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Abstract

We build a macro-finance model with an occasionally binding financing constraint where real interest rates have opposite effects on current and future financial stability, with the contemporaneous impact driven by valuation effects (akin to those triggering the 2023 banking turmoil) and the future impact driven by reach-for-yield by intermediaries. We use this model to illustrate the concept of the financial stability interest rate, r**, which we propose as a quantitative summary statistic for financial vulnerabilities. We provide a measure of r** for the U.S. economy and discuss its evolution over the past fifty years.

Key words: r**, financial crises, financial stability, occasionally binding credit constraint

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1 Introduction

The concept of natural real rate of interest, also known as r*, which dates back to Wicksell (1898), is associated with the notion of macroeconomic stability. r* plays an important role in policy discussions and there is a vast literature that tries to measure it and study its implications (see, for example, Laubach and Williams, 2003, Holston et al., 2017, and Del Negro et al., 2017a, 2019). In this paper we propose a complementary concept that we call the "financial stability real interest rate, r**." The idea of r** consists in measuring the level of the real interest rate that generates financial instability. The purpose of r** is to serve as a quantitative summary statistic for financial stability in the interest rate space, the one most relevant for monetary policymakers, just like r* is such a statistic for macroeconomic stability.

In order to define the financial stability real interest rate one first needs to develop a notion of financial stability. To this end, we consider an environment in which financial intermediaries in the economy face a credit constraint that gives rise to asset fire-sale dynamics. This credit constraint is occasionally binding, implying that the economy is characterized by two states: when the constraint is not binding the economy is in a financially tranquil period; when the constraint binds the economy experiences financial crisis. The financial stability real interest rate is the interest rate that, for a given state of the economy (and especially for a given degree of vulnerability of the financial system), would be consistent with the constraint being just binding.

For the purpose of illustrating how r** is constructed we build a model where i) the economy endogenously fluctuates between the two regimes, a tranquil one and a crisis one, as just discussed, and in which ii) changes in the real interest rate are a key driver of these fluctuations. The model builds upon the banking framework developed by Gertler and Kiyotaki (2010), Gertler and Karadi (2011) and Gertler and Kiyotaki (2015). In this framework, financial intermediaries channel funds from households to firms. The key imperfection is that banks have a limit in their ability to raise funds because of a moral hazard problem. This gives rise to the credit constraint that is at the core of our analysis. There are two important differences between our approach and the seminal work cited above. First, these authors assume that the constraint is always binding. We do not. As in Akinci and Queralto (2022), we use a global solution method and allow for the constraint to be occasionally binding so that the economy can display both a tranquil and a crisis state. Second, we assume that the tightness of the incentive compatibility constraint depends on the composition of the assets side of financial intermediaries' balance sheet between safe and risky loans. This implies that, for a given level of leverage, financial vulnerability is higher when the intermediaries' portfolio is tilted toward risky assets.

These features generate rich implications in terms of how interest rates affect financial stability in the short versus the medium-run. The *short-run* impact of movements in interest rates is driven by valuation effects, akin to those triggering the 2023 banking turmoil. These effects are at the core of the definition of r**. When the economy is in the tranquil regime, the idea of r** is similar to that of a "stress test": it measures how large a surprise increase in rate the economy can bear before tilting into a crisis. Vice versa, when the economy is in a crisis, r** measures what cut in rates is needed to make sure that the balance sheet constraint on financial intermediaries no longer binds.

The *medium-run* impact of real rates is driven by “reach-for-yield” by intermediaries. A prolonged period of low real interest rates leads to a shift in their portfolio toward risky assets. This makes the financial system eventually more vulnerable to shocks, and therefore more prone to enter the crisis regime, in line with recent empirical findings by Grimm et al. (2023). As a consequence, “low for long” real interest rates tend to reduce the gap between the real rate and the financial stability real interest rate r** because the latter falls, thereby making the economy more vulnerable to interest rate increases—a phenomenon that Brunnermeier (2016) calls “financial dominance.”

The result that prolonged periods of low interest rates may create financial stability risks is also present in recent literature, most prominently in the models in Coimbra and Rey (2017), Adrian and Duarte (2018), and Boissay et al. (2022). A difference with these papers is that in our model these results arise out of a standard macrofinance model, other than for the features discussed above. In particular, our model features the financial accelerator mechanism (e.g. Bernanke et al., 1999) that is common to many macrofinance models but is not present in these papers. This mechanism is key in generating the short-run effects of interest rate changes on financial stability discussed above. Another key difference, especially relative to Adrian and Duarte (2018) and Boissay et al. (2022), is that we focus on a real version of the model. In order to further simplify the exposition, we consider an environment where the real interest rate is exogenous. We leave the discussion of the rich interactions between monetary policy and the financial (in)stability real interest rate to future research.[1]

We provide an empirical measure of r** for the US economy and discuss its evolution over the past 50 years. The financial stability rate r** is a latent variable– we do not directly observe it. In order to measure it in the data we adopt the following strategy. First, we figure out in the model which variables that are observable in the data map into r**. Because our model is very non linear, we use machine learning techniques in order to construct such mapping. Then, we find the empirical counterpart of these variables and use them to obtain a measure of r** in the data. We provide an external validation of our r** measure by computing the time-varying sensitivity of financial conditions to interest rate shocks (in the data, these correspond to exogenous shocks to interest rates) and argue that this time variation is very much in line with the r** measure we construct.

The next section describes the model, section 3 discusses our calibration strategy and section 4 presents the quantitative properties of r**. In section 5 we construct the empirical measure of r**. Section 6 concludes.

2 Model

We propose a framework that builds upon the macrofinance model developed in Gertler and Kiyotaki (2010). In this setting, financial intermediaries make risky loans to nonfinancial firms and collect deposits from domestic households. In our setup, intermediaries also hold a perfectly safe asset. Because of an agency problem, intermediaries may be constrained in their access to external funds.

[1]Coimbra and Rey (2017) highlight the importance of heterogeneity in risk taking across financial intermediaries, which we ignore.

A key feature of our analysis is to allow this constraint to be occasionally binding, as in Akinci and Queralto (2022). In tranquil times, intermediaries' constraints do not bind: credit spreads are small and the economy's behavior is similar to a frictionless neoclassical framework. When the constraint binds the economy enters into financial stress mode: credit spreads become large and volatile, and investment and credit drop, consistent with the evidence. A second crucial feature of our model that differs from existing literature is that the degree of agency frictions facing a given intermediary depends on the composition of its assets: when the intermediary's portfolio is heavily tilted toward the safe asset, agency frictions are less severe than if it holds a large share of risky assets.

2.1 Households

Each household is composed of a constant fraction $(1-f)$ of workers and a fraction f of "bankers." Workers supply labor to the firms and return their wages to the household. Each banker manages a financial intermediary (which we will sometimes refer to as a "bank" for brevity) and similarly transfers any net earnings back to the household. Within the family there is perfect consumption insurance.

Households do not hold capital directly, but deposit funds in intermediaries. The deposits held by each household are in banks other than the one owned by the household. Bank deposits are riskless one-period securities. Consumption, C_t, deposits, D_t, and labor supply, L_t, are given by maximizing the discounted expected future flow of utility

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}, L_{t+i}),$$

subject to the budget constraint

$$C_t + D_t \leq W_t L_t + R^d_{t-1} D_{t-1} + \Pi_t$$

for all t, where W_t is the real wage, R^d_t is the gross real interest rate received from holding one-period deposits, and Π_t is total profits distributed to households from their ownership of both intermediaries and firms.

2.2 Intermediaries

In each period, the bank uses its own equity capital or net worth, denoted n_t, and deposits issued to households, d_t, to purchase securities issued by nonfinancial firms, s_t, at price Q_t, as well as safe assets b_t. In turn, nonfinancial firms use the proceeds to finance purchases of physical capital.

2.2.1 Agency friction and incentive constraints

We follow Gertler and Kiyotaki (2010) in assuming that intermediaries are "specialists" who are efficient at evaluating and monitoring nonfinancial firms and at enforcing contractual obligations with these borrowers. For this reason, firms rely solely on intermediaries to obtain funds and there are no contracting frictions between intermediaries and firms. However, as in Gertler and Kiyotaki (2010), we introduce an agency problem whereby the banker managing the bank may decide to default on its obligations. As a consequence, intermediaries may be credit constrained, depending on whether they are perceived to have the incentive to disregard their contractual obligations.

Specifically, after having borrowed external funds but before repaying its creditors, the banker may decide to default on its obligations and divert a fraction $\Theta(x_t)$ of their assets. In this case, the bank is forced into bankruptcy and its creditors recover the remaining funds. To ensure that the bank does not divert funds, the incentive constraint

$$V_t \geq \Theta(x_t)(Q_t s_t + b_t) \tag{1}$$

must hold, where V_t stands for the continuation value of the bank. This constraint requires that the banker's continuation value be higher than the value of the diverted funds.

An important difference with Gertler and Kiyotaki (2010) is that in our model the fraction $\Theta(x_t)$ is not constant but varies with $x_t \equiv \frac{b_t}{Q_t s_t + b_t}$, the share of safe assets in the banker's portfolio.[2] We assume that the function $\Theta(\cdot)$ satisfies $\Theta'(x_t) < 0$: as the banker's portfolio becomes more risky, the agency friction worsens. The rationale for this assumption is that risky loans are more opaque and hard to monitor relative to safe assets, which leads creditors to turn more cautious when the banker's portfolio becomes riskier. The assumption is also in line with the spirit of Basel's capital requirements, according to which assets are weighted differently depending on their riskiness.

We also assume that $\Theta''(x_t) > 0$: when x_t is very low, further diminishing it worsens the friction more than if x_t is high. The motivation for this assumption is that if x_t is already very low, further tilting the balance sheet toward risky assets entails entering segments of the asset market that are particularly sensitive to agency and information frictions. In broad terms, the dependence of $\Theta(\cdot)$ on x_t captures the notion that financial frictions worsen as the riskiness of intermediaries' portfolios increases, and progressively so.

2.2.2 The intermediaries' problem

The bank pays dividends only when it exits. If the exit shock realizes (with probability $1-\sigma$), the banker exits at the *beginning* of $t+1$, and simply waits for its asset holdings to mature and then pays the net proceeds to the household. The objective of the bank is to maximize expected terminal payouts to the household. Formally, the bank chooses state-contingent sequences $\{s_t, b_t, d_t\}_{t=0}^{\infty}$ to maximize

$$V_t = \mathbb{E}_t \sum_{i=0}^{\infty} \sigma^i \Big\{ \Lambda_{t,t+1+i} \Big[(1-\sigma)(R_{K,t+1+i} Q_{t+i} s_{t+i} + R_{t+i} b_{t+i} - R^d_{t+i} d_{t+i}) \Big] + \Lambda_{t,t+i} \zeta_{t+i} b_{t+i} \Big\}, \tag{2}$$

where $\Lambda_{t,t+1}$ is the household's stochastic discount factor given by the marginal rate of substitution between consumption at dates $t+1$ and t, R_t is the pecuniary gross return on the safe asset, and the

[2] Gertler et al. (2012), where bankers can issue outside equity in addition to deposits, assumes that the fraction Θ depends on banks' liability composition.

