Enrolled Actuaries Basic Examination

EA-1

Date: Thursday, May 12, 2022

INSTRUCTIONS TO CANDIDATES

1. Special conditions generally applicable to all questions on this examination are found on the next page.

2. On this examination the symbol "a" will be used to represent an annuity. On this examination the symbol "ℓ_x" will be used to represent the number of lives at age x.

3. This examination consists of 33 multiple-choice questions worth a total of 100 points. The point value for each question is shown in parentheses at the beginning of the question.

4. Your score will be based on the point values of questions that you answer correctly. No credit will be given for omitted answers and no credit will be lost for wrong answers; hence, you should answer all questions even those for which you have to guess.

5. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.

6. While every attempt is made to avoid defective questions, sometimes they do occur. If you believe a question is defective, the supervisor or proctor cannot give you any guidance beyond the instructions on the computer screen.

7. Use the scratch paper booklets provided by Prometric for your scratch work. Extra scratch paper booklets are available if you run out of scratch paper in the booklet provided to you.

Answer Key EA-1 Spring 2022
February 28, 2022

Question	Answer		Question	Answer
1	A		18	E
2	E		19	E
3	D		20	B
4	C		21	B
5	C		22	E
6	E		23	D
7	D		24	C
8	D		25	C
9	A		26	C
10	E		27	C
11	A		28	D
12	E		29	C
13	D		30	B
14	D		31	B
15	C		32	A
16	C		33	E
17	D			

CONDITIONS GENERALLY APPLICABLE TO ALL EA-1 EXAMINATION QUESTIONS

If applicable, the following conditions should be considered a part of the data for each question, unless otherwise stated or implied.

(1)   The normal retirement age is 65.

(2)   Retirement pensions commence at normal retirement age and are paid monthly for life at the beginning of each month.

(3)   There are no pre-retirement death or disability benefits.

(4)   Actuarial equivalence is based on the mortality table and interest rate assumed for funding purposes.

(5)   Interest rates that are compounded more frequently than annually are expressed as nominal rates.

(6)   Where multiple lives are involved, future lifetimes are assumed to be independent of each other.

(7)   The term "gross single premium" is equivalent to "contract single premium;" the term "net single premium" is equivalent to "single benefit premium;" the term "gross annual premium" is equivalent to "annual contract premium;" the term "net annual premium" is equivalent to "annual benefit premium."

(8)   There are no policy loans in effect.

(9)   For a bond, the face amount and the redemption value are the same.

(10)   Interest rate equals yield rate.

(11)   The term "duration" means "Macaulay duration".

If applicable, the preceding conditions should be considered a part of the data for each question, unless otherwise stated or implied.

Data for Question 1 (3 points)

A table of $s_{\overline{n}|}$ at 5.0% is to be constructed on a computer using the following recursive formula:

$$s_{\overline{1}|} = 1$$
$$s_{\overline{n+1}|} = 1.05 s_{\overline{n}|} + 1 \qquad n \geq 1$$

However, an error occurred and the following incorrect recursive formula was used:

$$s_{\overline{1}|} = 1$$
$$s_{\overline{n+1}|} = 1.05 s_{\overline{n}|} + 0.1 \qquad n \geq 1$$

X = the value produced for $s_{\overline{20}|}$ using the incorrect recursive formula.

Question 1

In what range is X?

(A)   Less than 5.60

(B)   5.60 but less than 5.70

(C)   5.70 but less than 5.80

(D)   5.80 but less than 5.90

(E)   5.90 or more

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Data for Question 2 (3 points)

Selected annuity values:

$$\ddot{a}_{\overline{n+2|}} = 14.030$$
$$\ddot{s}_{\overline{n|}} = 52.344$$

$X=$ the effective annual interest rate.

Question 2

In what range is X?

(A)   Less than 5.00%

(B)   5.00% but less than 5.25%

(C)   5.25% but less than 5.50%

(D)   5.50% but less than 5.75%

(E)   5.75% or more

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Data for Question 3 (3 points)

Terms of a perpetuity:

Frequency of payments   Annual, payable at the end of each year
Payment amounts $1 at the end of years 1, 4, 7, …
                         $2 at the end of years 2, 5, 8, …
                         $3 at the end of years 3, 6, 9, …

Interest rate: 5.0% per year, compounded semiannually

X = the present value of the perpetuity.

Question 3

In what range is X?

