Coverage Maximization of Heterogeneous UAV Networks

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***Abstract*—In this paper we study the deployment of a UAV (unmanned aerial vehicle) network that consists of multiple UAVs to provide emergent communication services to people trapped in a disaster area, where each UAV is equipped with a base station that has limited computing capacity and power supply, and thus can only serve a limited number of users. Unlike most existing studies focusing on homogenous UAVs, we consider the deployment of heterogeneous UAVs, where different UAVs have different computing capacities. We study a problem of deploying K heterogeneous UAVs in the air to form a connected UAV network such that the number of users served by the UAVs is maximized, subject to the constraint that the number of users served by each UAV is no greater than its service capacity, assuming that the maximum number of users can be served by a UAV is given. We then propose a novel $O(\sqrt{\frac{s}{K}})$-approximation algorithm for the problem, where s is a given positive integer, e.g., $s = 3$. We finally evaluate the performance of the approximation algorithm. Experimental results show that the number of users served by all UAVs in the approximate solution is improved by 22% compared with the solutions delivered by state-of-the-arts.**

***Index Terms*—UAV communication networks; UAV deployment problem; heterogeneous UAVs; approximation algorithms.**

I. Introduction

Terrestrial LTE base stations usually are statically deployed. However, their static deployments limit their ability in key 5G applications with surging traffic demands at some hotspot locations (e.g., battlefields and concerts). In addition, the deployed base stations may have been destroyed in natural disasters, e.g., earthquakes, tsunamis, flooding, etc. Emergent communication services are definitely needed for rescue teams to rescue people trapped in disaster areas [7], [15].

The employment of Unmanned Aerial Vehicles (UAVs) or drones, e.g., DJI Matrice 300 RTK UAVs, has gained great attention in public safety communications [6], [7], [16], [17], [18], [20], [21], [25], [26], [32], [33], [34], [35], [43], [44], [46]. By installing an LTE base station on a UAV, the UAV can provide wireless communication services to ground users in the air [4], [27]. The LTE base station usually consists of two modules: SkyRAN and SkyCore, where SkyRAN provides

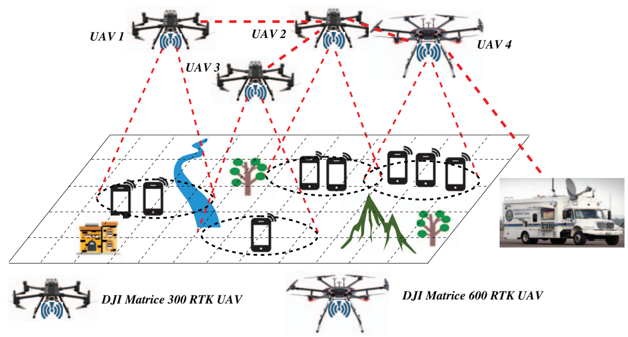

Fig. 1. A heterogeneous UAV network provides communication services to people trapped in a disaster area, where UAVs 1, 2, and 3 are DJI Matrice 300 RTK, while UAV 4 is DJI Matrice 600 RTK and it is connected to the Internet through the relay of an emergency communication vehicle.

wireless connectivity to ground users, while SkyCore is responsible for user mobility, management, control functions, as well as routing [27], [31]. In addition, some mobile operators, e.g., AT&T and Verizon, conducted experiments about UAVs with mounted LTE base stations [27]. A UAV communication network that consists of multiple UAVs can be easily deployed to provide emergent communication service in a disaster area, see Fig. 1. Both rescue teams and the people trapped can communicate with each other by leveraging the deployed UAV network.

In spite of the aforementioned promising applications of UAV networks, there are many challenges to realize these applications. Particularly, since the payload of a UAV usually is very limited, e.g., the maximum payload of a DJI Matrice 300 RTK UAV is only 2.7 kg [23], many functions in the SkyCore module must run a low-end, light-weight server with a very resource-constrained CPU and a small-capacity battery, where the server is mounted on the UAV [3], [4]. This could significantly increase the processing (control and data plane) latency of its traffic, thereby reducing network throughput [27]. Thus, a UAV usually needs to restrict the maximum number of users to access it, i.e., there is a *service capacity* for the

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UAV [37], [45], e.g., 200 users. Otherwise, if too many users access the UAV, each user will experience a very long service delay, e.g., a few seconds, and the network throughput also significantly decreases [27].

The deployment of such resource-constrained UAV networks recently has attracted a lot of attentions [8], [19], [29], [36], [37], [38], [39], [45], and most of existing studies assumed that the UAVs are *homogenous*. Different from these existing studies, in this paper we consider the deployment of *heterogeneous UAVs*. Since different UAVs may be purchased at different time and some UAVs bought a few years ago may not be available in the current market, it is very likely that different UAVs have different capacities, in terms of payloads, battery capacities, etc. For example, consider two popular UAVs for emergency communications: DJI Matrice 600 RTK UAV and DJI Matrice 300 RTK UAV. The former UAV has a maximum payload of 5.5 kg [22] but it is out of production now, while the latter one has a maximum payload of 2.7 kg only and it is still available in the market [23].

Due to different maximum payloads and energy capacities of different UAVs, the base stations mounted on different UAVs may be different, too. The base station on a DJI Matrice 600 RTK UAV may be more powerful than the one on a DJI Matrice 300 RTK UAV, in terms of computing ability and/or battery capacity, thus the former UAV is able to serve more users, i.e., has a larger *service capacity*. Fig. 1 illustrates such a heterogeneous UAV network.

In this paper, we consider the deployment of a UAV network that consists of multiple heterogenous UAVs in a disaster area, so as to provide emergent communication services to ground users in the area. We study a novel *maximum connected coverage problem*, which is to deploy K heterogeneous UAVs to serve users such that the number of users served by the deployed UAVs is maximized, subject to that (i) the number of users served by each UAV is no greater than its service capacity; (ii) the data rate of each user served by a UAV is no less than his/her minimum data rate requirement; and (iii) the UAV communication network must be connected, as the data from the users served by one UAV may need to be sent to the users served by another UAV, e.g., the communications between trapped people and rescue teams.

The heterogenous UAV deployment problem is very challenging, since the objective of the problem, i.e., serving more users, conflicts with the network connectivity constraint. On one hand, to serve as many users as possible, the UAVs should be deployed over places with high-density users. However, such places may be far away from each other. The UAV network may not be connected. On the other hand, to ensure the connectivity of the deployed UAV network, the UAVs should not be deployed too far away from each other, since the communication range between any two UAVs is limited, e.g., a few hundred meters. Then, the coverage areas of two UAVs may be overlapped, i.e., some users can be served by the two UAVs simultaneously. In addition, since different UAVs have different service capacities, the UAVs with large service capacities should be deployed over the places with high-density users, while the UAVs with low services capacities may be more likely to act as relays between the UAVs with large service capacities. However, existing studies in [8], [19], [29], [37], [39], [45] for homogeneous UAVs deployment does not consider the different UAV service capacities, and a UAV with a low service capacity may be deployed to serve ground users, while a UAV with a large service capacity may serve as a relay in their delivered solutions. Therefore, less users may be served in the solutions delivered by the existing studies, and thus a new UAV deployment algorithm is definitely needed for the heterogenous UAVs deployment problem.

The **novelty of this paper** lies in not only incorporating the heterogeneous service capacities of different UAVs into consideration, but also devising a novel approximation algorithm for the heterogeneous UAV deployment problem. Specifically, the proposed algorithm delivers an $O(\sqrt{\frac{s}{K}})$-approximate solution, where K is the number of UAVs, s is a given positive integer, e.g., $s = 3$. Notice that the approximation ratio $O(\sqrt{\frac{s}{K}})$ is better when the value of s is larger, this however incurs a larger time complexity.

The **main contributions** of this paper are summarized as follows. Unlike most existing studies that considered homogenous UAVs, in this paper we consider the deployment of a UAV network, which consists of multiple heterogeneous UAVs with different user service capacities, transmission powers and battery capacities. We first formulate a novel maximum connected coverage problem for deploying a UAV network. We then devise an $O(\sqrt{\frac{s}{K}})$-approximation algorithm for the problem. We finally study the performance of the proposed algorithm through experiments. Experimental results show that the number of users served by the proposed algorithm is up to 22% more than those by existing algorithms, which indicates that more trapped people may be early rescued and the casualty can be significantly reduced.