(time-varying) exogenous variable ζ_{t+i} captures the direct utility derived from holding safe assets, which follows an *iid* process with mean $\overline{\zeta} > 0$ and variance σ^2_ζ. While this direct utility shock plays no role for any of the qualitative results in the paper, quantitatively the shock captures movements in credit spreads that are due to "noise," that is, that are not necessarily related to the proximity of the economy to the financial crisis region. We model this noise as the result of time variation in bankers' preferences for safe assets, as in Krishnamurthy and Vissing-Jorgensen (2012).[3]

Maximization of (2) is subject to the incentive constraint (1) and the budget constraint

$$Q_t s_t + b_t + R^d_{t-1} d_{t-1} \leq R_{K,t} Q_{t-1} s_{t-1} + R_{t-1} b_{t-1} + d_t, \tag{3}$$

which states that the bank's expenditures (consisting of asset purchases, $Q_t s_t + b_t$, and repayment of deposit financing, $R^d_{t-1} d_{t-1}$) cannot exceed its revenues, stemming from payments of previous-period asset holdings, $R_{K,t} Q_{t-1} s_{t-1} + R_{t-1} b_{t-1}$, and deposits d_t.

The bank's balance sheet identity,

$$Q_t s_t + b_t \equiv n_t + d_t, \tag{4}$$

which is equivalent to a definition of net worth n_t—stating that the bank's assets are funded by the sum of net worth and deposits—can be combined with (3) to yield the law of motion of the bank's net worth:

$$n_t = (R_{K,t} - R^d_{t-1}) Q_{t-1} s_{t-1} + (R_{t-1} - R^d_{t-1}) b_{t-1} + R^d_{t-1} n_{t-1}. \tag{5}$$

We use the method of undetermined coefficients to solve the banker's problem. We guess that the value function satisfies $V_t(n_t) = \alpha_t n_t$, where α_t is a coefficient to be determined. Define $\Omega_{t+1} \equiv 1 - \sigma + \sigma \alpha_{t+1}$, and let

$$\mu_t \equiv \mathbb{E}_t[\Lambda_{t+1} \Omega_{t+1} (R_{K,t+1} - R^d_t)], \tag{6}$$

$$\mu_{B,t} \equiv \mathbb{E}_t[\Lambda_{t+1} \Omega_{t+1}](R_t - R^d_t), \tag{7}$$

$$\nu_t \equiv \mathbb{E}_t[\Lambda_{t+1} \Omega_{t+1}] R^d_t. \tag{8}$$

Note that Ω_{t+1}, capturing the value to the bank of an extra unit of net worth the following period (in case the banker does not exit), augments the banker's stochastic discount factor (SDF), which becomes $\Lambda_{t,t+1} \Omega_{t+1}$. This effective SDF captures the tightness of the incentive constraint in the following period, on top of the household's discount factor.

We also define the banker's *leverage ratio* ϕ_t as the ratio of total assets to net worth:

$$\phi_t \equiv \frac{Q_t K_t + B_t}{N_t}. \tag{9}$$

[3] In this paper we ignore the important relationship between liquidity and financial crisis (e.g., Kiyotaki and Moore, 2019), which provides a rationale for central bank facilities both during the Great Recession (Del Negro et al., 2017b) and the 2023 banking turmoil.

Given these definition, the banker's optimization problem in recursive form can be written as follows:

$$\alpha_t = \max_{x_t, \phi_t} \quad [\mu_t(1 - x_t) + (\mu_{B,t} + \zeta_t)x_t]\phi_t + \nu_t \tag{10}$$

subject to

$$[\mu_t(1 - x_t) + (\mu_{B,t} + \zeta_t)x_t]\phi_t + \nu_t \geq \Theta(x_t)\,\phi_t. \tag{11}$$

The banker's individual net worth n_t drops out of (10) and (11), as the banker's objective and constraints are linear in n_t. The incentive constraint (11) can then be expressed as a *leverage constraint*, stating that ϕ_t cannot exceed a given threshold:

$$\phi_t \leq \frac{\nu_t}{\Theta(x_t) - [\mu_t(1 - x_t) + (\mu_{B,t} + \zeta_t)x_t]}. \tag{12}$$

Taking first-order conditions with respect to x_t of the corresponding Lagrangian, we obtain the following condition:

$$\begin{aligned} \mu_{B,t} - \mu_t &= \mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}(R_{K,t+1} - R_t)] \\ &= \zeta_t + \frac{\overline{\lambda}_t}{1 + \overline{\lambda}_t}[-\Theta'(x_t)], \end{aligned} \tag{13}$$

where $\overline{\lambda}_t \geq 0$ denotes the Lagrange multiplier on the incentive constraint. Equation (13) states that positive discounted excess returns on risky relative to safe assets are positively linked to both the marginal utility derived from the safe asset (ζ_t) and to the tightness of the incentive constraint (recall that $\Theta'(x_t) < 0$).

Differentiating the Lagrangian with respect to ϕ_t, we obtain

$$\frac{\overline{\lambda}_t}{(1 + \overline{\lambda}_t)}\Theta(x_t) = \mu_t(1 - x_t) + (\mu_{B,t} + \zeta_t)x_t \equiv \overline{\mu}_t, \tag{14}$$

linking the Lagrange multiplier $\overline{\lambda}_t$ positively with the "total" excess returns on intermediaries' assets (inclusive of the preference shock ζ_t), which we define as $\overline{\mu}_t$.

The solution for overall banker leverage ϕ_t is as follows. If $\overline{\mu}_t = 0$, the constraint is not binding, and the banker is indifferent as to its leverage choice. If $\overline{\mu}_t > 0$, the banker leverages up as much as allowed by the incentive constraint. From (12), maximum leverage, denoted $\overline{\phi}_t$, is

$$\overline{\phi}_t \equiv \frac{\nu_t}{\Theta(x_t) - \overline{\mu}_t}. \tag{15}$$

Observe that $\overline{\phi}_t$ is decreasing in $\Theta(x_t)$, and therefore falls as the banking sector's porftolio shifts toward risky assets (i.e. as x_t falls).

Since the banker's problem is linear, we can easily aggregate across intermediaries. For surviving intermediaries, the evolution of net worth is given by (5). We assume entering bankers receive a small exogenous equity endowment, given by fraction ξ/f of the value of the aggregate capital stock. Thus the law of motion of aggregate net worth is

$$N_t = \sigma\left[\left(R_{K,t} - R^d_{t-1}\right) Q_{t-1}K_{t-1} + (R_{t-1} - R^d_{t-1})B_{t-1} + R^d_{t-1}N_{t-1}\right] + (1-\sigma)\xi Q_{t-1}K_{t-1}, \quad (16)$$

where we have used the market-clearing condition $K_t = S_t$.[4]

2.2.3 Credit spreads and the financial constraint

The model highlights how the behavior of credit spreads depends on both the tightness of the financial constraint and the preference shock ζ_t. We define the credit spread as the (annualized) expected return on nonfinancial firms' securities, $\mathbb{E}_t(R_{K,t+1})$, minus the rate on the safe asset, R_t. When the leverage constraint is far from being binding, intermediaries can fully arbitrage away excess returns. Expression (13) becomes

$$\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}(R_{Kt+1} - R_t)\right] = \zeta_t, \quad (17)$$

implying that spreads are driven by the preference shock ζ_t.[5] In such circumstances, the model's dynamics are the same as in standard frictionless models. A higher real rate R_t, for example, raises the required expected return on investment since $\mathbb{E}_t(R_{K,t+1})$ tracks R_t, triggering a fall in Q_t and I_t.

By contrast, when the leverage constraint binds ,

$$\mathbb{E}_t\left[\Lambda_{t,t+1}\Omega_{t+1}(R_{Kt+1} - R_t)\right] > \zeta_t.$$

Intermediaries are constrained by their net worth in their investment decisions, and therefore cannot fully arbitrage away the returns between risky and safe assets. In this regime, the economy is driven by the financial accelerator and fire-sale dynamics, as in standard macrofinance models. A negative shock that lowers asset prices erodes net worth and tightens the constraint further, triggering another round of decline in Q_t and pushing investment down in the process. As a consequence, credit spreads will be large and very volatile.

2.3 Nonfinancial firms

There are two categories of nonfinancial firms: final goods firms and capital goods producers. Within final goods firms we also distinguish between "capital leasing" firms and final goods producers.

[4] We use N_t, S_t, B_t to refer to the aggregate counterparts of n_t, s_t, b_t.

[5] When the constraint is far from binding, the effective intermediary SDF $\Lambda_{t,t+1}\Omega_{t+1}$ essentially coincides with the household's discount factor.

2.3.1 Final goods firms

There are two types of final goods firms: capital leasing firms and final goods producers. The first type of firm purchases capital goods from capital goods producers, stores them for one period, and then rents them to final goods firms. The second type uses physical capital rented from capital leasing firms and labor to produce final output. Importantly, capital leasing firms have to rely on intermediaries to obtain funding to finance purchases of capital. In addition, final goods producers need to rely on intermediaries to finance working capital.

In period $t-1$, a representative capital leasing firm purchases K_{t-1} units of physical capital at price Q_{t-1}. It finances these purchases by issuing S_{t-1} securities to intermediaries which pay a state-contingent return $R_{K,t}$ in period t. At the beginning of period t, the firm rents out this capital to final goods firms at price Z_t, and then sells the undepreciated capital $(1-\delta)K_{t-1}$ in the market at price Q_t. The payoff to the firm per unit of physical capital purchased is thus $[Z_t + (1-\delta)Q_t]$. Given frictionless contracting between firms and intermediaries, it follows that the return on the securities issued by the firm is given by $R_{K,t} = \frac{Z_t+(1-\delta)Q_t}{Q_{t-1}}$. Capital leasing firms make zero profits.