(A)   Less than $25

(B) $25 but less than $30

(C) $30 but less than $35

(D) $35 but less than $40

(E) $40 or more

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Data for Question 4 (3 points)

Provisions of an annuity-immediate:

        Periodic payment $150
        Payment frequency   Quarterly
        Term   5 years

Interest rate: 8.00% per year, compounded semi-annually

$X=$ the present value of this annuity.

Question 4

In what range is X?

(A)   Less than $2,405

(B) $2,405 but less than $2,455

(C) $2,455 but less than $2,505

(D) $2,505 but less than $2,555

(E) $2,555 or more

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Data for Question 5 (4 points)

Terms of a loan:

Term   36 months
Repayments   Level monthly, payable at the end of each month
Interest rate 18.0% per year, compounded monthly

X = the ratio of cumulative interest repaid to cumulative principal repaid immediately following the 24th payment.

Question 5

In what range is X?

(A)   Less than 0.40

(B)   0.40 but less than 0.42

(C)   0.42 but less than 0.44

(D)   0.44 but less than 0.46

(E)   0.46 or more

Data for Question 6 (3 points)

Terms of a loan:

Amount borrowed	$10,000
Repayments	Level annual, payable at the end of each year
Interest rate	4.0% per year, compounded annually

The interest portion of the fourth payment is $225.

X = the principal portion of the fourth payment.

Question 6

In what range is X?

(A) Less than $1,225

(B) $1,225 but less than $1,325

(C) $1,325 but less than $1,425

(D) $1,425 but less than $1,525

(E) $1,525 or more

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Data for Question 7 (3 points)

Terms of a 10-year bond:

        Face amount $1,000
        Redemption amount $1,000
        Coupon rate   4.0% per year, payable semi-annually
        Yield rate   5.0% per year, compounded semi-annually

X = the total investment return to the purchaser over the lifetime of the bond.

Question 7

In what range is X?

(A)   Less than $350

(B) $350 but less than $400

(C) $400 but less than $450

(D) $450 but less than $500

(E) $500 or more

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Data for Question 8 (3 points)

Given the following information for a calendar year:

Date	Fund balance	Deposit	Withdrawal
1/1	$100,000	--	--
1/31	--	--	$24,000
4/1	--	X	--
6/30	--	--	$30,000
10/1	--	X	--
12/31	$100,000	--	--

The annual rate of return using the dollar-weighted method was 15.0%.

Question 8

In what range is X?

(A)   Less than $18,500

(B) $18,500 but less than $19,500

(C) $19,500 but less than $20,500

(D) $20,500 but less than $21,500

(E) $21,500 or more

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A $1,000 mortgage is repaid over 20 years by level annual payments at the end of each year.

The repayments are computed at 4.0% per year, compounded annually.

X = the modified duration of the mortgage.

Question 9

In what range is X?

(A)   Less than 9.0

(B)   9.0 but less than 9.3

(C)   9.3 but less than 9.6

(D)   9.6 but less than 9.9

(E)   9.9 or more

An insurance company receives $10,000 to fund a 5-year annuity-immediate at 4.0%.

The insurer will invest this $10,000 in 1-year and 5-year zero coupon bonds,
each with yield rate 4.0%.

X= the amount that should be invested in the 5-year zero coupon bonds in order for the duration
of the investment to equal the duration of the 5-year annuity.

Question 10

In what range is X?

(A)   Less than $4,600

(B) $4,600 but less than $4,660

(C) $4,660 but less than $4,720

(D) $4,720 but less than $4,780

(E) $4,780 or more

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Data for Question 11 (3 points)

Provisions of a bond:

Face amount	$1,000
Redemption value	$1,000
Coupon rate	5.0% per year, payable annually
Term to maturity	10 years
Yield rate	4.0% per year, risk-free rate to maturity

For any bond in good standing at the beginning of a year, the probability of default during that year is assumed to be 3.0%.

If default occurs, there are no future payments.

X = the expected present value of the payments under the bond.

Question 11

In what range is X?

(A)   Less than $850

(B) $850 but less than $865

(C) $865 but less than $880

(D) $880 but less than $895

(E) $895 or more

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Data for Question 12 (3 points)

Smith's initial salary is $100,000, payable in full at the beginning of the year.

Smith's salary for all future years is paid in full at the beginning of the year.

At the beginning of each year, Smith deposits 1.0% of his annual salary into a bank account that earns an annual effective rate of interest of 5.0%.

At the end of each year, Smith receives a 3.0% increase in salary.

Smith makes no withdrawals over a 21-year period.

X = the accumulated value in Smith's bank account at the end of the 21st year.

Question 12

In what range is X?