The organization of this paper is as follows. Section II introduces system models and defines the problem precisely. Section III proposes a novel approximation algorithm for the problem. Section IV evaluates the performance of the proposed algorithm empirically. Section V reviews related work, and Section VI concludes this paper.

II. Preliminaries

A. System model

Communication infrastructures in a disaster, e.g., an earthquake, a debris flow, or a flooding, may not work any more, due to damages or power outage caused by the disaster. To help people evacuate from a disaster area, it is important to provide temporarily emergent communications to them. A promising solution is to deploy a UAV communication network.

Fig. 1 in Section I illustrates that four UAVs in a UAV network act as aerial base stations to provide communication services (e.g., LTE or WiFi) to people above a disaster area. There is at least one of the UAVs serving as a *gateway UAV*, which means that it is connected to the Internet with the help of satellites or emergency communication vehicles. With the help of the UAV network, a person trapped in the disaster area can communicate with a nearby UAV using his/her smartphone, and he/she is able to send/receive critical information, such as voice and video, to/from the rescue team.

We treat the disaster zone as a 3-dimensional space with length α, width β, and height γ, e.g., $\alpha = \beta = 3$ km and $\gamma = 500$ m. Assume that there are n users $u_1, u_2, \ldots, u_n$ in the disaster area, and let U be the set of the n users, i.e., $U = \{u_1, u_2, \ldots, u_n\}$. A user u_i has a minimum data rate requirement r_i^{min}, e.g., 2 kbps, if it is served by a UAV base station. Denote by $(x_i, y_i, 0)$ the coordinate of a user u_i with $1 \leq i \leq n$. Assume that locations of the n users are given, where the location information can be derived by applying an existing target detection method [11], [12] for the photos/vidoes taken by the on-board cameras on the UAVs.

We consider the employment of K (≥ 2) *heterogeneous* UAVs to provide communication services (e.g., LTE or WiFi) to affected people in the disaster area. Each UAV is equipped with a base station to serve as an aerial base station in the air [4]. Due to different maximum payloads and energy capacities of different UAVs, the base stations equipped on different UAVs may be different. For example, since the maximum payload (i.e., 5.5 kg) [22] of a DJI Matrice 600 RTK UAV is larger than the payload (i.e., 2.7 kg) [23] of a DJI Matrice 300 RTK UAV, the base station on the former UAV may be more powerful, in terms of computing ability and/or battery capacity, thus is able to serve more users than the one on the latter UAV.

Denote by C_k the *service capacity* of the kth UAV with $1 \leq k \leq K$, which means that the UAV can provide communication services to at most C_k users simultaneously, e.g., $C_k = 100$ users. Notice that the service capacities of different UAVs usually are different. Following most existing studies [5], [19], [37], [40], [45], we assume that all UAVs hover at the same altitude H_{uav} to provide communication services to ground users, where H_{uav} is the optimal altitude for the maximum coverage from the sky and the value of H_{uav} can be calculated by the algorithms in [2], [39], e.g., $H_{uav} = 300$ meters. On the other hand, a ground user will receive weaker signals from a UAV if the UAV hovers at a higher or lower altitude than the optimal altitude H_{uav}, which was both analytically and empirically validated in [2].

Since the base stations mounted on the K UAVs may be different, the transmission powers of the base stations on the UAVs are different, too. Denote by P_t^k the transmission power of the base station on the kth UAV with $1 \leq k \leq K$.

For the sake of convenience, we divide the plane at altitude H_{uav} into equal size squares with a given side length λ, e.g., $\lambda = 50$ meters. Assume that both the length α and width β of the disaster area are divisible by the side length λ. Thus, the UAV hovering/service plane at altitude H_{uav} are partitioned into $m = \frac{\alpha}{\lambda} \times \frac{\beta}{\lambda}$ grids. Let $v_1, v_2, \ldots, v_m$ be the center locations of the m grids, respectively. Also, let $V = \{v_1, v_2, \ldots, v_m\}$. We assume that no more than one UAV can hover in a grid to avoid UAV collisions [45]. That is, two or more UAVs are not allowed to hover in the same grid.

B. Wireless channel models

We adopt similar UAV-to-user and UAV-to-UAV wireless channel models as those in [2], [37], [45]. For the sake of convenience, we briefly introduce them as follows. On one hand, UAV-to-user wireless channels are complicated, as there may be obstacles, e.g., a building, between a UAV in the air and a user on the ground. Following existing studies, the UAV-to-user wireless channels are composed of Line-of-Sight (LoS) links and Non-Line-of-Sight (NLoS) links [2], [45]. Specifically, the pathloss $PL_{i,j}$ between a ground user u_i and a UAV deployed at an aerial hovering location v_j is $PL_{i,j} = P_{LoS} \cdot L_{LoS} + P_{NLoS} \cdot L_{NLoS}$, where P_{LoS} is the LoS link probability and can be calculated by the method in [2], $P_{NLoS} = 1 - P_{LoS}$, L_{LoS} and L_{NLoS} are the average pathlosses for LoS and NLoS links, respectively. In addition, $L_{LoS} = 20\log_{10}\frac{4\pi f_c d_{ij}}{c} + \eta_{LoS}$, $L_{NLoS} = 20\log_{10}\frac{4\pi f_c d_{ij}}{c} + \eta_{NLoS}$, where $20\log_{10}\frac{4\pi f_c d_{ij}}{c}$ is the free space passloss, f_c is the carrier frequency, d_{ij} is the Euclidean distance between nodes u_i and v_j, c is the velocity of light, η_{LoS} and η_{NLoS} are the average shadow fadings in LoS and NLoS wireless connections, respectively. The signal-to-noise ratio (SNR) received by user u_i from the UAV at location v_j then is $SNR_{ij} = 10^{\frac{P_t^j + g_t^j - PL_{i,j} - P_N}{10}}$, where P_t^j and g_t^j are the transmission power and antenna gain of the base station on the UAV, and P_N is the noise power.

The average data rate r_{ij} of user u_i from the UAV at hovering location v_j then is $r_{ij} = B_w \log_2(1 + SNR_{ij})$, where B_w is the channel bandwidth allocated to user u_i, e.g., $B_w = 180\ kHz$ if the OFDMA technique is used [28], [37].

Assume that the kth UAV at altitude H_{uav} can communicate with a ground user if their Euclidean distance is no greater than a given communication range R_{user}^k, where $1 \leq k \leq K$. This indicates that the communication coverage radii R_{user}^ks of different UAVs may be different, due to their different transmission powers and/or antenna gains.

On the other hand, UAV-to-UAV wireless channels can be modelled as the free space path loss [2], since there are usually no obstacles between any two UAVs in the air. We assume that any two UAVs can communicate with each other if their Euclidean distance is no more than a given communication range R_{uav}. Notice that the value of R_{user}^k usually is smaller than R_{uav} [19], i.e., $R_{user}^k \leq R_{uav}$.

C. Problem definition

We represent the UAV network by a graph $G = (U \cup V, E)$, where U is the set of n to-be-served users in the disaster area, V is the set of the m candidate UAV hovering locations at altitude H_{uav}. There is an edge (v_j, v_k) in the edge set E between two hovering locations v_j and v_k if their Euclidean distance is no more than the UAV communication range R_{uav}, and there is an edge (u_i, v_k) in E between a ground user u_i and a UAV hovering location v_k if their distance is no more than the communication coverage radius R_{user}^k of the UAV.

Note that there may be limited number of available UAVs just after a disaster and they may not be able to serve all users in the disaster area, since there may be many trapped people to be served. Moreover, it may take several days to purchase new UAVs and install new base stations on them. Thus, a critical problem is to quickly deploy available UAVs to serve as many users as possible, especially within the first 72 golden hours after the disaster [37].

In this paper, we consider a *maximum connected coverage problem* in G, which is to choose K hovering locations $v_1, v_2, \ldots, v_K$ among the m candidate hovering locations in V ($K \leq m$), place K UAVs at the K chosen locations, respectively, and assign users to the K deployed UAVs, such that the number of users served by the K deployed UAVs is maximized, subject to following constraints that (i) each user $u_i \in U$ is served by at most one UAV within its communication range R^k_{user} and its data rate is no less than its minimum data rate requirement r^{min}_i; (ii) the number of users served by the kth UAV is no greater than its service capacity C_k with $1 \leq k \leq K$; and (iii) the deployed UAV communication network is connected.

