbert Strang Wellesley Cambridge Press
- 2. Advanced topics in Applied Mathematics by Sudhakar Nair, Cambridge Press
- 3. Applied Mathematics: A Very Short Introduction by Alain Goriely Oxford Publications