(A) Less than $30,000

(B) $30,000 but less than $35,000

(C) $35,000 but less than $40,000

(D) $40,000 but less than $45,000

(E) $45,000 or more

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An annuity is purchased that pays $2,000 per year at the end of each year for 6 years.

Term structure of interest rates at date of purchase:

Term (years)	Spot rate
1	1.00%
2	2.00%
3	2.75%
4	3.25%
5	3.50%
6	3.75%

Three years from the date of purchase, the spot rates are expected to be 200 basis points higher than the current spot rates for all periods.

X = the present value of the annuity immediately after the third payment has been made.

Question 13

In what range is X?

(A)   Less than $5,300

(B) $5,300 but less than $5,400

(C) $5,400 but less than $5,500

(D) $5,500 but less than $5,600

(E) $5,600 or more

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<u>Data for Question 14 (3 points)</u>

An investor pays $950 for an investment that returns:

  $500 at the end of year 3, and
  $700 at the end of year 4.

The term structure of interest rates:

<u>Term (years)</u>	<u>Spot rate</u>
2	5.00%
4	7.00%

X= the implied one-year forward rate for year 4.

<u>Question 14</u>

In what range is X?

(A)   Less than 7.1%

(B)   7.1% but less than 7.8%

(C)   7.8% but less than 8.5%

(D)   8.5% but less than 9.2%

(E)   9.2% or more

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Data for Question 15 (4 points)

Eligibility conditions under a pension plan:

Normal retirement	Age 65 with 5 years of service
Early retirement	Age 60 with 15 years of service

Data for active participant Smith:
Age at hire   46

The following rates of retirement are used in the actuarial valuation of this plan:

Age at which first eligible for early retirement: 40%

All other ages:

Age	Rate
61	10%
62	35%
63-64	20%
65	100%

Retirements occur at the beginning of the year.

No other decrements apply from age 60 to age 65.

$X=$ the weighted average assumed retirement age for Smith at Smith's date of hire.

Question 15

In what range is X?

(A)   Less than 62.22

(B)   62.22 but less than 62.52

(C)   62.52 but less than 62.82

(D)   62.82 but less than 63.12

(E)   63.12 or more

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Data for Question 16 (2 points)

An excerpt from select and ultimate mortality table with a 3-year select period follows:

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
41	0.10	0.12	0.14	0.16	44
42	0.11	0.13	0.15	0.17	45
43	0.12	0.14	0.16	0.18	46
44	0.13	0.15	0.17	0.19	47
45	0.14	0.16	0.18	0.20	48

$X = {}_5q_{[41]}$

Question 16

In what range is X?

(A) Less than 0.495

(B) 0.495 but less than 0.515

(C) 0.515 but less than 0.535

(D) 0.535 but less than 0.555

(E) 0.555 or more

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Smith and Jones (both age 101) purchase a joint-life annuity-immediate with annual payments of $20,000.

Select and ultimate mortality values are calculated using the following formulas:

$$\ell_{[x]+t} = 105 - x - \frac{t}{2}, \qquad t \le 4$$

$$\ell_{x+t} = 107 - x - t, \qquad t \ge 4$$

Interest rate: 3.0% per year, compounded annually

X = the single premium for this annuity.

Question 17

In what range is X?

(A)   Less than $35,000

(B) $35,000 but less than $36,500

(C) $36,500 but less than $38,000

(D) $38,000 but less than $39,500

(E) $39,500 or more

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Data for Question 18 (4 points)

Selected commutation functions using an interest rate of 4.0% per year, compounded annually:

x	C_x	D_x
70	3.42	
71	3.71	
72	4.37	
73	5.35	513.94
74	6.60	
75	8.04	

$$X = 1{,}000 \; {}_{5|}q_{70}$$

Question 18

In what range is X?

(A)   Less than 14.00

(B)   14.00 but less than 15.00

(C)   15.00 but less than 16.00

(D)   16.00 but less than 17.00

(E)   17.00 or more

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Data for Question 19 (2 points)

Consider the following statements:

I.   The net annual premium for whole life insurance with $1,000 payable at the end of the year of death to a life age x is

$$1,000 \frac{M_x}{N_x}.$$

II.   The net single premium for an annual $1,000 life annuity-immediate to a life age x is

$$1,000 \frac{N_x}{D_x}.$$

III.   The net single premium for an annual $1,000 n-year certain and life annuity-immediate to a life age x is

$$1,000 \left(a_{\overline{n}|} + \frac{N_{x+n}}{D_x} \right).$$

Question 19

Which, if any, of the above statements is (are) true?

