

# $\pi$ IN THE SKY<sup>7</sup>

## Answer Key

### Mars Maneuver

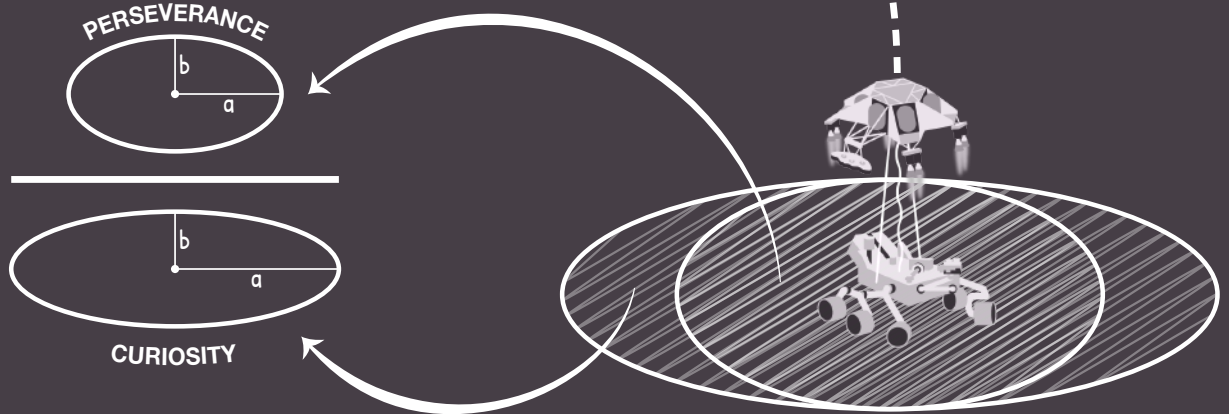
What percentage of Curiosity's landing ellipse is Perseverance's landing ellipse?

1. Divide the area of Perseverance's landing ellipse by the area of Curiosity's landing ellipse, using the formula for the area of an ellipse. (Note:  $\pi$  cancels out  $\pi$ .)

$$A_{\text{ellipse}} = \pi ab$$
$$(\pi \cdot 3.5 \cdot 6.5) / (\pi \cdot 3.5 \cdot 10) \Rightarrow (3.5 \cdot 6.5) / (3.5 \cdot 10) = 0.65$$

2. Convert to a percentage.

$$0.65 \cdot 100 = 65\%$$



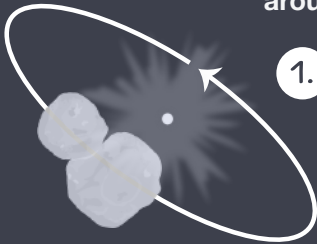


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### Cold Case

Learn a bit more about Arrokoth by calculating how long it takes the object to make one trip around the Sun.



- Using the formula for circumference, compute distance traveled by Arrokoth in one orbit.

$$C = 2\pi r$$

$$C = 2\pi(6,600,000,000 \text{ km} + 150,000,000 \text{ km})$$

$$C = 2\pi(6,750,000,000 \text{ km}) \approx 42,411,500,823 \text{ km}$$

- Convert radius kilometers to meters, then compute Arrokoth's orbital velocity.

$$V = \sqrt{\frac{GM_{\text{Sun}}}{r}}$$

$$V = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{s}^{-2}) \cdot (2 \times 10^{30} \text{ kg})}{6.75 \times 10^{12} \text{ m}}}$$

$$V \approx 4,446 \text{ m/s}$$

- Convert circumference kilometers to meters, then use  $d=rt$  compute the time it takes Arrokoth to complete one orbit.

$$t \approx (42,411,500,823,000 \text{ m}) / (4,446 \text{ m/s}) \approx 9,539,248,948 \text{ s}$$

- Convert seconds to years.

$$(9,539,248,948 \text{ s}) \cdot (1 \text{ min} / 60 \text{ s}) \cdot (1 \text{ hour} / 60 \text{ min}) \cdot$$

$$(1 \text{ day} / 24 \text{ hours}) \cdot (1 \text{ year} / 365 \text{ days}) \approx 300 \text{ years}$$





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### Coral Calculus

Using the absorption coefficient and Beer-Lambert law formulas, calculate the water's depth.

Estimate distance on the blue and red ends of the spectrum:

1. Solve for the blue light and red light absorption coefficients.

$$\alpha = (4\pi k) / \lambda$$

$$\text{blue light: } \alpha = (4\pi \cdot 1.01\text{E-}09) / (0.00000045 \text{ m}) \approx 0.028/\text{m}$$

$$\text{red light: } \alpha = (4\pi \cdot 1.60\text{E-}08) / (0.00000065 \text{ m}) \approx 0.309/\text{m}$$

2. Rearrange the Beer-Lambert law formula,  $T = e^{(-\alpha \cdot d)}$ , to solve for d.

$$\ln(T) = \ln(e^{(-\alpha \cdot d)})$$

$$\ln(T) = -\alpha \cdot d$$

$$d = \ln(T) / (-\alpha)$$

3. Solve for d on the blue and red ends of the spectrum.

$$\text{blue light: } d = \ln(0.76) / (-0.028) \approx 9.73 \text{ m}$$

$$\text{red light: } d = \ln(0.045) / (-0.309) \approx 10.04 \text{ m}$$

4. Because light passes through the water twice, divide the total distances by 2.

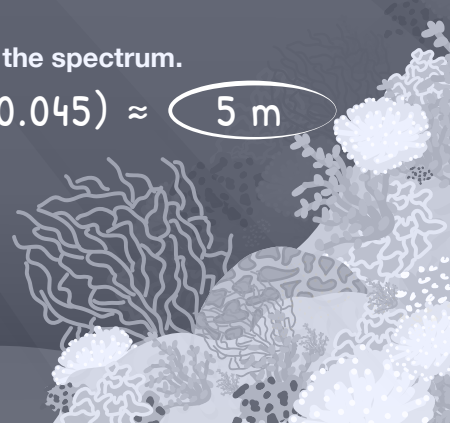
$$\text{blue light: } 9.73 \text{ m} / 2 \approx 4.87 \text{ m}$$

$$\text{red light: } 10.04 \text{ m} / 2 \approx 5.02 \text{ m}$$

5. Find the weighted mean of the distances from both ends of the spectrum.

$$((0.76 \cdot 4.87) + (0.045 \cdot 5.02)) / (0.76 + 0.045) \approx 5 \text{ m}$$

5 m





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### Planet Pinpointer

Given the angle of the disk's apparent size is 169 arcseconds, determine the actual distance across it using the formula for small angle approximation.



1. Convert arcseconds to degrees.

$$1 \text{ arcsec} = (1/3,600)^\circ$$

$$169 \text{ arcsec} \cdot 1^\circ/3,600 \text{ arcsec} \approx 0.0469^\circ$$

2. Multiply degrees by  $\pi/180^\circ$  to convert degrees to radians.

$$0.0469^\circ \cdot (\pi / 180^\circ) \approx 0.000819 \text{ radians}$$

Use the formula for small angle approximation to find the distance across the Beta Pictoris debris disk.

- 3.

$$D = d\theta$$

$$D = (6 \cdot 10^{14} \text{ km}) \cdot 0.000819 \approx 500 \text{ billion km}$$