We note that the users in the disaster zone may move around. In this scenario, an optimal deployment of the UAVs may become sub-optimal sometimes later. We thus need to re-deploy the UAVs by adopting the strategy in [37] and invoking the proposed algorithm later in Section III, where the most recent user location information can be detected and predicted from the photos taken by the on-board cameras of the UAVs [11], [12].

D. The optimal assignment of users with given deployed UAVs

Given K hovering locations $v_1, v_2, \ldots, v_K$, assume that the kth UAV with service capacity C_k has already been deployed at location v_k in the air with $1 \leq k \leq K$. We here consider a *maximum assignment problem*, which is how to assign users in U to the K deployed UAVs such that the number of served users is maximized, subject to the constraint that the number of users served by the UAV at each location v_k is no greater than its service capacity C_k. The problem serves as a subproblem of the maximum connected coverage problem considered in this paper in the previous Section II-C.

There are two major differences between the maximum assignment problem and the maximum connected coverage problem defined in Section II-C. The first difference is that the K UAVs have been deployed in the former problem, while the to-be-deployed locations of the K UAVs are unknown in the latter problem. The second difference is that the deployed UAV communication network may be disconnected in the former problem, whereas the deployed UAV network must be connected in the latter one.

We now propose an optimal algorithm for the maximum assignment problem, which will serve as a subroutine of the proposed algorithm for the maximum connected coverage problem considered in this paper. Given a set S of K hovering locations $v_1, v_2, \ldots, v_K$ with $|S| = K$, the kth UAV with service capacity C_k has already been deployed at location v_k with $1 \leq k \leq K$. A flow graph $G' = (\{s\} \cup U \cup S \cup \{t\}, E')$ is first constructed, where nodes s and t are the source and sink nodes in G', respectively. There is an edge $\langle s, u_i \rangle$ in E' from s to each user $u_i \in U$ with a capacity of one. There is an edge $\langle u_i, v_k \rangle$ in E' from a user $u_i \in U$ to a location $v_k \in S$ if their Euclidean distance is no more than the communication range R^k_{user} of the kth UAV, and the data rate r_{ik} of user u_i is no less than its minimum data rate r^{min}_i. The capacity of edge $\langle u_i, v_k \rangle$ is one. Finally, there is an edge $\langle v_k, t \rangle$ in E' from each location $v_k \in S$ to sink node t, and the edge capacity is the service capacity C_k of the UAV deployed at location v_k.

Having constructed the flow graph G', we find an integral maximum flow in G' from s to t, by applying the algorithm in [1]. We obtain a feasible solution to the maximum assignment problem from the flow, where a user u_i is assigned to the UAV at location v_k if the flow of edge $\langle u_i, v_k \rangle$ is one.

Lemma 1: Given a set U of users, a set S of K hovering locations $v_1, v_2, \ldots, v_K$, the kth UAV with service capacity C_k has already been deployed at location v_k in S with $1 \leq k \leq K$. There is an algorithm for the maximum assignment problem in G, which delivers an optimal solution in time $O(Kn^2)$, where $K = |S|$ and $n = |U|$.

Proof: The proof is omitted, due to space limitation. ∎

E. Notions of submodular functions and matroids

Let N be a set of finite elements and f be a function with $f : 2^N \mapsto \mathbb{R}^{\geq 0}$. For any two subsets A and B of N with $A \subseteq B$ and any element $e \in N \setminus B$, f is *submodular* if $f(A \cup \{e\}) - f(A) \geq f(B \cup \{e\}) - f(B)$ [9], and f is *monotone submodular* if $f(A) \leq f(B)$.

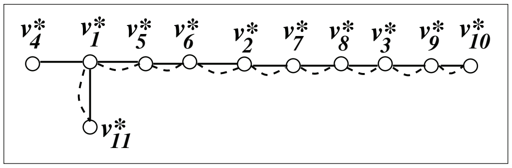
A *matroid* $\mathcal{M}$ is a pair $(N, \mathcal{I})$, where N is a set of elements and $\mathcal{I}$ is a family of subsets of N with the following three properties [9]: (i) $\emptyset \in \mathcal{I}$; (ii) the hereditary property: for any two sets A and B with $A \subseteq B \subseteq N$, if $B \in \mathcal{I}$, then $A \in \mathcal{I}$; and (iii) the augmentation property: for any two sets A and B in $\mathcal{I}$, if A contains more elements than B (i.e., $|A| > |B|$), then there is an element $e \in A \setminus B$ such that $B \cup \{e\}$ is contained in $\mathcal{I}$, too.

III. Approximation Algorithm for the Maximum Connected Coverage Problem

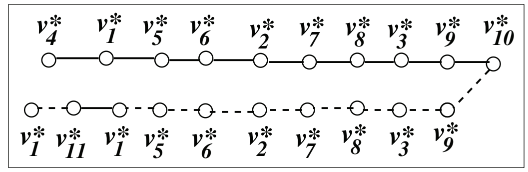
In this section, we study the maximum connected coverage problem in a large-scale disaster area. In this case, we must carefully choose the hovering locations of the K UAVs, such that not only the number of users served by deployed UAVs is maximized, but also the communication network formed by the UAVs is connected. We propose a novel $O(\sqrt{\frac{s}{K}})$-approximation algorithm for the problem with a time complexity $O(K^2 n^2 m^{s+1})$, where s is a given positive integer, K is the number of UAVs, n is the number of users in the disaster area, and m is the number of candidate hovering locations. It can be seen that the approximation ratio $O(\sqrt{\frac{s}{K}})$ of the proposed algorithm is better (i.e., larger) if the value of s is larger, which however incurs a larger time complexity.

A. Overview of the proposed algorithm

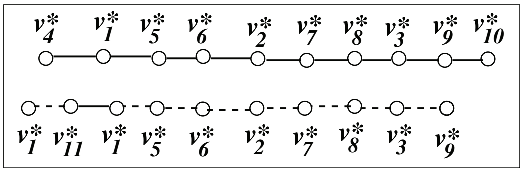
Assume that, in an optimal solution, the K UAVs are deployed at K hovering locations $v^*_1, v^*_2, \ldots, v^*_K$, respectively.



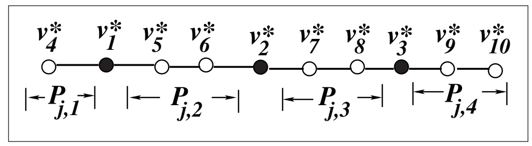
(a) Duplicate $9(=K-2)$ edges in tree T^* with $K=11$



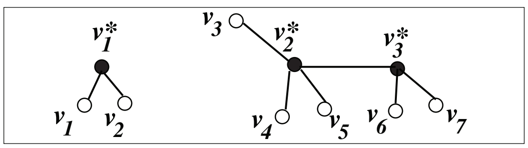
(b) A Eulerian path P_{Euler} that visits each edge in the graph of Fig. 2(b)



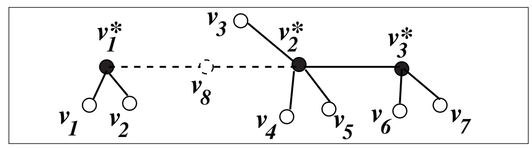
(c) Split path P_{Euler} into $\Delta = \lceil \frac{2K-2}{L} \rceil = \lceil \frac{20}{10} \rceil = 2$ subpaths P_1 and P_2, where $L = 10$.



(d) Subpath P_j consists of s nodes $v_1^*, v_2^*, \ldots, v_s^*$ and $s+1$ segments $P_{j,1}, P_{j,2}, \ldots, P_{j,s+1}$, where $s = 3$.



(e) A $\frac{1}{3}$-approximate solution with nodes $v_1, v_2, \ldots, v_7$, and $v_1^*, v_2^*, \ldots, v_3^*$ of the L nodes in P_j



(f) Place a relay node v_8 to obtain a connected UAV network

Fig. 2. An illustration of the basic idea of the proposed algorithm.