Final goods firms produce output Y_t using capital and labor: $Y_t = A_t F(K_{t-1}, L_t)$, where A_t is a TFP shock. We assume a working capital requirement, following Neumeyer and Perri (2005), whereby firms need to borrow a fraction Υ of the wage bill before production takes place. These loans are obtained from bankers at the beginning of the period, and pay gross return $R_{W,t} = R_t^d + \frac{\mu_t}{\mathbb{E}_t[\mathbf{\Omega_{t+1}}]}$. The first-order conditions for labor and for physical capital are

$$A_t F_1(K_t, L_t) = Z_t, \tag{18}$$

$$A_t F_2(K_t, L_t) = W_t \left[1 + \Upsilon(R_{W,t} - 1)\right]. \tag{19}$$

2.3.2 Capital goods producers

Capital goods producers, owned by households, produce new investment goods using final output, and they sell those goods to firms at the price Q_t. The quantity of newly produced capital, $\Gamma(I_t)$, is an increasing and concave function of investment expenditure to capture convex adjustment costs. The objective of the capital producer is then to choose $\{I_t\}$ to maximize profits distributed to households:

$$\max_{I_t} \quad Q_t \Gamma(I_t) - I_t \tag{20}$$

The resulting first-order condition yields a positive relation between Q_t and I_t:

$$Q_t = \left[\Gamma'(I_t)\right]^{-1} \tag{21}$$

In the aggregate, the law of motion for capital is given by

$$K_t = \Gamma(I_t) + (1-\delta)K_{t-1}. \tag{22}$$

2.4 Interest rate determination

We assume that the safe rate, R_t, evolves (mostly) exogenously. Accordingly, R_t satisfies

$$R_t = \overline{R} + \mathcal{R}_t + f(x_t - \overline{x}), \tag{23}$$

where $\overline{R}$ and $\overline{x}$ are parameters, and $\mathcal{R}_t$ follows the stochastic process

$$\log(\mathcal{R}_t) = \rho_R \log(\mathcal{R}_{t-1}) + \epsilon_{R,t}, \tag{24}$$

with $\epsilon_{R,t} \sim N(0, \sigma_R)$. The (endogenous) term $f(x_t - \overline{x})$ is a small portfolio cost we introduce for technical reasons, as it helps ensure stationarity of safe asset holdings B_t (Schmitt-Grohe and Uribe, 2003). In the reminder of the paper we will refer to r_t as the logarithm of R_t, espressed in annualized terms.

2.5 Resource constraint, market clearing, and equilibrium

The resource constraint and the balance of payments equations, respectively, are given by:

$$Y_t = C_t + I_t + \mathbf{T}_t \tag{25}$$

$$T_t = B_t - R_{t-1}B_{t-1} \tag{26}$$

where $\mathbf{T}$ stands for net exports (or net transfers under an equivalent formulation where safe assets are provided by the government sector). An equilibrium is defined as stochastic sequences for the eight quantities $Y_t, C_t, I_t, \mathbf{T}_t, B_t, L_t, K_t, N_t,$ five prices $R_{K,t}, Q_t, R_t, R_t^d, W_t,$ and six banking sector coefficients $\mu_t, \mu_{B,t}, \nu_t, \alpha_t, \phi_t, x_t$ such that households, intermediaries, and firms solve their optimization problems, and all markets (for short-term debt, securities, new capital goods, final goods, and labor) clear, given exogenous stochastic sequences for A_t, ζ_t, and $\mathcal{R}_t$.

2.6 Constructing r**

The model has three endogenous state variables, in addition to three exogenous states associated with the shock processes. The endogenous state variables are the beginning-of-period values of the capital stock, K_{t-1}, bankers' holdings of safe assets, B_{t-1}, and bankers' aggregate deposits issued to households, $D_{t-1} = Q_{t-1}K_{t-1} + B_{t-1} - N_{t-1}$. Thus the period-$t$ state vector is

$$\mathbf{S}_t \equiv \{K_{t-1}, B_{t-1}, D_{t-1}, A_t, \zeta_t, r_t\} \tag{27}$$

The financial stability interest rate, r**, is defined as the threshold real rate above which financial instability arises; i.e., the real interest rate that makes the financial constraint *just* bind— keeping all other states variables of the economy unchanged. As such, r** can be viewed as a *threshold*: real interest rates below r** ensure that the economy remains in the financial stability regime. Specifically, we compute the size of the real interest rate shock $\epsilon_{R,t}$ that, holding constant the other elements of $\mathbf{S}_t$, makes the constraint (12) just binding. This counterfactual value of r_t—counterfactual because the actual economy has not been subject to this shock—is what we refer to as r_t^{**}.[6] If the economy is in the unconstrained region, the counterfactual shock $\epsilon_{R,t}$ is positive—that is, $r_t^{**} > r_t$—and *vice versa* in the unconstrained region.

A few remarks are in order. First, the mechanism by which a shock to the real interest rate makes the constraint (12) just binding is a standard valuation effect. Changes in the real rate affect the discount rate at which future dividends from capital are evaluated, and therefore the value of capital Q_t. In turn, changes in Q_t affect the net worth of intermediaries and hence the tightness of the constraint (12).[7]

Second, the purpose of r** is not to convey the idea that low real interest rates are good for financial stability—in fact, in this economy they can be harmful in the medium run as discussed later. r** simply maps the current state of the economy $\mathbf{S}_t$, and in particular its degree of financial vulnerability, into the interest rate space—the space that is relevant for monetary policy makers.

Third, the way this mapping is constructed in practice is by asking: how large a shock to the real rate can the economy take before entering the financial instability region if it is currently out of it, and *vice versa*, how large a cut does it need to get out of a crisis if it is in it? This implies that the object of interest is the size of the counterfactual shock, which we will refer to the r**-r gap in the remainder of the paper.

3 Functional forms and parameter values

In this section we describe, in turn, the functional forms and the model's calibration. The functional forms of preferences, production function, and investment adjustment cost are the following:

$$U(C_t, L_t) = \frac{\left(C_t - \chi \frac{L_t^{1+\epsilon}}{1+\epsilon}\right)^{1-\gamma} - 1}{1-\gamma} \tag{28}$$

$$F(K_t, H_t) = A_t (K_{t-1})^{\eta} L_t^{1-\eta} \tag{29}$$

$$\Gamma(I_t) = a_1 (I_t)^{1-\vartheta} + a_2 \tag{30}$$

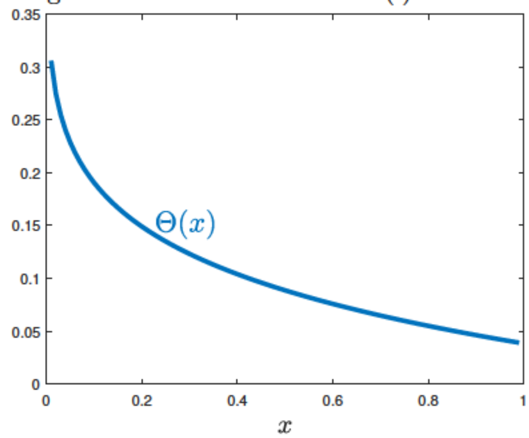
$$\Theta(x_t) = \theta \left(1 - \frac{\lambda}{\kappa} x_t{}^{\kappa}\right) \tag{31}$$

The utility function, equation (28), is defined as in Greenwood et al. (1988), which implies non-separability between consumption and leisure. This assumption eliminates the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor independent of consumption. The parameter γ is the coefficient of relative risk aversion, and ϵ determines the wage elasticity of labor supply, given by $1/\epsilon$. The production function, equation (29), takes the Cobb-Douglas form. The coefficient η is the elasticity of output with respect to capital. Equation

[6]In constructing r** we assume that R_t follows the low of motion implied by (24) and (23) in the counterfactual economy.

[7]When the financing constraint is binding the effect of interest rate shocks on intermediaries' balance sheets is amplified by the financial accelerator, as emphasized by numerous authors (eg Bernanke et al., 1999; Gertler and Karadi, 2011).

Figure 1: Functional form of $\Theta(\cdot)$ function



(30) defines the the investment technology, with the ϑ corresponding to the elasticity of the price of capital with respect to investment.

Equation (31) defines the relationship between the ratio of safe-to-risky assets, x_t, and $\Theta(x)$, which affects the tightness of the incentive compatibility constraint. As discussed before, we assume that Θ is convex—relatively flat around high levels of the safe asset ratio, but much steeper when x is low.

We calibrate preference, production, and financial sector parameters to standard values when possible, and report them in Table 1. We set the discount factor, β, to 0.995, which implies an annual real neutral rate of interest rate of 2%. The following four parameters are standard values in business cycle literature: The risk aversion parameter, γ, the capital share, η, and the depreciation of capital, δ, are set to 2, 0.33, and 0.025, respectively.

We set the Frisch labor supply elasticity (given by $1/\epsilon$) to 4, a value at the higher end of a wide range of values used in the literature. As in Gertler and Kiyotaki (2010), this relatively high value represents an attempt to compensate for the absence of frictions such as nominal wage and price rigidities, which are typically included in quantitative DSGE models. While our framework excludes these frictions to preserve simplicity, they likely have a role in accounting for employment and output volatility, so we partly compensate for their absence by setting a relatively high elasticity of labor supply.

We follow Gertler et al. (2019) in choosing the parameters governing the investment technology. More specifically, we set ϑ, which corresponds to the elasticity of the price of capital with respect to investment rate, equal to 0.25, a value within the range of estimates from panel data. We then choose a_1 and a_2 to hit two targets: first, a ratio of quarterly investment to the capital stock of 2 percent and, second, a value of the price of capital Q equal to unity in the risk-adjusted steady state.

The reference safe asset ratio, $\overline{x}$ is set to 0.2. This value roughly corresponds to the average holdings of safe assets by U.S. depository institutions (relative to total assets), defining safe assets as the sum of cash, reserves, federal funds, and Treasury and agency-backed securities. We then let

Table 1: Calibrated Model Parameters

Parameter	Symbol	Value	Source/Target
Conventional			
Discount factor	β	0.995	Interest rate 2%, ann.)
Risk aversion	γ	2	Standard RBC value
Capital share	η	0.33	Standard RBC value
Capital depreciation	δ	0.025	Standard RBC value
Elasticity of R to x	φ	0.005	Standard RBC value
Reference safe asset ratio	$\overline{x}$	0.2	
Labor disutility	χ	2.5	Steady state labor of 33%
Inverse Frisch elast.	ϵ	1/4	Gertler and Kiyotaki (2010)
Elasticity of Q w.r.t. I	ϑ	0.25	Gertler, Kiyotaki, Prestipino (2019)
Investment technology	a_1	1.1261	$Q = 1$
Investment technology	a_2	-0.1696	$\Gamma(I) = I$
Financial Intermediaries			
Survival rate	σ	0.925	Exp. survival of 3.5 yrs
Transfer rate	ξ	0.20	{ Frequency of crises around 3%,
Fraction divertable	θ	0.69	Leverage of 6}
Elasticity of $\Theta(x)$ w.r.t. x	κ	0.124	
	λ	0.117	
Shock Processes			
Persistence of interest rate	ρ_R	0.95	
SD of interest rate innov. (%)	σ_R	0.06	
Persistence of TFP	ρ_A	0.90	
SD of TFP innov. (%)	σ_A	0.44	
Steady State level of liquidity shock	$\bar{\zeta}$	0.00125	
SD of liquidity shock innov. (%)	σ_ζ	0.0313	

$\overline{R}$ adjust such that x equals the target $\overline{x}$ in the steady state.

