π IN THE SKY[7]

Answer Key

Mars Maneuver

What percentage of Curiosity's landing ellipse is Perseverance's landing ellipse?

1. **Divide the area of Perseverance's landing ellipse by the area of Curiosity's landing ellipse, using the formula for the area of an ellipse. (Note: π cancels out π.)**
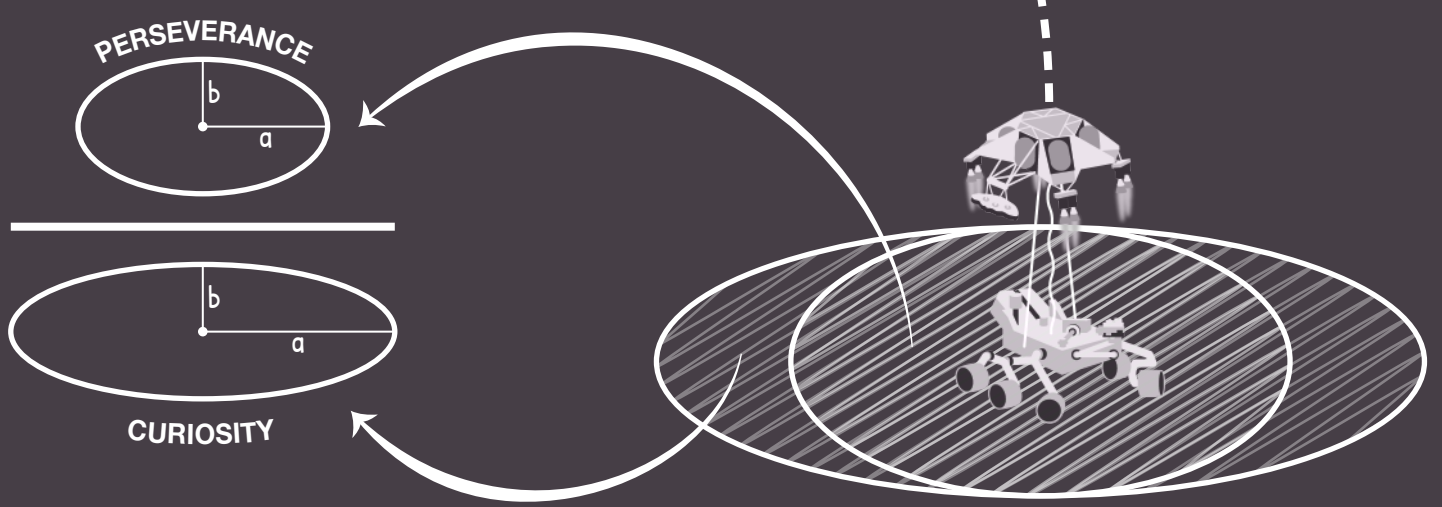
$A_{ellipse} = \pi ab$

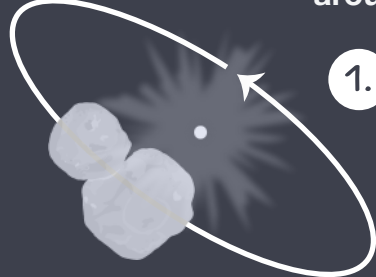$(\pi \cdot 3.5 \cdot 6.5) / (\pi \cdot 3.5 \cdot 10) \Rightarrow (3.5 \cdot 6.5) / (3.5 \cdot 10) = 0.65$

2. **Convert to a percentage.**

$0.65 \cdot 100 =$ 65%

PERSEVERANCE
b
a

CURIOSITY
b
a

Cold Case

Learn a bit more about Arrokoth by calculating how long it takes the object to make one trip around the Sun.

1. **Using the formula for circumference, compute distance traveled by Arrokoth in one orbit.**

$C = 2\pi r$

$C = 2\pi(6,600,000,000 \text{ km} + 150,000,000 \text{ km})$

$C = 2\pi(6,750,000,000 \text{ km}) \approx 42,411,500,823 \text{ km}$

2. **Convert radius kilometers to meters, then compute Arrokoth's orbital velocity.**