Let $V^* = \{v_1^*, v_2^*, \ldots, v_K^*\}$. Recall that in the maximum connected coverage problem, the induced subgraph $G[V^*]$ by V^* is connected. Denote by T^* a spanning tree of $G[V^*]$, where T^* consists of the K nodes in V^* and $K-1$ edges. A Eulerian path P_{Euler} with $2K-3$ edges can be obtained by duplicating any $K-2$ edges in T^*, see Fig. 2(a) and Fig. 2(b).

For any positive integer L with $L \geq s$ (the optimal value of L will be calculated later in Section III-D), the Eulerian path P_{Euler} can be split into Δ subpaths (or segments) $P_1, P_2, \ldots, P_\Delta$, such that the number of nodes in each subpath P_j is equal to L with $1 \leq j \leq \Delta - 1$, and the number of nodes in the last subpath P_Δ is no greater than L, where $\Delta = \lceil \frac{2K-3+1}{L} \rceil = \lceil \frac{2K-2}{L} \rceil$, see Fig. 2(c). It can be seen that there is one subpath P_j among the Δ subpaths such that the number of users served by the UAVs in P_j is no less than $\frac{1}{\Delta}$ of the number of users served by the UAVs in tree T^*.

Consider any s nodes $v_1^*, v_2^*, \ldots, v_s^*$ in subpath P_j. It can be seen that P_j consists of the s nodes and $s+1$ segments $P_{j,1}, P_{j,2}, \ldots, P_{j,s+1}$, see Fig. 2(d). Denote by p_i the number of nodes in segment $P_{j,i}$ with $1 \leq i \leq s+1$. For example, Fig. 2(d) shows that $p_1 = 1$, $p_2 = p_3 = p_4 = 2$ with $s = 3$. Let D be the sum of nodes in the $s+1$ segments, i.e., $D = \sum_{i=1}^{s+1} p_i = L - s$, where there are L nodes in P_j.

The key of the proposed algorithm is that, we observe that the L nodes in subpath P_j form a feasible solution to a submodular maximization problem, subject to the constraints of $\rho(=2)$ matroids $\mathcal{M}_1$ and $\mathcal{M}_2$, where $\mathcal{M}_1$ and $\mathcal{M}_2$ will be introduced later in Section III-B and Section III-C, respectively. We then can obtain a $\frac{1}{\rho+1}$ $(= \frac{1}{3})$ approximate solution V' with L nodes, by applying the algorithm in [9], where the s nodes $v_1^*, v_2^*, \ldots, v_s^*$ must be contained in V'. Assume that $V' = \{v_1, v_2, \ldots, v_D, v_1^*, v_2^*, \ldots, v_s^*\}$, e.g., see Fig. 2(e) with $D = L - s = 10 - 3 = 7$. It can be seen that the number of users served by the UAVs deployed at locations in set V' is no less than $\frac{1}{3}$ of the number of users served by the UAVs in P_j, thus no less than $\frac{1}{3\Delta}$ of the number of users served by the UAVs in the optimal solution T^*, where $\Delta = \lceil \frac{2K-2}{L} \rceil$.

Notice that the induced subgraph by $G[V']$ may not be connected, e.g., see Fig. 2(e). We then place extra relaying nodes to obtain a connected UAV subnetwork, such that the nodes in V' are contained in the subnetwork. Fig. 2(f) shows that node v_8 is added as a relay between nodes v_1^* and v_2^*. Notice that the number of nodes in the connected subnetwork must be no greater than the number K of UAVs.

Algorithm outline: In the following, we first define matroids $\mathcal{M}_1$ and $\mathcal{M}_2$ in Sections III-B and Section III-C, respectively, where the definition of $\mathcal{M}_2$ depends on the value of L, and the $s+1$ numbers $p_1, p_2, \ldots, p_{s+1}$. We then calculate the optimal values of L and $p_1, p_2, \ldots, p_{s+1}$ in Section III-D. We further devise the approximation algorithm in Section III-E.

B. Definition of matroid $\mathcal{M}_1$

Let X be the set of K UAVs, i.e., $X = \{1, 2, \ldots, K\}$. Given the K UAVs in X and m hovering locations in V, we construct a set N of $K \times m$ elements, where N is the Cartesian product of sets X and V, i.e., $N = \{<k, v_j> \mid 1 \leq k \leq K,\ \forall v_j \in V\}$. It can be seen that an element $<k, v_j>$ in N indicates that the kth UAV with service capacity C_k will be deployed at location v_j.

Given any subset A of N, denote by $f(A)$ the number of users served by the UAVs in A, which can be calculated by invoking the algorithm in Section II-D. For example, assume that $A = \{< 1, v_1 >, < 2, v_2 >\}$, which means that UAVs 1 and 2 are deployed at locations v_1 and v_2, respectively. Following the study in [24], function $f(A)$ is **submodular**.

We define a set system $\mathcal{M}_1 = (N, \mathcal{I}_1)$ on set N, where $\mathcal{I}_1$ is a family of subsets of N such that, for each set $A \in \mathcal{I}_1$ ($A \subseteq N$), the number of pairs in A sharing the *same UAV* is no greater than one. In other words, each UAV cannot be placed at more than one location. For example, $A_1 = \{< 1, v_1 >\}$ is contained in $\mathcal{I}_1$, while $A_2 = \{< 1, v_1 >, \ < 1, v_2 >\}$ is not contained in $\mathcal{I}_1$ as UAV 1 cannot be deployed at the two different locations v_1 and v_2. The proof for the claim that $\mathcal{M}_1$ is a matroid is omitted, due to space limitation.

C. Definition of matroid $\mathcal{M}_2$

Consider any s nodes $v_1^*, v_2^*, \ldots, v_s^*$ in subpath P_j, where there are L nodes in P_j, see Fig. 2(d). Subpath P_j consists of the s nodes and $s+1$ segments $P_{j,1}, P_{j,2}, \ldots, P_{j,s+1}$. Recall that there are p_i nodes in segment $P_{j,i}$ with $1 \le i \le s+1$.

For any node v_l in P_j, denote by d_l the minimum number of hops in P_j between node v_l and nodes in the set $\{v_1^*, v_2^*, \ldots, v_s^*\}$. For example, Fig. 2(d) shows that the shortest hop between node v_5^* and nodes in set $\{v_1^*, v_2^*, v_3^*\}$ is only one. Let $h_{max} = \max\{p_1, p_{s+1}, \max_{i=2}^{s}\{\lceil \frac{p_i}{2} \rceil\}\}$, where h_{max} means the maximum shortest hops between nodes in P_j and nodes in the set $\{v_1^*, v_2^*, \ldots, v_s^*\}$. For example, in Fig. 2(d), we know that $p_1 = 1$, $p_2 = p_3 = 2$, and $p_4 = 2$ with $s = 3$. Then, $h_{max} = 2$.

For each integer h with $0 \le h \le h_{max}$, denote by Q_h the number of nodes in P_j that are at least h hops away from the nodes in set $\{v_1^*, v_2^*, \ldots, v_s^*\}$. For example, Fig. 2(d) shows that $Q_0 = 10$ since all the ten nodes in P_j are at least zero hop away from the nodes in $\{v_1^*, v_2^*, v_3^*\}$, $Q_1 = 7$ since the seven nodes $v_4^*, v_5^*, \ldots, v_{10}^*$ are at least one hop away from the nodes in $\{v_1^*, v_2^*, v_3^*\}$, and $Q_2 = 1$ since only node v_{10}^* is at least two hops away from the nodes in $\{v_1^*, v_2^*, v_3^*\}$.

We now formally define the value of Q_h with $0 \le h \le h_{max}$. Initially, $Q_0 = L$. When $1 \le h \le h_{max}$, we then have

$$\begin{aligned} Q_h \quad &= \quad \max\{p_1 - (h-1), 0\} + \sum_{i=2}^{s} \max\{p_i - 2(h-1), 0\} \\ &+ \max\{p_{s+1} - (h-1), \ 0\}, \ 1 \le h \le h_{max}. \quad (1) \end{aligned}$$

Considering the L nodes in P_j, we define a family $\mathcal{I}_2$ of subsets of V, such that for any subset V' in $\mathcal{I}_2$, the shortest hop between any node in V' and the nodes in $\{v_1^*, v_2^*, \ldots, v_s^*\}$ is no more than h_{max}, and there are no more than Q_h nodes in V' that are at least h hops away from the nodes in set $\{v_1^*, v_2^*, \ldots, v_s^*\}$, where $0 \le h \le h_{max}$. The proof that $\mathcal{M}_2$ is a matroid is omitted, due to space limitation.