We then need to assign values to the five parameters relating to financial intermediaries: the survival rate of bankers, σ, the transfer to entering bankers, ξ, and the parameters governing the $\Theta(\cdot)$ function: θ, λ, and κ. We calibrate σ to 0.925, implying that bankers survive for about 3.5 years on average. This value of banks' survival rate is within the range of values found in the literature. The start-up transfer rate ξ, which ensures that entering bankers have some funds to start operations, is set to target a leverage ratio of around 6 in the risk-adjusted steady state. This target is an estimate of the leverage ratio of the aggregate financial sector (broadly defined). We then set the three parameters governing the asset diversion function to hit three targets: a frequency of severe financial crises of 3 percent annually, an asset diversion fraction that is nearly zero as x approaches unity, and a Θ function that is very flat at high values of x (see figure 1). The second target is based on the presumption that a portfolio composed of purely safe assets is nearly impossible to divert. The third target captures that notion that when the banker's asset portfolio is already very safe, there are almost no gains (in terms of reduced agency frictions) of marginally making it safer. As we will discuss next, given these calibration targets the model economy produces infrequent financial crisis consistent with the empirical evidence documented in Schularick and Taylor (2012) using historical data for several developed countries.

Finally, as we have direct observations on real interest rates, we fix the persistence and standard deviation of innovation for the interest rate shocks, ρ_R and σ_R, to the real interest rate from U.S. data. We then choose the standard deviation of the TFP shock so that the model matches the standard deviation of output growth in the United States, equal to about 2 percent annually since the mid 80s. The mean of process for the liquidity shock, $\overline{\zeta}$, is calibrated to 0.00125 to deliver a steady state liquidity premium of 50 basis points, and we set its standard deviation to deliver a small volatility of ζ_t (equal to one-fourth of its mean).

4 Model results

We now turn to the key quantitative properties of the model, and then discuss the drivers and dynamics of the financial stability rate, r**.

4.1 Quantitative properties of the model

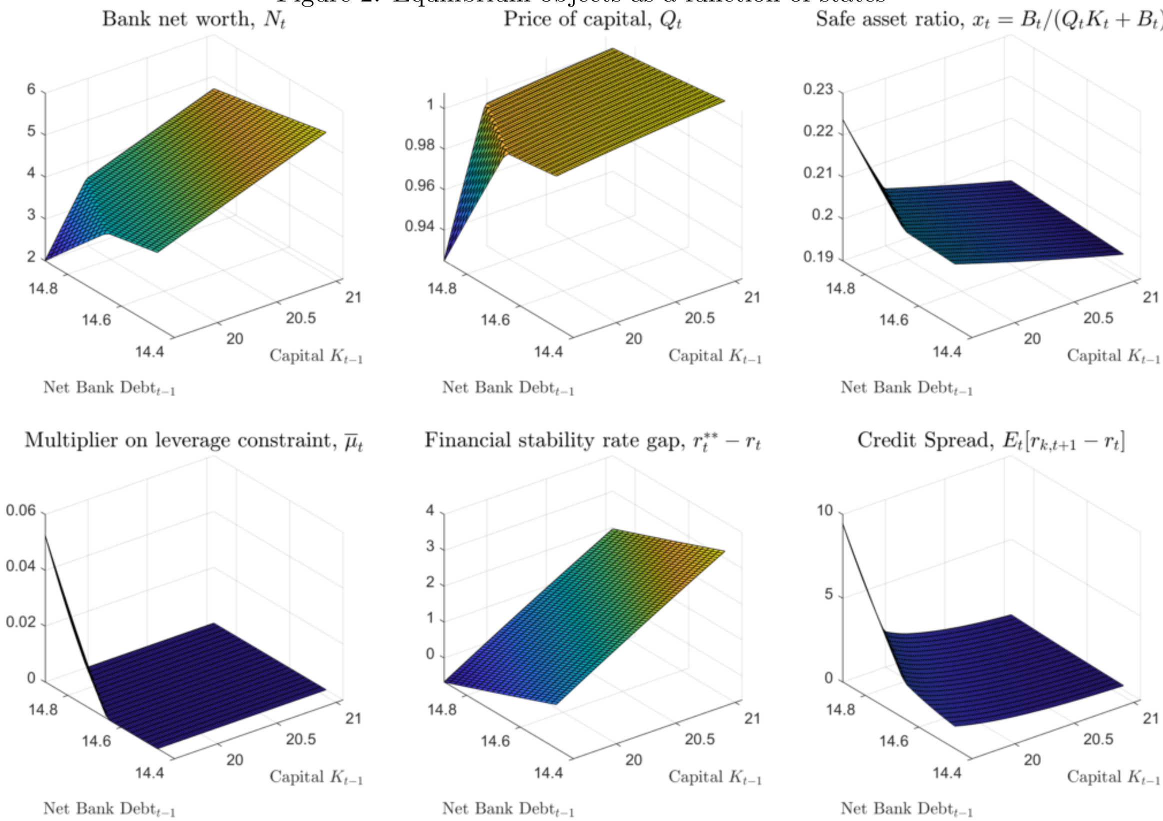
In this section we show that while the model is simple, it is quantitatively realistic, especially in capturing the dynamics surrounding financial crises. Quantitative realism is important as it enables us to use the model for constructing an empirical measure of the financial stability rate in the data, which is one of the main objectives of our paper.

The model economy displays nonlinearity and state-dependence, which is induced by the leverage constraint, as well as amplification via the financial accelerator mechanism that occurs when the constraint binds. In order to illustrate both of these features of the model, we first show banks' behavior as a function of some of the endogenous states in our calibrated economy. Figure 2 displays the three-dimensional policy functions with aggregate banking sector debt and aggregate capital as arguments. It is apparent that when the leverage constraint becomes binding, as reflected by positive values of the excess return $\overline{\mu}$ (which is linked positively to the Lagrange multiplier on the constraint), the responses of banks' net worth, asset prices, and the holdings of safe assets to a given change in the states are larger compared to the region in which the constraint is slack (i.e. $\overline{\mu} = 0$). The constrained region is not only characterized by very low values of bankers' capital or by very high values of banking sector debt, but also by a combination of relatively low values the former and relatively high values of the latter. The threshold of banking sector debt for which the constraint becomes binding, and hence the level of the financial stability interest rate gap, r**- r, is a function of the level of bankers' assets.[8]

It is worth emphasizing that the model features a form of precautionary behavior in bankers' choice of the safe asset ratio x_t (shown in the right column on the top row). When the economy is far from the constrained region (where the bank capital is high and the net bank debt is low), the safe asset ratio of the banker is quite small. Interestingly, as the banker is approaching the constrained region (either via lower capital or higher bank debt), even before the constraint starts

[8]Note that r** gap chart looks very linear, while all the other charts are very nonlinear. This is because the power of changes in the real interest rate affecting the financing conditions varies with the extent to which the economy is constrained, as discussed later.

Figure 2: Equilibrium objects as a function of states

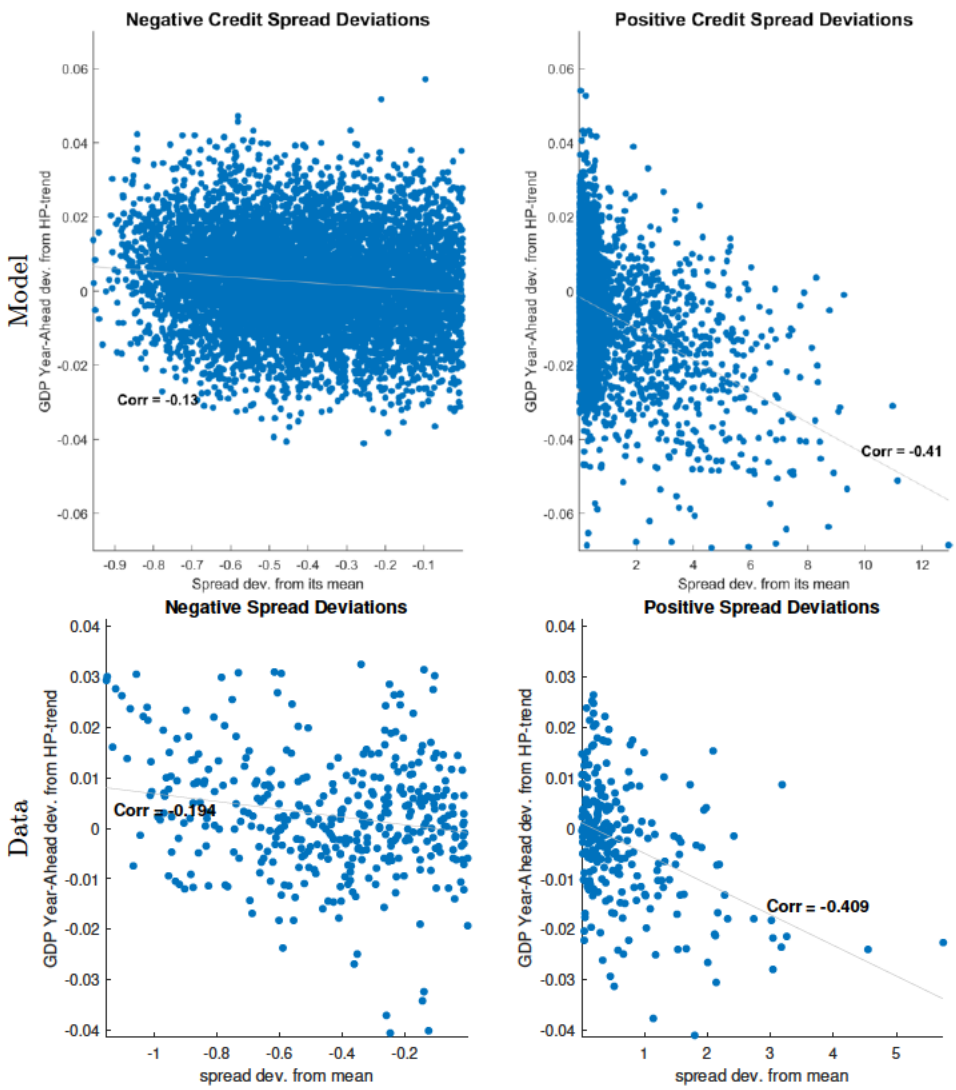


Note: Endogenous variables as a function of two of the model state variables—the beginning-of-period value of the capital stock, K_{t-1}, and bankers' net indebtedness (defined as total deposits minus excess returns on the safe asset). All other states kept at their risk-adjusted-steady-state value.

to bind, banks start to accumulate safe assets and de-lever on risky capital (so that x rises) in an attempt to avoid the crisis. Nonetheless, crises occasionally happen in the model, as either the precautionary behavior arrives too late or is not strong enough to avoid the crisis. A similar behavior occurs within the constrained region, with x_t now increasing more steeply as the economy enters the constrained region. This is because now the value to the banks of relaxing the constraint rises sharply, inducing them to tilt their balance sheet toward safe assets. One can also see from the figure that credit spreads start to rise as the economy moves towards the binding region, and they eventually rise much more steeply along with sharply deteriorating equity values and falling asset prices when the crisis happens.