(A)   I and II only

(B)   I and III only

(C)   II and III only

(D)   I, II, and III

(E)   The correct answer is not given by (A), (B), (C), or (D) above.

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Smith (age 30) purchases a single premium annuity on 1/1 with the following characteristics:

Payments	$1 annually, payable at the end of each year.
Term	Payments are for life with a final payment on the 12/31 following Smith's death.

Selected actuarial values:

$$N_{30} = 42,738$$
$$N_{31} = 40,437$$

Interest rate: 5.0% per year, compounded annually.

X = the single premium for this annuity.

Question 20

In what range is X?

(A)   Less than $17.65

(B) $17.65 but less than $17.70

(C) $17.70 but less than $17.75

(D) $17.75 but less than $17.80

(E) $17.80 or more

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Data for Question 21 (4 points)

The terms of three annuity contracts are as follows:

	Contract 1	Contract 2	Contract 3
Age at issue	40	70	50
Age at first annuity payment	50	70	50
Type of annuity	20-year temporary life annuity	Life	Life
Frequency of payments	Annual	Annual	Annual
Amount of each payment	$2,000	$500	$1,000
Net single premium	$14,096	$3,646	X

All annuity payments are made at the beginning of the year.

Selected commutation functions used to calculate the net single premium for each contract:

x	D_x
40	17,138
50	10,243
70	2,439

Question 21

In what range is X?

(A)   Less than $13,500

(B) $13,500 but less than $14,000

(C) $14,000 but less than $14,500

(D) $14,500 but less than $15,000

(E) $15,000 or more

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Data for Question 22 (3 points)

A mortality table produces the following values:

$$e_{30:\overline{4|}} = 3.6$$
$$e_{34:\overline{6|}} = 5.0$$
$$e_{30:\overline{10|}} = 8.0$$

$$X = {}_{4}q_{30}$$

Question 22

In what range is X?

(A)   Less than 0.05

(B)   0.05 but less than 0.07

(C)   0.07 but less than 0.09

(D)   0.09 but less than 0.11

(E)   0.11 or more

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Smith and Jones are both age 35. They are considering the following annuities:

- Annuity A pays $100 at the end of each year while exactly one of Smith or Jones is alive. The present value of Annuity A is $1,100.
- Annuity B pays $600 at the end of each year while Smith is alive, and $300 at the end of each year to Jones after Smith dies. The present value of Annuity B is $7,500.
- Annuity C pays $100 at the end of each year while both Smith and Jones are alive.

X= the present value of Annuity C.

Question 23

In what range is X?

(A)   Less than $380

(B) $380 but less than $400

(C) $400 but less than $420

(D) $420 but less than $440

(E) $440 or more

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Data for Question 24 (3 points)

Smith is age 20. Jones is age 30.

Mortality is calculated using the following formula:

$$\ell_x = 80 - x \qquad 0 \le x \le 80$$

X = the probability that the second death occurs between 15 and 20 years from now.

Question 24

In what range is X?

(A)   Less than 0.0565

(B)   0.0565 but less than 0.0575

(C)   0.0575 but less than 0.0585

(D)   0.0585 but less than 0.0595

(E)   0.0595 or more

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Data for Question 25 (4 points)

A multiple decrement model for a retirement plan includes three decrements:

mortality (d), retirement (r), and withdrawal (w).

Selected values from this model follow:

x	$\ell_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(r)}$	$d_x^{(w)}$
55	10,000	16	200	100
56	9,684	16	190	85
57	9,393		175	70
58		17		
59		18	150	50
60		18	300	35

Given the following values:

- The probability that a 55-year old plan member is still active in the plan at age 60 is 0.86760.
- The probability that a 57-year old plan member withdraws or dies before age 59 is 0.01746.
- The probability that a 55-year old plan member withdraws between age 57 and age 60 is 0.01800.

$X=$ the probability that a 55-year old plan member is still active in the plan at age 58.

Question 25

In what range is X?

(A)   Less than 0.900

(B)   0.900 but less than 0.910

(C)   0.910 but less than 0.920

(D)   0.920 but less than 0.930

(E)   0.930 or more

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Selected values from a two-decrement model:

x	$q_x^{(1)}$	$q_x^{(2)}$
46	0.0244	0.1000
47	0.0273	0.0900
48	0.0309	0.0800
49	0.0345	0.0700
50	0.0390	0.0600

$$X = {}_{2|2}q_{46}^{(2)}$$

Question 26

In what range is X?