D. Calculate the optimal values of L and $p_1, p_2, \ldots, p_{s+1}$

Consider any feasible solution V' in matroid $\mathcal{M}_2$, the induced subgraph by $G[V']$ may not be connected, see Fig. 2(e). We then need to place extra relaying nodes to make it become a connected UAV subnetwork, such that nodes in V' are contained in the subnetwork. The number of deployed UAVs in the connected subnetwork is no greater than

$$\begin{aligned} g(L, p_1, p_2, \ldots, p_{s+1}) = s + \sum_{i=2}^{s} p_i + \frac{p_1(p_1+1)}{2} + \\ \sum_{i=2}^{s} \frac{p_i^2 + 2p_i + (p_i \bmod 2)}{4} + \frac{p_{s+1}(p_{s+1}+1)}{2}, \quad (2) \end{aligned}$$

and its proof is contained in Lemma 2 of Section III-F.

To serve more users, the value of L should be as large as possible. However, the number $g(L, p_1, p_2, \ldots, p_{s+1})$ of deployed UAVs should be no greater than the number K of available UAVs.

In the following, we calculate the optimal values of L and $p_1, p_2, \ldots, p_{s+1}$. Denote by L_{max} the maximum value of L, and denote by $p_1^*, p_2^*, \ldots, p_{s+1}^*$ the optimal numbers of $p_1, p_2, \ldots, p_{s+1}$, respectively, subject to the constraint that $g(L_{max}, p_1^*, p_2^*, \ldots, p_{s+1}^*)$ is no greater than K.

We calculate the maximum value of L_{max} by binary search. It can be seen that $s \le L_{max} \le K$. Given a guess L of L_{max}, following Eq. (2), the number $g(L, p_1, p_2, \ldots, p_{s+1})$ of deployed UAVs depends on the values of L, and $p_1, p_2, \ldots, p_{s+1}$. Denote by $p_1^L, p_2^L, \ldots, p_{s+1}^L$ the optimal values of $p_1, p_2, \ldots, p_{s+1}$, respectively, for the fixed L, such that the number $g(L, p_1, p_2, \ldots, p_{s+1})$ of deployed UAVs is minimized, where $\sum_{i=1}^{s+1} p_i^L = L - s$, $0 \le p_i^L \le L - s$ with $1 \le i \le s+1$. We calculate the values of $p_1^L, p_2^L, \ldots, p_{s+1}^L$ as follows.

Given the value of L, we later show that, when the number $g(L, p_1^L, p_2^L, \ldots, p_{s+1}^L)$ of deployed UAVs is minimized, the difference of p_1^L and p_{s+1}^L is no greater than one, i.e., $|p_1^L - p_{s+1}^L| \le 1$, and the difference of p_i^L and $p_{i'}^L$ is also no greater than one, i.e., $|p_i^L - p_{i'}^L| \le 1$ with $2 \le i, i' \le s$. Without loss of generality, we assume that $p_2^L \ge p_3^L \ge \cdots \ge p_s^L$. Then, $p_2^L - p_s^L \le 1$. Assume that there are j integers among the $s-2$ integers $p_2^L, p_3^L, \ldots, p_{s-1}^L$ so that they are larger than p_s^L by one. Let $p = p_s^L$. Then, $p_2^L = p_3^L = \cdots = p_{j+1}^L = p+1$ while $p_{j+2}^L = p_{j+3}^L = \cdots = p_s^L = p$. Since the difference of p_1^L and p_{s+1}^L is no greater than one, let $p_1^L = \lfloor \frac{L-s-\sum_{i=2}^{s} p_i^L}{2} \rfloor = \lfloor \frac{L-s-(s-1)p-j}{2} \rfloor$, and $p_{s+1}^L = \lceil \frac{L-s-(s-1)p-j}{2} \rceil$.

It can be seen that the value of p is in the interval $[0, L-s]$ and the value of j is in the interval $[0, s-2]$. Then, we can calculate the minimum number $g(L, p_1^L, p_2^L, \ldots, p_{s+1}^L)$ of deployed UAVs and the values of $p_1^L, p_2^L, \ldots, p_{s+1}^L$, by considering all combinations of p and j.

The algorithm for calculating L_{max} and $p_1^*, p_2^*, \ldots, p_{s+1}^*$ is presented in `Algorithm` 1. It can be seen that the time for finding the optimal value L_{max} and the optimal numbers $p_1^*, p_2^*, \ldots, p_{s+1}^*$ is only $O(s^2 K \log K)$.

E. Approximation algorithm

Given a positive integer s, the proposed algorithm first calculates the optimal values of L_{max} and $p_1^*, p_2^*, \ldots, p_{s+1}^*$, by invoking `Algorithm` 1 in Section III-D.

For any subset V_j^* of V with s nodes, the proposed algorithm finds a connected subgraph G_j of G, where $1 \le j \le \binom{m}{s}$,

Algorithm 1 Calculate the maximum value of L_{max} and the optimal numbers $p_1^*, p_2^*, \ldots, p_{s+1}^*$

Input: The number K of UAVs and the value of s
Output: The values of L_{max} and $p_1^*, p_2^*, \ldots, p_{s+1}^*$
Let $L_{max} \leftarrow s$; /* an initial value of L_{max} */
Let $L_{lb} \leftarrow s$, $L_{ub} \leftarrow K$; /* L_{lb} and L_{ub} are lower and upper bounds on L_{max}, respectively */
while $L_{lb} + 1 < L_{ub}$ **do**
  Let $L \leftarrow \lfloor \frac{L_{lb}+L_{ub}}{2} \rfloor$; /* L is a guess of L_{max} */
  Let $g(L, p_1^L, p_2^L, \ldots, p_{s+1}^L) \leftarrow +\infty$;
  for $0 \le p \le L - s$, $0 \le j \le s-2$ **do**
    if $(s-1)p + j \le L - s$ **then**
      /* Ensure that the sum of $p_2, p_3, \ldots, p_s$, i.e., $(s-1)p+j$, is no greater than $L - s$; */
      Set $p_2^L = p_3^L = \cdots = p_{j+1}^L = p+1$, $p_{j+2}^L = p_{j+3}^L = \cdots = p_s^L = p$, $p_1^L = \lfloor \frac{L-s-(s-1)p-j}{2} \rfloor$, and $p_{s+1}^L = \lceil \frac{L-s-(s-1)p-j}{2} \rceil$;
      Calculate the number $g(L, p_1, p_2, \ldots, p_{s+1})$ of deployed UAVs by Eq. (2);
      if $g(L, p_1, p_2, \ldots, p_{s+1}) < g(L, p_1^L, p_2^L, \ldots, p_{s+1}^L)$ **then**
        Let $p_i^L \leftarrow p_i$ with $1 \le i \le s+1$;
      end if
    end if
  end for
  if $g(L, p_1^L, p_2^L, \ldots, p_{s+1}^L) \le K$ **then**
    Let $L_{lb} \leftarrow L$; /* L becomes the updated lower bound on L_{max} */
    Let $L_{max} \leftarrow L$ and $p_i^* \leftarrow p_i^L$ with $1 \le i \le s+1$;
  else
    Let $L_{ub} \leftarrow L$; /* L becomes the updated upper bound on L_{max} */
  end if
end while
return the values of L_{max} and $p_1^*, p_2^*, \ldots, p_{s+1}^*$.

$m = |V|$ and $\binom{m}{s}$ is the number of different ways of choosing s nodes from set V with m nodes. The solution to the problem then is the subgraph G_{j^*} among the $\binom{m}{s}$ subgraphs such that the number of served users is maximized and the number of nodes in the subgraph is no greater than K, where $1 \le j^* \le \binom{m}{s}$. In the following, we show how to find a connected subgraph G_j.