We next evaluate our model's quantitative performance in matching the following two facts associated with the relationship between financial stress episodes and the real economy (see Akinci and Queralto (2022) for a more extensive characterization of these empirical regularities associated with crises in the data). First, we demonstrate that the model captures the asymmetric relationship between credit spreads and economic activity. Second, we show that the average financial crisis in the model is consistent with the evolution of real and financial variables around actual crises

Figure 3: Credit spreads and economic activity



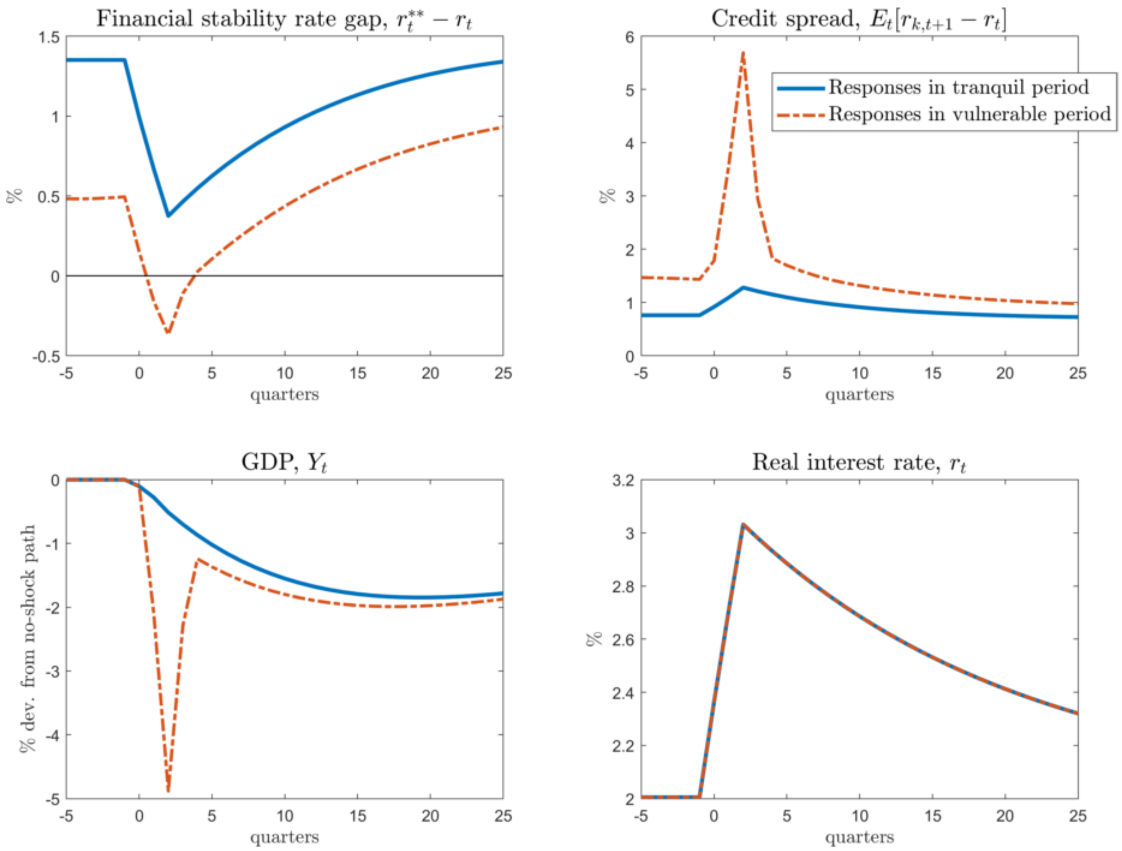
Note: The upper (lower) left (right) panel shows the relationship between year-ahead real GDP, expressed as a deviation from its HP trend, and the negative (positive) deviations of the credit spread from its mean in the model (data). Data sources: Haver Analytics, Gilchrist and Mojon (2014), Bank of England, Gilchrist and Zakrajsek (2012), authors' calculations.

episodes.[9]

The quantitative model displays strong nonlinearities, consistent with both previous theoretical macro-finance contributions (for instance, He and Krishnamurthy, 2013; Brunnermeier and Sannikov, 2014) as well as evidence from the empirical literature (for example, Adrian et al., 2019 and Boyarchenko et al., 2022). Figure 3 illustrates the asymmetric relation between credit spreads and

[9]Our calibrated model can also account for the fact that credit spreads display occasional spikes. As shown in figure A1 in the appendix, which displays histograms of credit spreads both from model-simulated observations (left panel) and from actual data (right panel), credit spreads display occasional large spikes in the data and the model delivers a right skewed distribution of credit spreads, as observed in the data.

Figure 4: Responses to an interest rate shock in financially vulnerable and non vulnerable periods

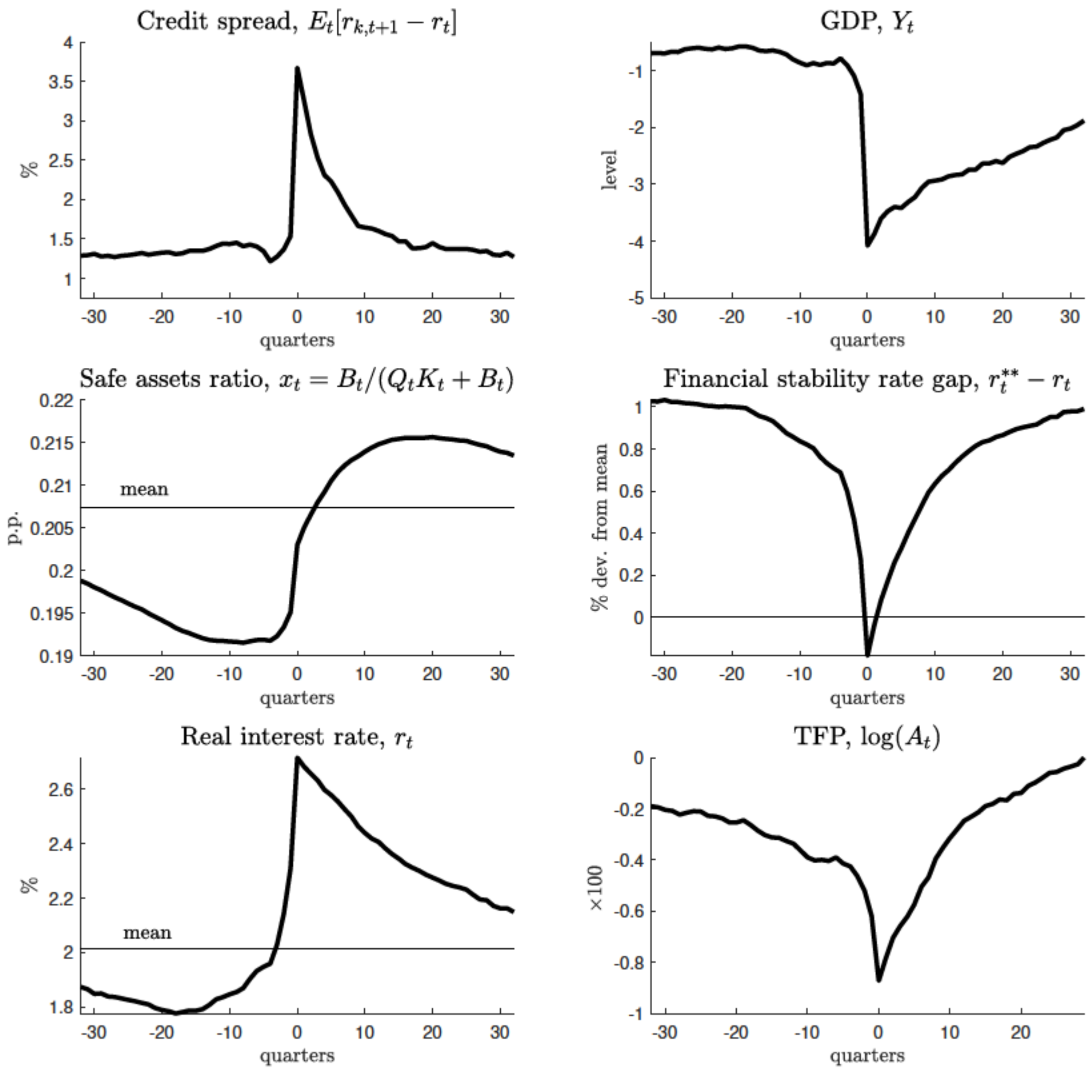


Note: The figure shows the effects of a positive interest rate shock for different initial states of the economy. In the blue solid lines, the economy is initially far away from the constrained region. In the red dashed lines, the economy is initially close to the constrained region.

economic activity predicted by the model: when financial stress is relatively elevated, higher spreads tend to be more strongly associated with weaker real activity than in more tranquil times. In particular, when spreads are above their sample mean there exists a correlation between credit spreads and real economic activity (calculated as year-ahead deviation of real output from its HP trend) of about -0.41, compared with -0.13 obtained when spreads are below the sample mean. These patterns are consistent with the empirical results shown in the lower panel of the same figure (and are reminiscent of similar evidence in Stein, 2014). Key to explaining the model's ability to generate this asymmetry is the occasionally binding financial constraint: a binding constraint is associated with higher spreads, since banks are prevented from arbitraging away the difference between risky and safe returns, and at the same time with an amplified response of real activity to shocks via the financial accelerator, as we show below.

Figure 4 illustrates further the state-dependence induced by the leverage constraint, as well as the amplification via the financial accelerator mechanism that occurs when the constraint binds. The figure shows the responses of the r**-r gap, credit spreads, and output to a one percentage point

Figure 5: Average financial crisis in the model



Note: A financial crisis event in the model is defined as an event in which banks' constraint binds for at least two consecutive quarters. We simulate the economy for a large number of periods and compute average paths before, during, and after financial crisis events (the crisis starts at time 0 for each path).

increase in real interest rates. The blue solid lines display the responses when the economy is in a tranquil period (r** is 1.4 percentage points above r before the shock hits). By construction the one percentage points shock is not large enough to push the economy into the constrained region (the r**-r gap remains positive throughout) and, as a consequence, has only modest effects on output and spreads. The red dashed lines display the responses when the economy is much closer to the financially unstable region (r** is only 0.5 percentage points above r). In this case, the increase in interest rates is enough to tip the economy into this region. As discussed in Section 2, the financial accelerator mechanism then kicks in, and the response of both spreads and the real economy to the shock is much larger than in the financially stable region: the spread jumps by about 400 basis points (annually), and output declines by about 5 percent. Note that while the impact on all other variables is larger when the economy is close to the financially vulnerable region, the decline in the r**-r gap is smaller. This is not surprising in light of its definition: the r**-r gap measures the size of interest rate shock needed to make the constraint just binding. When the economy is vulnerable, the effect of an interest rate shock on the economy is larger, and hence smaller sized shocks are required to make the constraint just binding. Last, the response of both output and spreads in figure 4 is short-lived because, by design, the economy quickly returns into the tranquil region. As we discuss next, the response is much more long-lasting when the economy remains in the vulnerable region for longer, as is the case during typical financial crisis.

