22MA801 **Analysis and Algebra** **3 1 0 4**

Course Overview

This course is offered to PhD students in mathematics to gain an operational understanding of Real Analysis, Abstract Algebra, and Linear Algebra at a level commensurate with their progress in the program. Priority is given to understanding the concepts and applying them to problems of various difficulties so as to get a flavor of mathematical thinking and problem-solving techniques.

Course Outcomes

CO1: Produce rigorous proofs of results that arise in the context of mathematical analysis and topology.

CO2: Analyze finite and infinite dimensional vector spaces and subspaces over a field and study their properties.

CO3: Analyze proofs and demonstrate examples of various topics in groups, rings, and fields.

CO4: Solve problems of various difficulty levels in analysis and algebra.

Course Syllabus

Unit – 1

Analysis: Elementary set theory, finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum. Sequences and series, convergence, limsup, liminf. Bolzano Weierstrass theorem, Heine Borel theorem. Continuity, uniform continuity, differentiability, mean value theorem. Sequences and series of functions, uniform convergence. Riemann sums and Riemann integral, Improper Integrals. Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Topology: basis, dense sets, subspace and product topology, separation axioms, connectedness, and compactness.

Unit – 2

Linear Algebra: Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations. Algebra of matrices, rank and determinant of matrices, linear equations. Eigenvalues and eigenvectors, Cayley-Hamilton theorem. Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms

Unit – 3

Algebra: Permutations, combinations, pigeon-hole principle, inclusion-exclusion principle, derangements. Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's Ø-function, primitive roots. Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation groups, Cayley's theorem, class equations, and Sylow theorems. Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain. Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory.

Text Books

1. Principles of mathematical analysis, Walter Rudin, McGraw Hill Education Third Edition, 2017.
2. Introduction to Topology and Modern Analysis, George F. Simmons, McGraw Hill, 2017.
3. Contemporary Abstract Algebra, Joseph Gallian, Cengage India, Ninth Edition,2019.
4. Linear Algebra, Kenneth Hoffman and Ray Kunze, Pearson, Second Edition, 2018.

Reference Books

1. Schaum's Outline of Linear Algebra, Seymour Lipschutz, McGraw Hill Education Third Edition, 2017.
2. Topic in Algebra, I.N. Herstein, Wiley Publications, 2006.
3. Algebra, Michael Artin, Pearson, Second Edition, 2010.
4. Topology, James Munkers, Pearson Education, Second Edition, 2021.
5. Mathematical Analysis, Tom M. Apostol, Narosa, 2002.

Evaluation Pattern

Internal (70%): Assignments (problem-solving), Viva, Presentation

External (30%): Exam