For any subset V_j^* of V with s nodes in V, let $V_j^* = \{v_1^*, v_2^*, \ldots, v_s^*\}$. We define a submodular maximization problem, subject to the constraints of $\rho(=2)$ matroids $\mathcal{M}_1$ and $\mathcal{M}_2$, where $\mathcal{M}_1$ was defined in Section III-B, while $\mathcal{M}_2$ was defined in Section III-C by replacing L with L_{max} and replacing p_i with p_i^* $(1 \le i \le s+1)$.

We find an approximate solution V_j' with no more than L_{max} nodes to the submodular maximization problem under the constraints of matroids $\mathcal{M}_1$ and $\mathcal{M}_2$ as follows.

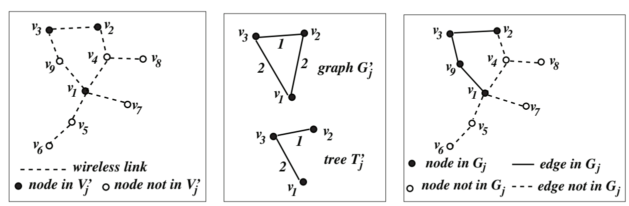
For the sake of convenience, we assume that $C_1 \ge C_2 \ge \cdots \ge C_K$, where C_k is the service capacity of the kth UAV with $1 \le k \le K$. The proposed algorithm consists of L_{max} iterations, and in the kth iteration we deploy the kth UAV with service capacity C_k at a hovering location, where $L_{max} \le K$.

Assume that before the kth iteration, UAVs $1, 2, \ldots, k-1$ have already been deployed at hovering locations $v_1, v_2, \ldots, v_{k-1}$, respectively, i.e., $V_j' = \{v_1, v_2, \ldots, v_{k-1}\}$. Also, denote by n_{k-1} the number of users served by the deployed $k-1$ UAVs, which can be calculated by invoking the algorithm in Section II-D.

In the kth iteration, we deploy the kth UAV at a hovering location v_k such that the increased number of users served by the UAV in maximized. Specifically, denote by $V_{feasible}^k$ the set of nodes in $V \setminus V_j'$ such that the set $\{v_l\} \cup V_j'$ is contained in matroid $\mathcal{M}_2$, where v_l is in $V \setminus V_j'$, i.e., $V_{feasible}^k = \{v_l \mid (\{v_l\} \cup V_j') \in \mathcal{M}_2,\ v_l \in V \setminus V_j'\}$. For each hovering location $v_l \in V_{feasible}^k$ that has not been deployed a UAV in the first $k-1$ iterations, we calculate the number $n_{k,l}$ of users served the k UAVs $1, 2, \ldots, k$, assuming that the kth UAV is deployed at location v_l. We then identify the location v_k in $V_{feasible}^k$ such that the new increased number of users served is maximized, i.e., $v_k = \arg\max_{v_l \in V_{feasible}^k} \{n_{k,l} - n_{k-1}\}$, where n_{k-1} is the number of users served by the deployed first $k-1$ UAVs in the first $k-1$ iterations. The procedure continues until the hovering locations for UAVs $1, 2, \ldots, L_{max}$ are found, where $L_{max} \le K$. The set of hovering locations for the L_{max} UAVs then is $V_j' = \{v_1, v_2, \ldots, v_{L_{max}}\}$.

It must be mentioned that the s nodes $v_1^*, v_2^*, \ldots, v_s^*$ in V_j^* must be contained in V_j', as only nodes in V_j^* are zero hop away from V_j^* itself, and the number of nodes in V_j' that are zero hop away from V_j^* is $Q_0 - Q_1 = s$.

Recall that the kth UAV is deployed at hovering location v_k with $1 \le k \le L_{max}$ and $V_j' = \{v_1, v_2, \ldots, v_{L_{max}}\}$. Notice that the induced subgraph $G[V_j']$ by V_j' may not be connected, see Fig. 3(a). We construct a connected subgraph G_j of G such that the nodes in V_j' are contained in G_j as follows.



(a) nodes in set V_j' with $L_{max} = 3$

(b) graph $G_j' = (V_j', E_j')$ and an MST T_j' in G_j'

(c) graph G_j, where V_j is the set of nodes in G_j

Fig. 3. An illustration of constructing a connected subgraph in the approximation algorithm.

A weighted graph $G_j' = (V_j', E_j')$ is first constructed from set V_j', where there is an edge (v_k, v_l) in E_j' between any two nodes v_k and v_l in V_j', and its edge weight $w(v_k, v_l)$ is the minimum number of hops in G between them. A minimum spanning tree (MST) T_j' in G_j' is then found, see Fig. 3(b). Denote by q_j' the number of nodes in T_j'. For each edge (v_k, v_l) in tree T_j', there is a corresponding shortest path $P_{k,l}$ in graph G between nodes v_k and v_l.

A *connected subgraph* G_j of G can be obtained from T_j', which is the union of the $(q_j' - 1)$ shortest paths in G, i.e., $G_j = \{P_{k,l} \mid (v_k, v_l) \in T_j'\}$, see Fig. 3(c). Denote by V_j the set of nodes in G_j. Also, let $q_j = |V_j|$. If the number q_j of nodes in G_j is greater than K, then G_j is not a feasible solution to the problem. Otherwise ($q_j \le K$), we deploy UAVs at the location node in G_j as follows.

Following the construction of G_j, it can be seen that the nodes in V'_j are contained in G_j (i.e.,V'_j is a subset of V_j), where $V'_j = \{v_1, v_2, \dots, v_{L_{max}}\}$ and the kth UAV with service capacity C_k has already been deployed at location node v_k with $1 \le k \le L_{max}$. We deploy UAVs $L_{max}+1, L_{max}+2, \dots, q_j$ at nodes in $V_j \setminus V'_j$ in an arbitrary way, e.g., in a greedy way.

The algorithm for the problem is presented in `Algorithm` 2.

Algorithm 2 Approximation algorithm for the maximum connected coverage problem in a disaster area (`approAlg`)

Input: A set U of users, a set V of candidate hovering locations, and K UAVs with service capacities $C_1, C_2, \dots, C_K$, respectively
Output: A solution to the maximum connected coverage problem

Calculates the optimal values of L_{max} and $p_1^*, p_2^*, \dots, p_{s+1}^*$, by invoking the algorithm in Section III-D;
Let $Q_0 \leftarrow L_{max}$ and define Q_h by Eq. (1), $1 \le h \le h_{max}$;
Let $n^* \leftarrow 0$; /* the maximum number of served users */
for each subset V_j^* of V with s nodes **do**
    Sort the K UAVs by their service capacities in decreasing order, and assume that $C_1 \ge C_2 \ge \dots \ge C_K$;
    Let $V'_j \leftarrow \emptyset$; /* no UAVs are deployed initially */
    Let $n_0 \leftarrow 0$; /* no users are served initially */
    for $1 \le k \le L_{max}$ **do**
        Find the set $V^k_{feasible}$ of feasible location nodes for deploying the kth UAV, where $V^k_{feasible} \leftarrow \{v_l \mid (\{v_l\} \cup V'_j) \in \mathcal{M}_2,\ v_l \in V \setminus V'_j\}$;
        Deploy the kth UAV at a location node v_k in $V^k_{feasible}$ such that the increased number of users served by the UAV in maximized, i.e., $v_k \leftarrow \arg\max_{v_l \in V^k_{feasible}} \{n_{k,l} - n_{k-1}\}$;
        Let $V'_j \leftarrow V'_j \cup \{v_k\}$;
    end for
    Construct a graph $G'_j = (V'_j, E'_j)$, where there is an edge $(v_k, v_l) \in E'_j$ between any two nodes v_k and v_l in V'_j, and its edge weight $w(v_k, v_l)$ is the minimum number of hops between v_k and v_l in G;
    Find a Minimum Spanning Tree (MST) T'_j in G'_j;
    Construct a connected subgraph G_j of G, where $G_j = \{P_{k,l} \mid (v_k, v_l) \in T'_j\}$ and $P_{k,l}$ is the shortest path in G between nodes v_k and v_l. Let V_j be the set of nodes in G_j and $q_j = |V_j|$;
    if $q_j \le K$ **then**
        Deploy UAVs $L_{max}+1, L_{max}+2, \dots, q_j$ at location nodes in $V_j \setminus V'_j$ in an arbitrary way;
        Calculate the number n_j^* of users served by the deployed UAVs in G_j;
        if $n_j^* > n^*$ **then**
            /* Find a better way of deploying UAVs */
            Let $n^* \leftarrow n_j^*$ and $j^* \leftarrow j$;
        end if
    end if
end for
Assign users in U to the UAVs deployed in subgraph G_{j^*}, by invoking the algorithm in Section II-D;
return the deployment of UAVs in G_{j^*} and the assignment of users in U.