(A)   Less than 0.0955

(B)   0.0955 but less than 0.1055

(C)   0.1055 but less than 0.1155

(D)   0.1155 but less than 0.1255

(E)   0.1255 or more

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Data for Question 27 (2 points)

For a two decrement model, you are given:

$$\ell\,'^{(1)}_x = 50 - x, \quad 0 \le x \le 50$$
$$\ell\,'^{(2)}_x = 60 - x, \quad 0 \le x \le 60$$

$$X = q^{(1)}_{40}$$

Question 27

In what range is X?

(A)   Less than 0.090

(B)   0.090 but less than 0.095

(C)   0.095 but less than 0.100

(D)   0.100 but less than 0.105

(E)   0.105 or more

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Data for Question 28 (3 points)

Smith (age 65) is entitled to a life annuity of $10,000 per year, starting at Smith's age 65.

Immediately before receiving the first payment, Smith elects an actuarially-equivalent annuity with Jones (age 66) as Smith's beneficiary.

This actuarially-equivalent annuity pays X at the beginning of each year starting at Smith's age 65 and reduces to $0.75X$ upon the first death.

Selected actuarial factors:

$$\ddot{a}_{65} = 13.8350$$
$$\ddot{a}_{66} = 13.4808$$
$$\ddot{a}_{65:66} = 11.1861$$

Question 28

In what range is X?

(A)   Less than $9,050

(B) $9,050 but less than $9,150

(C) $9,150 but less than $9,250

(D) $9,250 but less than $9,350

(E) $9,350 or more

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Data for Question 29 (4 points)

Under a pension plan, Smith (age 55) is entitled to a monthly life annuity benefit of $750 at the beginning of each month, commencing immediately.

Smith elects a Social Security level income option that is actuarially-equivalent to this life annuity.

Under this option, Smith's total monthly income from the pension plan plus the amount Smith will receive from Social Security remains level for Smith's lifetime.

Smith's monthly Social Security benefit will be $1,400 commencing at age 62.

Selected actuarial values:

$$\ddot{a}_{55}^{(12)} = 13.728$$
$$\ddot{a}_{62}^{(12)} = 12.218$$
$$_{7}E_{55} = 0.656$$

$X=$ the monthly benefit payable to Smith from the pension plan from age 55 to age 62 under the Social Security level income option.

Question 29

In what range is X?

(A)   Less than $1,520

(B) $1,520 but less than $1,550

(C) $1,550 but less than $1,580

(D) $1,580 but less than $1,610

(E) $1,610 or more

Data for Question 30 (2 points)

Selected values from a mortality table:

x	ℓ_x
102	333.33
103	250.00
104	100.00
105	0

Interest rate: 7.0% per year, compounded annually.

$X = A_{102}$

Question 30

In what range is X?

(A)   Less than 0.870

(B)   0.870 but less than 0.890

(C)   0.890 but less than 0.910

(D)   0.910 but less than 0.930

(E)   0.930 or more

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Data for Question 31 (2 points)

Mortality is calculated using the following formula:

$$\ell_x = 100 - 0.9x - 0.001x^2 \qquad 0 \le x \le 100$$

$$X = 1{,}000 \ _{20|10}q_{40}$$

Question 31

In what range is X?

(A)   Less than 150

(B)   150 but less than 200

(C)   200 but less than 250

(D)   250 but less than 300

(E)   300 or more

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Data for Question 32 (4 points)

Terms of a life annuity:

Form of payment	One-year temporary annuity immediate
Frequency of payments	Quarterly
Amount of each quarterly payment	$25,000
Annuitant's age	x

Interest rate: 5.0% per year, compounded annually.

Selected values from the mortality table used in the calculation:

$$\ell_x = 246,746$$
$$\ell_{x+1} = 209,742$$

Deaths are assumed to be uniform over each integral age.

X= the present value of this annuity.

Question 32

In what range is X?

(A)   Less than $88,000

(B) $88,000 but less than $88,060

(C) $88,060 but less than $88,120

(D) $88,120 but less than $88,180

(E) $88,180 or more

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Data for Question 33 (2 points)

Selected values from a mortality table:

$$q_{98} = 0.60$$
$$q_{99} = 1.00$$

Deaths are assumed to be uniform over each integral age

$$X = {}_{0.50|0.75}q_{98.25}$$

Question 33

In what range is X?

(A)   Less than 0.25

(B)   0.25 but less than 0.30

(C)   0.30 but less than 0.35

(D)   0.35 but less than 0.40

(E)   0.40 or more

END OF EXAMINATION

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