Figure 5 shows how the calibrated model can produce quantitatively realistic crisis dynamics. The figure displays the *average* path in the economy before, during, and after entering a financial crisis (time 0), where a crisis event is defined as the leverage constraint (12) being binding for at least two consecutive quarters. First, the average crisis in the model is associated with a sizable increase in spreads and a persistent decline in output, in line with economic developments during the Great Recession (top panels). Specifically, output decreases by about 4 percent, with an almost 20 percent fall in investment (not shown). Leading up to the crisis, the economy becomes progressively more financially vulnerable, with both the safe asset ratio x_t and consequently the r**-r gap steadily decreasing (middle panels). This higher vulnerability is in part driven by below-mean values of the real interest rate for several quarters before the crisis, which contribute to the risk-taking behavior of intermediaries as discussed in the next section. Thus, while the crisis is ultimately triggered by exogenous forces (a sharp upward movement in the real rate, along with deteriorating TFP; bottom panels), the pre-conditions for its occurrence reflect endogenous choices. These choices place the economy in a fragile region where it is more vulnerable to shocks, as shown in figure 4.

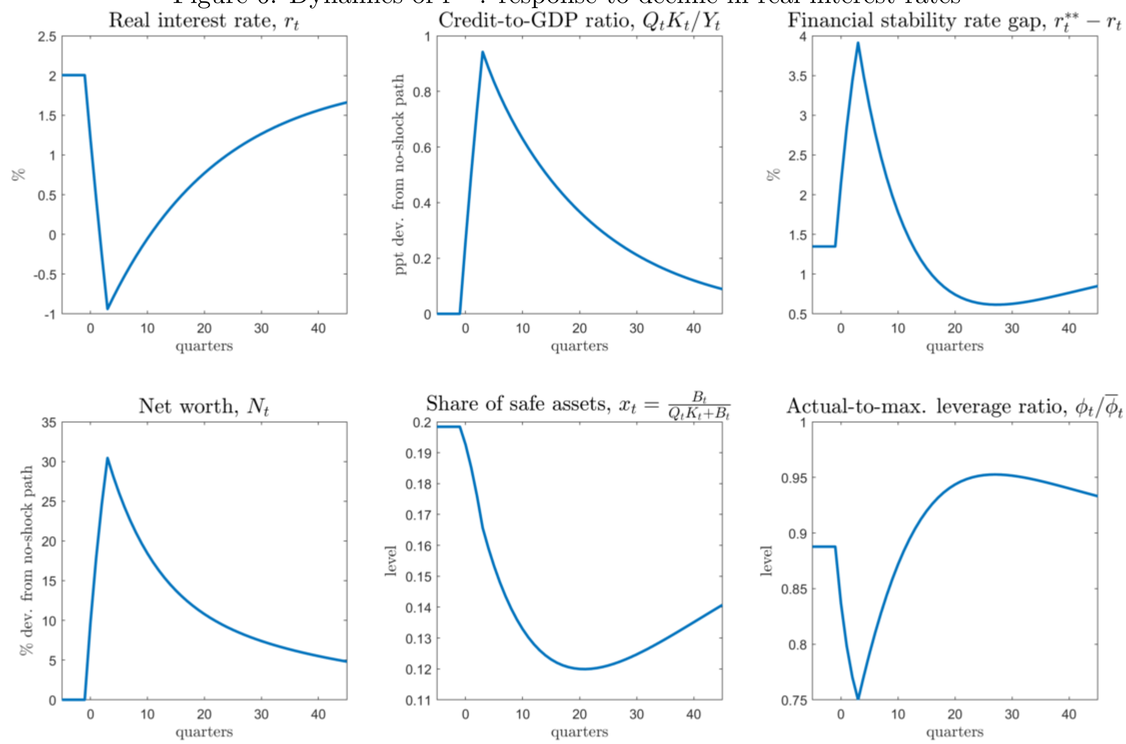
4.2 Financial stability and interest rates: dynamics of r**

In this section we stress the very different short- and medium-run implications of persistent declines in interest rates in the calibrated version of the model, and characterize the dynamic properties of the financial stability interest rate r**.

We start by showing in figure 6 the dynamic evolution of variables characterizing the financial sector, such as intermediaries' net worth, the share of safe assets in their balance sheets, the actual-to-maximum leverage ratio (i.e., the distance to the endogenous leverage constraint), and the credit to GDP ratio, in response to an unexpected 3 percentage points fall in the real rate of interest. The figure also shows the dynamics of financial stability interest rate gap, r**- r. Before the shock hits, the economy is at the risk-adjusted state state, which features a real rate of interest of 2 percent. In the experiment, the real rate then falls and returns to the steady state only gradually, following the law of motion in (24).

In the near term, the reduction in real rates leads to an improvement in financial conditions via the valuation effect discussed in section 2: the price of capital Q (not shown) rises on impact, leading to higher net worth, lower leverage, and therefore a higher r**- r gap. These short-run dynamics are simply the reverse of what is shown in figure 4. The focus of figure 6 is the medium-term impact of

Figure 6: Dynamics of r**: response to decline in real interest rates

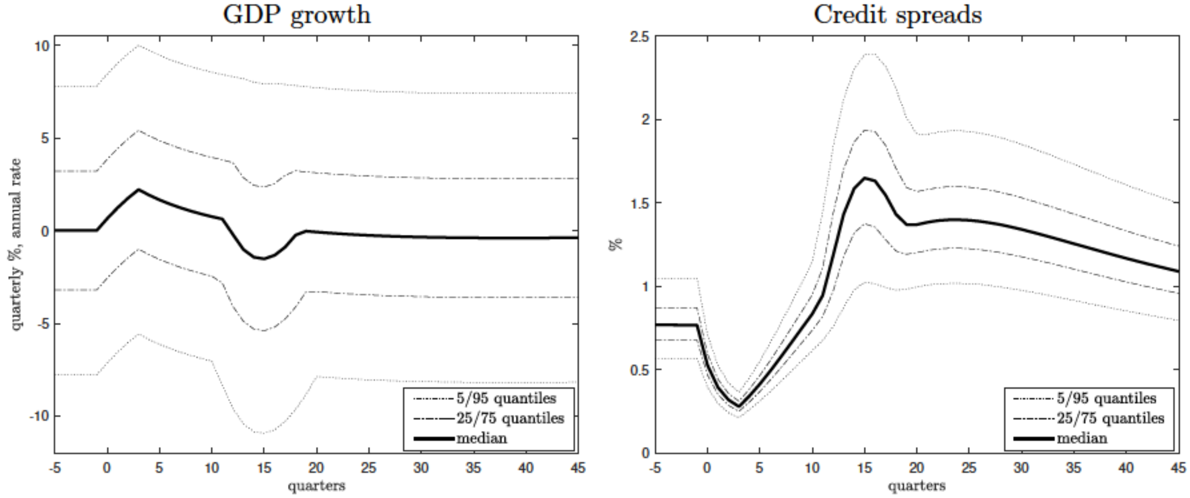


Note: Responses to an unexpected 3 percentage points fall in the real interest rate, r_t. The economy is at the risk-adjusted steady in the initial period. Variables indicated % dev. computed as percent deviations relative to their risk-adjusted steady state values.

the persistent decline in real rates on the financial sector. Gradually, this decline triggers a "reach for yield" behavior by bankers, as they shift their portfolios from safe assets (B falls) towards riskier capital (K increases), leading to an increase in the credit-to-GDP ratio (as in Schularick and Taylor, 2012). This type of behavior arises naturally in the model as intermediaries try to arbitrage away the difference between expected R_k and the new, lower levels of R, and can do that at will while the leverage constraint is not binding. Thus, the ratio of safe-to-risky assets in intermediaries' portfolio, x_t, declines gradually but persistently. As discussed earlier, the degree of agency friction facing bankers depends on the asset composition of banks' balance sheet: frictions are more severe when x_t is low. Moreover, the $\Theta(\cdot)$ function is convex, so vulnerabilities increase at a faster pace as x_t falls. As a result, the ratio of actual-to-maximum leverage begins to rise gradually after about a year and after roughly three years it has surpassed its initial point, leaving the economy more vulnerable than in its initial state.[10].

[10] Recent literature has also emphasized the result that reductions in interest rates may eventually put the economy closer to the financial instability region, most prominently in the models in Boissay et al. (2022) and Coimbra and Rey (2017). The mechanisms behind this result in these models are quite different from ours, however. One key difference is that our model includes a positive effect of interest rate reductions on net worth, a salient feature in the long literature on the financial accelerator (eg Bernanke et al., 1999; Gertler and Kiyotaki, 2010) that is not present in the aforementioned papers. Standard financial accelerator models however generally do not imply higher financial vulnerabilities in the medium run after a rate reduction (in Gertler and Kiyotaki, 2010, $\Theta(\cdot)$ is constant, and intermediaries do not face a portfolio choice between safe and risky assets).

Figure 7: Growth-at-risk in the model



Note: Responses of the distribution of GDP growth and credit spreads to an unexpected 3 percentage points fall in the real interest rate, r_t. The 5^{th}, 25^{th}, 50^{th}, 75^{th} and the 95^{th} quantiles are reported. These responses are computed by hitting the economy with randomly drawn shocks during the transition, on top of the initial negative interest rate shock, while the responses in figure 6 are computed assuming there are no shocks other than the initial one.

The increased financial fragility in the economy is captured by the financial stability interest rate gap, r**- r. Before the shock hits the r**-r gap is 1.4 percentage points—the same initial level as in figure 4 for the non-vulnerable case. While the gap rises sharply on impact as r drops, it declines over time and ends up after about five years lower than where it started—specifically, at the same initial level as in figure 4 for the vulnerable case. As we have seen, this implies that in the medium term the economy ends up being more vulnerable to shocks and in particular to increases in interest rates. Brunnermeier (2016) refers to this phenomenon as "financial dominance."[11] While the model we are currently working with abstract from endogenous monetary policy, our analysis suggests that the extension with nominal rigidities may feature interesting intertemporal policy trade-offs between stabilizing the financial system now by lowering interest rates and making it potentially more vulnerable in the future. The impulse responses in figure 4 also show that the current level of the r**-r gap is a measure of *current* financial vulnerability but *not* a predictor of future vulnerabilities: the gap increases in the first few quarters after the shock in spite of the fact that the economy will eventually be more vulnerable. We discuss below similar results for credit spreads.