F. An upper bound on the number of nodes in connected subgraph G_j

Lemma 2: Given a subset $V_j^* = \{v_1^*, v_2^*, \dots, v_s^*\}$ of V with s nodes in V, $s+1$ nonnegative integers $p_1, p_2, \dots, p_{s+1}$, and a subset V' of V with no greater than L nodes such that the nodes in V_j^* are contained in V' (i.e., $V_j^* \subseteq V'$) and V' is contained in matroid $\mathcal{M}_2 = (V, \mathcal{I}_2)$ (i.e., $V' \in \mathcal{I}_2$), assume that there are no more than p_i intermediate nodes in the shortest path between nodes v_{i-1}^* and v_i^* in G with $2 \le i \le s$. Then, a connected subgraph G_j of G can be found such that the nodes in V' are contained in G_j and the number of nodes in G_j is no greater than $g(L, p_1, p_2, \dots, p_{s+1}) = s + \sum_{i=2}^{s} p_i + \frac{p_1(p_1+1)}{2} + \sum_{i=2}^{s} \frac{p_i^2 + 2p_i + (p_i \bmod 2)}{4} + \frac{p_{s+1}(p_{s+1}+1)}{2}$.

Proof: A connected subgraph G_j of G is constructed as follows. Since the s nodes in V_j^* are contained in V', the s nodes can be connected by adding nodes in the shortest paths between nodes v_{i-1}^* and v_i^* in G with $2 \le i \le s$. It can be seen that the number of added nodes is no greater than $\sum_{i=2}^{s} p_i$. On the other hand, for each node $v_l \in V' \setminus V_j^*$, a shortest path from v_l to a node $v_i^* \in V_j^*$ in G is added, where v_i^* is the nearest node among the nodes in V_j^* to v_l. The number of nodes in G_j is upper bounded as follows. Assume that there are k_h nodes in V' that are exactly h hops away from nodes in V_j^* with $0 \le h \le h_{max}$. Since V' is in matroid $\mathcal{M}_2$, we know that there are no more than Q_h nodes in V' that are at least h hops away from the nodes in V_j^*. Then,

$$\sum_{j=h}^{h_{max}} k_j \le Q_h, \quad 0 \le h \le h_{max}. \tag{3}$$

Consider a node $v_l \in V' \setminus V_j^*$ such that v_l is exactly h hops away from nodes in V_j^*, assume that v_i^* is the nearest node among the nodes in V_j^* to v_l. It can be seen that there are h nodes in the shortest path between v_l to v_i^* except v_i^*. Then, the number of nodes in G_j is no greater than

$$\begin{aligned}
& s + \sum_{i=2}^{s} p_i + \sum_{h=1}^{h_{max}} k_h \cdot h \\
= \quad & s + \sum_{i=2}^{s} p_i + \sum_{h=1}^{h_{max}} \Big(\sum_{j=h}^{h_{max}} k_j\Big) \\
\le \quad & s + \sum_{i=2}^{s} p_i + \sum_{h=1}^{h_{max}} Q_h, \quad \text{by Ineq. (3)} \\
= \quad & s + \sum_{i=2}^{s} p_i + \sum_{h=1}^{h_{max}} \max\{p_1 - (h-1),\ 0\} + \\
& \sum_{h=1}^{h_{max}} \sum_{i=2}^{s} \max\{p_i - 2(h-1), 0\} + \\
& \sum_{h=1}^{h_{max}} \max\{p_{s+1} - (h-1),\ 0\}, \ \text{by Eq. (1)} \\
= \quad & s + \sum_{i=2}^{s} p_i + \sum_{h=1}^{p_1} (p_1 - (h-1)) + \\
& \sum_{i=2}^{s} \sum_{h=1}^{h_{max}} \max\{p_i - 2(h-1), 0\} + \sum_{h=1}^{p_{s+1}} (p_{s+1} - (h-1)) \\
= \quad & s + \sum_{i=2}^{s} p_i + \frac{p_1(p_1+1)}{2} + \\
& \sum_{i=2}^{s} \sum_{h=1}^{h_{max}} \max\{p_i - 2(h-1), 0\} + \frac{p_{s+1}(p_{s+1}+1)}{2}.
\end{aligned} \tag{4}$$

It can be easily verified that $\sum_{h=1}^{h_{max}} \max\{p_i - 2(h-1), 0\} = \frac{p_i^2 + 2p_i + (p_i \bmod 2)}{4}$ with $2 \le i \le s$, by considering two cases: (i) p_i is even; and (ii) p_i is odd. The lemma then follows. ■

G. The analysis of the approximation ratio

Theorem 1: Given a UAV network $G = (U \cup V, E)$ and K UAVs with service capacities $C_1, C_2, \ldots, C_K$, respectively, and a positive integer s, there is an approximation algorithm, `Algorithm` 2, for the maximum connected coverage problem with a time complexity of $O(K^2 n^2 m^{s+1})$, and the approximation ratio of the algorithm is $\frac{1}{3\lceil \frac{2K-2}{L_1} \rceil} = O(\sqrt{\frac{s}{K}})$ and $L_1 = \lfloor \sqrt{4sK + 4s^2 - 8.5s} \rfloor - 2s + 2$, where n is the number of users in U ($n = |U|$) and m is the number of candidate hovering locations in V ($m = |V|$).

Proof: The proof is omitted, due to space limitation. ■

IV. Performance Evaluation

A. Experimental environment

Consider a disaster zone with a 3×3 km^2 square [45], in which 1,000 to 3,000 users are located. The user density follows a fat-tailed distribution, i.e., many users are located at a small portion of places while a few users are sparely located at many other places in the disaster zone [30]. The number K of UAVs varies from 2 to 20. The service capacity C_k of the kth UAV is randomly chosen from an interval of $[C_{min}, C_{max}]$, where $C_{min} = 50$ users, $C_{max} = 300$ users [37], and $1 \le k \le K$. Every UAV hovers at an altitude $H_{uav} = 300$ m to provide communication services to ground users [2]. The UAV communication range is $R_{uav} = 600$ m, while the user communication range is $R_{user}^k = 500$ m [45].

In addition to the proposed algorithm `approAlg`, we consider four benchmark algorithms. (i) Algorithm `MCS` [14] finds a $\frac{1-1/e}{5(\sqrt{K}+1)}$-approximate solution to cover as many users as possible by deploying K UAVs. (ii) Algorithm `MotionCtrl` [45] proposes a motion control solution to cover the maximum number of users by deploying a connected UAV network that consists of K UAVs. (iii) Algorithm `GreedyAssign` [13] first assigns each candidate hovering location a profit in a greedy way, then deploys a network consisting of K UAVs, such that the sum of profits in the network is maximized. (iv) Algorithm `maxThroughput` [37] finds a $\frac{1-1/e}{\sqrt{K}}$-approximation solution to a problem of placing K homogenous UAVs, so that the network throughput is maximized. All experiments were run on a server with an Intel(R) Core(TM) i5-10400 CPU (2.9 GHz) and 16 GB RAM.

B. Algorithm Performance

We first study the performance of different algorithms by increasing the number K of UAVs from 2 to 20, when there are $n = 3,000$ users and the parameter s in the proposed algorithm `approAlg` is set as 3. Fig. 4 shows that the number of served users by each algorithm increases with more deployed UAVs. In addition, the number of served users by algorithm `approAlg` is up to 22% larger than those by the other four algorithms when $K = 20$ UAVs. For examples, the numbers of

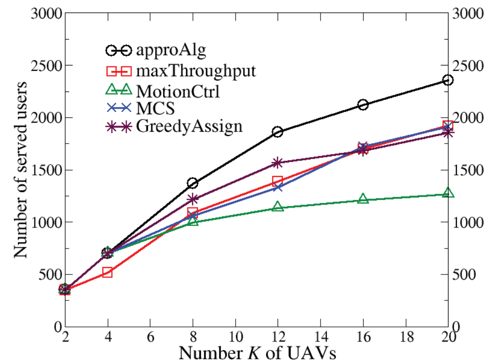


Fig. 4. The performance of different algorithms by increasing the number K of UAVs from 2 to 20, when there are $n = 3,000$ users and $s = 3$.