$V = \sqrt{\dfrac{GM_{Sun}}{r}}$

$V = \sqrt{\dfrac{(6.67 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{s}^{-2}) \cdot (2 \times 10^{30} \text{ kg})}{6.75 \times 10^{12} \text{ m}}}$
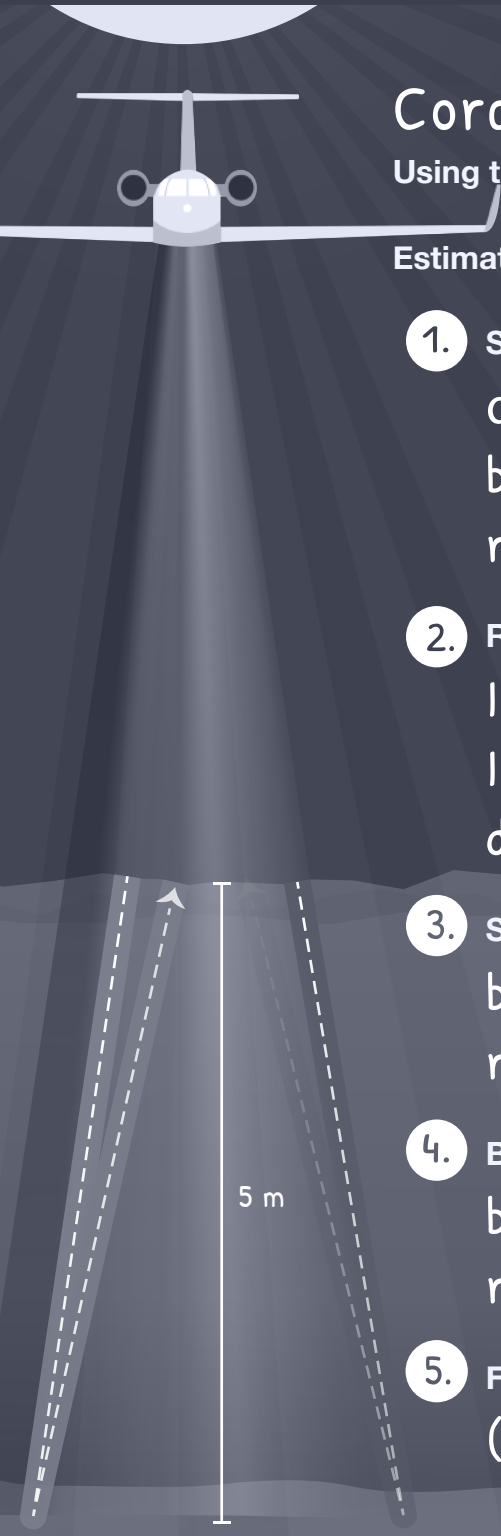
$V \approx 4,446 \text{ m/s}$

3. **Convert circumference kilometers to meters, then use d=rt compute the time it takes Arrokoth to complete one orbit.**

$t \approx (42,411,500,823,000 \text{ m}) / (4,446 \text{ m/s}) \approx 9,539,248,948 \text{ s}$

4. **Convert seconds to years.**

$(9,539,248,948 \text{ s}) \cdot (1 \text{ min} / 60 \text{ s}) \cdot (1 \text{ hour} / 60 \text{ min}) \cdot$
$(1 \text{ day} / 24 \text{ hours}) \cdot (1 \text{ year} / 365 \text{ days}) \approx$ 300 years

Coral Calculus

Using the absorption coefficient and Beer-Lambert law formulas, calculate the water's depth.

Estimate distance on the blue and red ends of the spectrum:

1. **Solve for the blue light and red light absorption coefficients.**

$\alpha = (4\pi k) / \lambda$

blue light: $\alpha = (4\pi \cdot 1.01\text{E-09}) / (0.00000045 \text{ m}) \approx 0.028/\text{m}$

red light: $\alpha = (4\pi \cdot 1.60\text{E-08}) / (0.00000065 \text{ m}) \approx 0.309/\text{m}$

2. **Rearrange the Beer-Lambert law formula, $T = e^{(-\alpha \cdot d)}$, to solve for d.**

$\ln(T) = \ln(e^{(-\alpha \cdot d)})$

$\ln(T) = -\alpha \cdot d$

$d = \ln(T) / (-\alpha)$

3. **Solve for d on the blue and red ends of the spectrum.**

blue light: $d = \ln(0.76) / (-0.028) \approx 9.73 \text{ m}$

red light: $d = \ln(0.045) / (-0.309) \approx 10.04 \text{ m}$

4. **Because light passes through the water twice, divide the total distances by 2.**
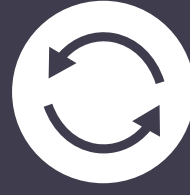
blue light: $9.73 \text{ m}/2 \approx 4.87 \text{ m}$

red light: $10.04 \text{ m}/2 \approx 5.02 \text{ m}$

5. **Find the weighted mean of the distances from both ends of the spectrum.**

$((0.76 \cdot 4.87) + (0.045 \cdot 5.02)) / (0.76 + 0.045) \approx$ 5 m

5 m

Planet Pinpointer

Given the angle of the disk's apparent size is 169 arcseconds, determine the actual distance across it using the formula for small angle approximation.

1. **Convert arcseconds to degrees.**

$1 \text{ arcsec} = (1/3,600)°$

$169 \text{ arcsec} \cdot 1°/3,600 \text{ arcsec} \approx 0.0469°$

2. **Multiply degrees by π/180° to convert degrees to radians.**

$0.0469° \cdot (\pi / 180°) \approx 0.000819 \text{ radians}$

3. **Use the formula for small angle approximation to find the distance across the Beta Pictoris debris disk.**

$D = d\theta$

$D = (6 \cdot 10^{14} \text{ km}) \cdot 0.000819 \approx$ 500 billion km

d

θ

D