Figure 6 does not include any measure of economic activity or spreads. The left panel of figure 7 reports the response of GDP growth in the economy, except that it displays the response of the entire distribution of possible outcomes. The cut in real rates has at first a positive impact on all quantiles of the distribution, as the higher investment translates into higher output. The

[11] Figure A2 in the appendix shows that these dynamics depend on the initial level of interest rate: a cut in real rates when these rates are already low implies a much more dramatic medium-term increase in financial vulnerability than if rates are high, a result that mirrors that in Coimbra and Rey (2017).

increased financial instability in the economy due to the portfolio shift of intermediaries has instead a very asymmetric impact: it mostly affects the lower quantiles but has no or little effect on the higher quantiles. To put it differently, the less-financially-stable economy is more vulnerable to bad shocks, but its response to good shocks is no different than before. This finding is reminiscent of the growth-at-risk literature (Adrian et al., 2019). In particular, Caldara et al. (2021) provides a Markov-switching interpretation of the growth-at-risk results that is in line with the workings of this non linear, two-regime model. In the model, a cut in interest rates eventually produces a higher likelihood of switching to the financially unstable regime, where the economy is more vulnerable to shocks, delivering the asymmetry. That is, if the economy is hit by positive shocks the cut in rates has no long-term consequences, but if it is hit by sufficiently large adverse shocks the implications can be dire: the economy transitions into a crisis.[12]

The right panel of figure 7 reports the response of credit spreads. Again, in the short run the entire distribution of spreads is compressed. But this low level of spreads is not necessarily good news in terms of financial stability, as in the medium term it is harbinger of higher fragility—a result that is in line with those in Coimbra and Rey (2017) and with the empirical evidence in López-Salido et al. (2017) and Krishnamurthy and Muir (2017). The results discussed above also share a number of striking similarities with the empirical evidence in Grimm et al. (2023), who study the impact of persistent declines in real rates (specifically, a fall of the real rate below the long run r*, as measured by Del Negro et al., 2019) on the likelihood of crisis events. In particular, the responses in Grimm et al. (2023) show both a positive short-run effect and a negative medium-run effect on financial vulnerability, where the latter takes place about three to four years after the initial decline in interest rates, as in the results just discussed.

In this very non-linear model, correlations are unlikely to provide a good description of the relationship between real interest rates and financial stability.[13] We therefore turn next to machine learning (ML) techniques, and in particular to Support Vector Machine (SVM) algorithms, to document this relationship in the model.[14] We do so because the flexibility of machine learning, coupled with the fact that we can of course simulate as much data as we want from the model, is likely to provide more reliable statistics than any parametric approach.

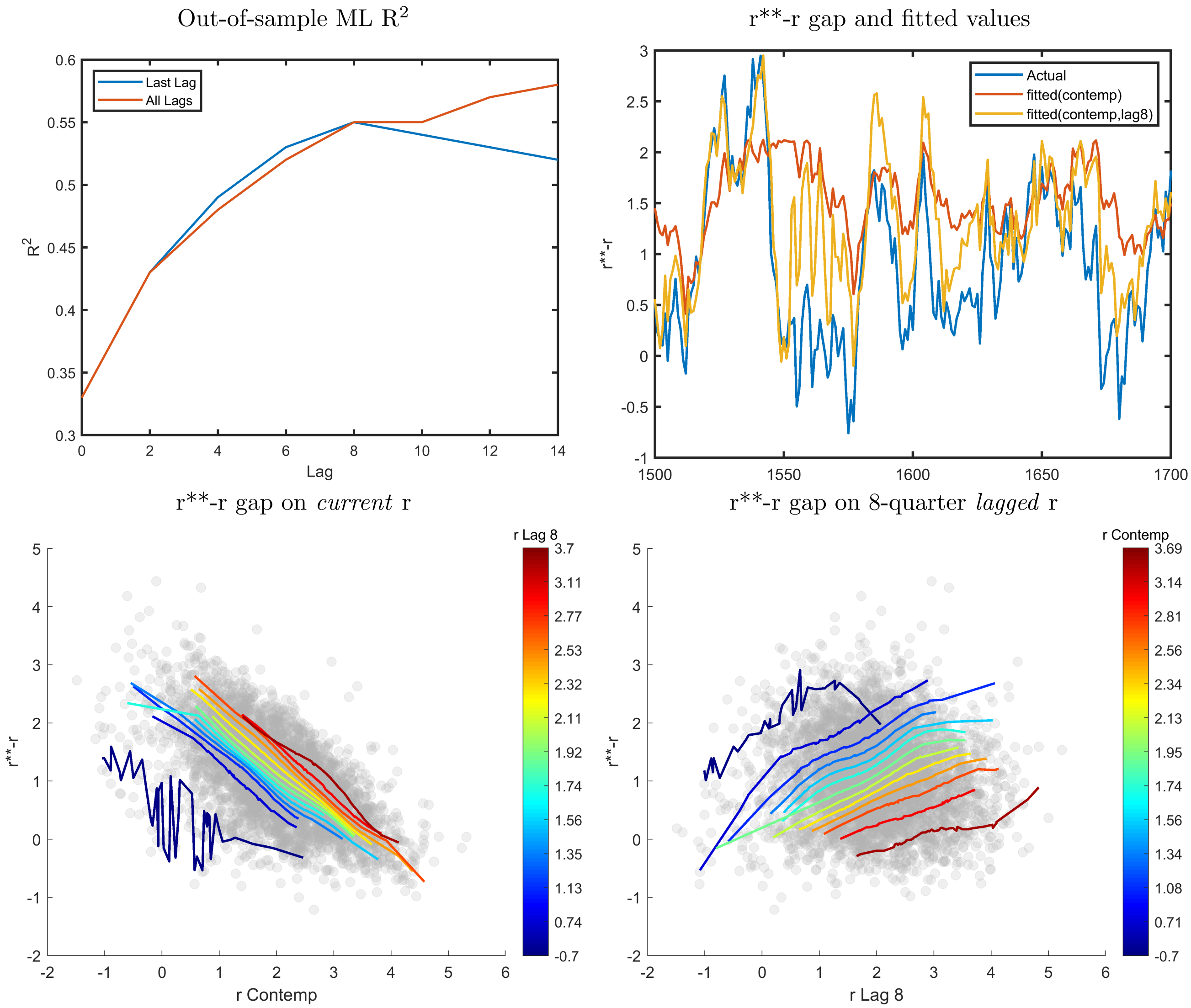
Figure 8 makes the point that the relationship between financial stability and interest rates is well captured by the current rate (short-term effect) and by the 8-quarter lag of the interest rate (medium-term effect). The left hand side panel of figure 8 shows the *out-of sample* R^2 obtained from

[12]Hubrich and Waggoner (2022) also provide empirical results from a Markov-switching vector autoregression that can be reinterepreted in light of this model, as discussed in the next section.

[13]For good measure, figure A3 in the Appendix shows the model-implied cross-correlogram obtained from model simulations between the real interest rate and i) the safe-assets ratio x_t, ii) spreads, and iii) r** - r. The correlation between the real interest rate r_{t+h} and x_t is positive for all h but peaks for $h \sim -10$, in line with the responses in figure 6. The correlation between r_{t+h} and spreads is s positive for $h = 0$ but is also negative (and reaches a trough) for $h \sim -10$. The correlation between r_{t+h} and the r**-r gap is the mirror image of the correlation in spreads. Low rates predict lower safe assets ratios in the banking system, higher spreads, and higher fragility (a low r**-r gap) about two years ahead.

[14]While there is a growing literature using machine learning (ML) to solve non linear models, to our knowledge we are the first paper using ML to understand and document how they work. We checked the robustness of the results to other machine learning approaches and found that these give similar answers. Specifically, we have tested multiple ML regression algorithms including Support Vector Machine (SVM) models, Ensemble models, Decision Tree Models, Gaussian and Naive Bayesian models. Out of these models, SVM using a Radial Basis Function (RBF) kernel worked best out-of-sample.

Figure 8: Financial stability, current, and lagged real rates



Note: Top left panel: Out-of-sample R^2 obtained from ML regression for lags 0 through h (red line) and 0 plus h only (blue line). Top right panel: r**-r gap (blue) and ML fitted values obtained using only contemporaneous real rates (orange line) and contemporaneous plus 8 lags (yellow) for a section of the simulated data. Bottom panels: estimated relationship between the r**-r gap and current real rates for given 8-quarter lagged rates (left panel) and for lagged rates for given current rates (right panel). The color of the fitted lines varies from dark purple to red depending on the value of the other regressor, while the light gray dots are observations.

ML regressions of the r**-r gap on the lags of the real interest rate, including the contemporaneous one (lag 0).[15] The orange line in the figure shows the R^2 obtained including *all* lags, from 0 to the lag indicated on the x-axis. The blue line shows the R^2 obtained from lag 0 and the lag indicated on the

[15] The R^2 is out-of-sample in the sense that the ML technique is trained on a different sample from that on which the R^2 is computed.

x-axis *only*. The figure shows that contemporaneous real rates explain about a third of movements in the r**-r gap. Lagged real rates add substantially to the explanatory power, reaching almost 60 percent using up to 8 lags or more. It is actually enough to include contemporaneous and 8-quarter lagged real rates to explain most of the variation in the gap. The top right hand panel of figure 8 plots the r**-r gap (blue line) and the fitted values obtained using only contemporaneous real rates (orange line) and contemporaneous plus 8 lags (yellow line) for a section of the simulated data. The figure shows that while the fit of the yellow line is far from perfect (lagged real rates are not the only thing that matters for financial stability) lagged interest rates help a lot explaining the low frequency patterns of the gap.

The bottom two panels of figure 8 show the estimated relationship between the r**-r gap and current real rates for given 8-quarter lagged rates (left panel) and for lagged rates for given current rates (right panel). The color of the fitted lines varies from dark purple to red depending on the value of the other regressor, while the light gray dots are observations. Consistent with the impulse responses in figure 6, the contemporaneous relationship is negative. Given past rates, current low rates help financial stability via the valuation effect discussed above. Interestingly, this relationship is weaker for very low values of the lagged rates: the financial system is so vulnerable that there is not as much that low current real rates can do to rescue it. But there is also an almost equally strong relationship (note that the size of the y-axis is the same in both panels) between lagged rates and financial stability, and of the opposite sign: when 8-quarter lagged rates are low, the economy tends to be close to the financial instability region.