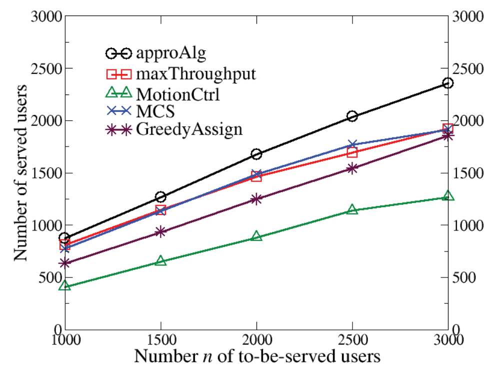
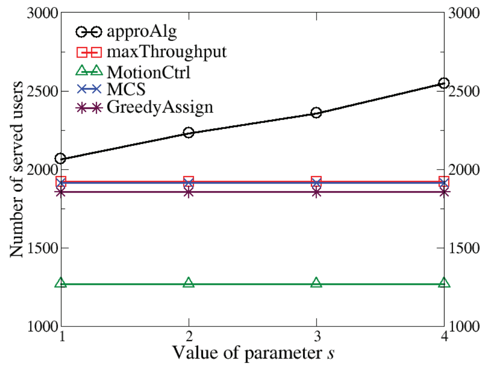


Fig. 5. The performance of different algorithms by increasing the number n of to-be-served users from 1,000 to 3,000, when $K = 20$ UAVs and $s = 3$.

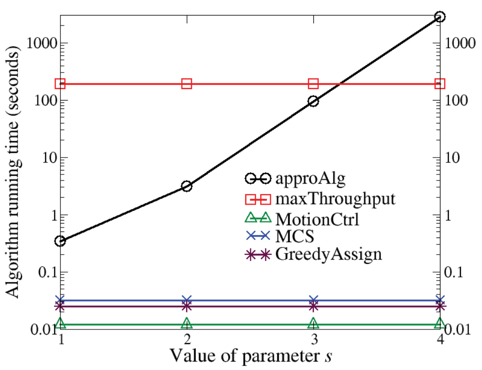
users served by algorithms `approAlg`, `maxThroughput`, `MotionCtrl`, `MCS`, and `GreedyAssign`, are 2,356, 1,920, 1,269, 1,913, 1,855, respectively, when $K = 20$.

We then investigate the algorithm performance by varying the number n of to-be-served users from 1,000 to 3,000, when there are $K (= 20)$ UAVs and $s = 3$ in algorithm `approAlg`. Fig. 5 shows that the number of served users by algorithm `approAlg` is about from 7% to 22% larger than those by algorithms `maxThroughput`, `MotionCtrl`, `MCS`, and `GreedyAssign`, when n increases from 1,000 to 3,000. Fig. 5 also demonstrates that more users are served by each of the five algorithms when there are more to-be-served users in the disaster area.

We finally study the tradeoff between the quality of the solution delivered by the proposed algorithm `approAlg` and its running time, by increasing the parameter s from 1 to 4. Fig. 6(a) shows that the number of served users by algorithm `approAlg` significantly increases with the growth of parameter s, and the number is from 7% to 33% larger than those by the other four algorithms when s grows from 1 to 4, where the approximation ratio of algorithm `approAlg` is $O(\sqrt{\frac{s}{K}})$ (see Theorem 1). Fig. 6(b) plots that the running time of algorithm `approAlg` also significantly increases with the growth of s, since its time complexity is $O(K^2 n^2 m^{s+1})$. Notice that in the application of deploying a UAV communication network to people trapped in a disaster area, we need the best tradeoff between the quality of the delivered solution (i.e., the number of served users) and the algorithm running time. It can be seen



(a) Number of served users by different algorithms



(b) Running time of different algorithms

Fig. 6. The performance of different algorithms by increasing the parameter s from 1 to 4, when there are $n(=3,000)$ users and $K(=20)$ UAVs.

from Fig. 6 that the running times of algorithm `approAlg` with $s = 1, 2, 3$ are acceptable, which are 0.34, 3.1, 95 seconds, respectively, while its running time with $s = 4$ usually are unacceptable, which is as high as about 47 minutes.

V. Related Work

The deployment of UAV networks recently has gained lots of attentions in public communications. Most existing studies considered the deployment of *homogenous UAVs*. For example, Zhao *et al.* [45] studied a problem of deploying a connected UAV network that consist of K UAVs to serve as many as users as possible, and they proposed a motion control algorithm for their problem. Liu *et al.* [19] investigated a similar problem in [45], and proposed an algorithm based on the deep reinforcement learning technique. Yang *et al.* [39] considered the problem of the flying trajectory planning of multiple UAVs, so as to provide emergent communication services to ground people. Shi *et al.* [29] considered the problem of finding UAV flying trajectories during a given period, in order to minimize the average pathloss between UAVs and users. Fahim *et al.* [8] studied the deployment of a single UAV to serve as many ground devices as possible. Xu *et al.* [37] recently studied a problem of deploying a connected UAV network that consists of K homogenous UAVs in the air for monitoring a disaster area, such that the sum of data rates of all users is maximized, subject to the constraint that the number of users served by each UAV is no greater than its service capacity. They proposed a $\frac{1-1/e}{\sqrt{K}}$-approximation algorithm, where e is the base of the natural logarithm.

There are several studies on finding a connected subgraph with no more K nodes in a graph such that the value of a given submodular function over the K found nodes is maximized. For instance, Kuo *et al.* [14] studied a problem of placing a connected wireless network that consists of K wireless routers such that the number of users served is maximized, and proposed a $\frac{1-1/e}{5(\sqrt{K}+1)}$-approximation algorithm. Khuller *et al.* [13] investigated a problem of finding a connected subgraph with K nodes in a graph, such that the number of neighboring nodes of the found K nodes is maximized. They proposed a $\frac{1-1/e}{12}$-approximation algorithm. However, the proposed algorithm is not applicable to the problem in this paper. Huang *et al.* [10] studied a problem of placing a connected sensor network that consists of K sensors, such that the number of targets monitored by the placed sensors is maximized, by designing a $\frac{1-1/e}{8(\lceil 2\sqrt{2}\theta\rceil+1)^2}$-approximation algorithm, where $0 < \theta \leq 1$. Yu *et al.* [41], [42] recently proposed an improved algorithm and the approximation ratio is improved to $\frac{1-1/e}{8(\lceil \frac{4}{\sqrt{3}}\theta\rceil+1)^2}$. It can be seen that both the approximation ratios $\frac{1-1/e}{8(\lceil 2\sqrt{2}\theta\rceil+1)^2}$ [10] and $\frac{1-1/e}{8(\lceil \frac{4}{\sqrt{3}}\theta\rceil+1)^2}$ [41], [42] are between $\frac{1-1/e}{128}$ and $\frac{1-1/e}{32}$, as $0 < \theta \leq 1$. On the other hand, notice that there usually are tens or hundreds of UAVs to the deployed. In this case, the approximation ratio $\frac{1-1/e}{\sqrt{K}}$ in [37] usually is larger than those in [10] and [41], [42], i.e., $\frac{1-1/e}{\sqrt{K}} \geq \frac{1-1/e}{32}$ when $K \leq 1,024$. However, the solutions in the aforementioned studies are inapplicable to the heterogenous UAVs deployment.

VI. Conclusions

Unlike most existing studies that considered homogenous UAVs, in this paper we investigated the deployment of heterogeneous UAVs to form a connected network, where different UAVs have different user service capacities. We studied a connected UAV network deployment problem with K heterogeneous UAVs, such that the number of users served by the deployed UAVs is maximized, subject to the constraint that the number of users served by each UAV is no greater than its service capacity. We then proposed an $O(\sqrt{\frac{s}{K}})$-approximation algorithm for the problem, where s is given positive integer, e.g., $s = 3$. We finally evaluated the performance of the approximation algorithm. Experimental results showed that the number of users served by the approximation algorithm is up to 22% larger than those by existing algorithms.

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