5 Measuring r**

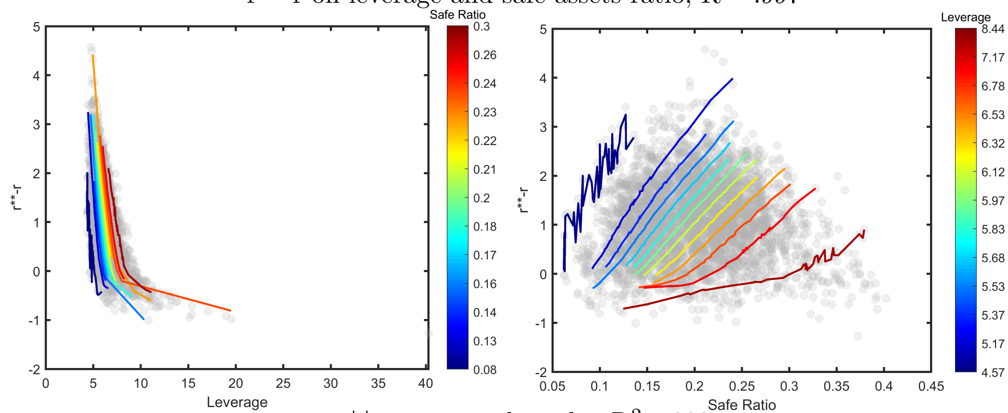
The previous sections defined r** and discussed its properties. This section provides a measure of r** for the US economy and discusses its evolution over the past 50 years. The financial stability rate r** is a latent variable—we do not directly observe it. In order to measure it in the data we adopt the following strategy. First, we figure out in the model which variables that are observable in the data map into r**, or, more precisely, into the r**-r gap (section 5.1). Then, we find the empirical counterpart of these variables and use them to obtain a measure of r** in the data (section 5.2). Last, we provide an external validation of the r** measure by i) providing an alternative measure that uses a completely different set of observables, and ii) computing the time-varying sensitivity of financial conditions to interest rate shocks (in the data, these correspond to exogenous shocks to interest rates) and arguing that this time variation is very much in line with the r** measure we construct.

5.1 Mapping r** into observables

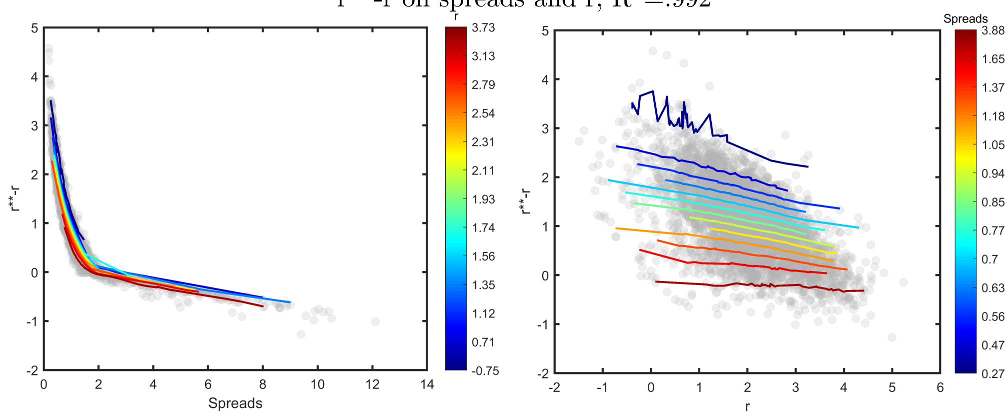
We use machine learning techniques to figure out in the model which variables best capture our proposed measure of financial instability, the gap between r** and r. Again, we do that because these techniques are flexible enough to provide an accurate description of the workings of our non linear model. Among all the model variables, we search for the two variables that in the model

Figure 9: The r**-r gap, financial constraints, and credit spreads

r**-r on leverage and safe assets ratio, R^2=.997



r**-r on spreads and r, R^2=.992



Note: Top panels: leverage and safe-ratio. Bottom panel: spreads and the real rate. Estimated relationship between the r**-r gap and one regressor, for given values of the other regressor (the color of the fitted lines varies from dark purple to red depending on the value of this other regressor). Light gray dots are observations.

provide the best out-of-sample fit of the r**-r gap (as we will see, two variables already provide near perfect fit). Not too surprisingly in light of how the constraint (12) works, we find that the best two variables are leverage and the safe assets ratio x, with an R^2 of .997. The top two panels of figure 9 depict the relationship between these variables and the gap. The relationship between leverage and r**-r is of course negative, with a slope that is very different depending on whether the economy is in the financially stable region or in the unstable one. In the first region the slope is very negative, implying that financial vulnerability increases substantially as leverage rises. In the second one the slope is less negative: past a certain threshold it matters less whether leverage increases further or not. The relationship between the safe assets ratio x and r**-r is positive, again not too surprisingly, with a slope that very much depends on the level of leverage: steep for low leverage, but much flatter for high leverage (a very levered economy is vulnerable almost regardless of x).

We could construct a measure of r** in the data by finding empirical counterparts for leverage and the safe asset ratio, x and plugging them into the ML regressions just discussed. We actually do this in section 5.3 below. There a few reasons why this is not our baseline approach. One is that leverage in the model corresponds to leverage of the entire financial system, while in the data we can only measure leverage for specific financial institutions. Even then we would face a challenging aggregation problem, as different institutions may have different leverage/portfolio combinations.[16] A second problem is that readily available measures of leverage or of safe assets for financial intermediaries (eg, the H.8 report, or the flow of funds) do not necessarily correspond to what these variables are in the model. Leverage in the model is defined by assets over equity, both of which are evaluated at market prices, for instance. Similarly, long term Treasuries are often lumped with short-term Treasuries in balance sheet data, as they may be considered equally safe according to credit risk measures, but are clearly not safe in terms of interest rate risk—as the 2023 banking turmoil has emphasized.

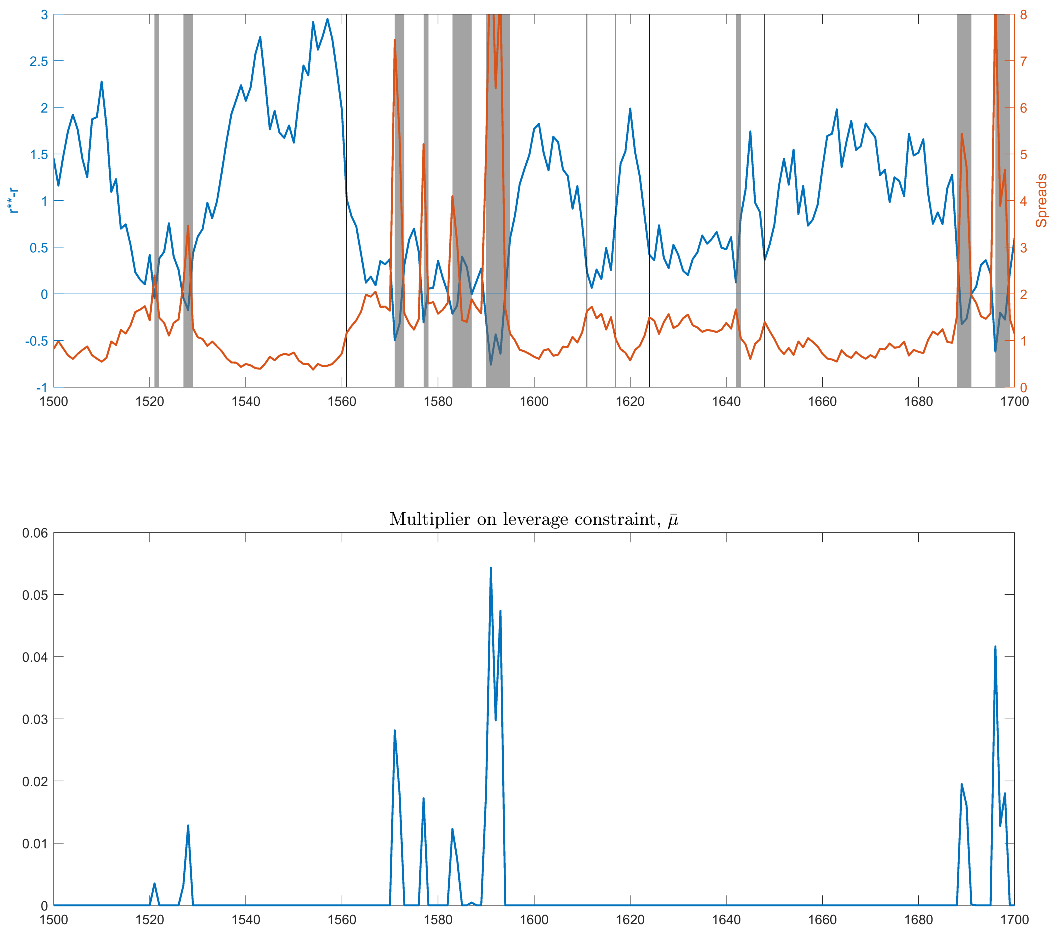
The next best couple of variables that describe the r**-r gap in the model are variables that are much easier to measure in the data: credit spreads and the level of the real interest rate. The out-of-sample R^2 of this couple of variables is almost as high (.992) as that of leverage and x. The bottom two panels of figure 9 depict the relationship between these variables and the gap. The relationship between spreads and the gap is quite tight (meaning that the fitted lines cover most of the observations), negative, and very non linear. The relationship between r and the r**-r gap is also negative, but its slope depends on the level of spreads. When spreads are low the relationship is stronger, and much weaker when spreads are high.

It is worth briefly elaborating on the tight (non linear) relationship between spreads and the r**-r gap in the model that emerges from the bottom left panel of figure 9, since it is key in understanding some of the empirical results discussed in the remainder of the paper. The top panel of figure 10 shows the path of credit spreads (blue line) and the r**-r gap (red line) in data simulated from the model; the bottom panel shows the value of the Lagrange multiplier on the leverage constraint, $\overline{\mu}$. By construction the r**-r gap is negative when $\overline{\mu}$ is positive. The figure highlights a few points. First, whenever the constraint binds spreads become very volatile. As documented above, when $\overline{\mu}$ is positive any shock to the economy has much larger effects on intermediaries' balance sheets, and hence on their ability to arbitrage away the difference between expected returns on capital and returns on safe assets. In fact, the shaded areas demarcate high spread volatility regions, constructed using a simple heuristic algorithm: these high volatile regions mostly coincide with periods where $\overline{\mu}$ is positive.[17] Second, in the financially constrained regions spreads move a lot more than the r**-r gap, and *vice versa* in tranquil times. This is consistent with the non linearity in the bottom left panel of figure 9. The source of this non linearity is again the financial accelerator. When the constraint is binding, the effect of an interest shock on the economy in general, and on spreads in particular, is much larger. Therefore the cut in rates needed to bring the economy back into the financially stable region—which is what the r**-r gap measures when it is negative—does not have

[16] Coimbra and Rey (2017) is one of the few papers emphasizing the role of intermediaries' heterogeneity.

[17] The heuristic algorithm works as follows. Call "spread jumps" changes in spread $\Delta spread_t$ that are above some quantile q of the distribution, i.e., $|\Delta spread_t| > q$. We then define a financial stress region as a sequence of jumps no more than two quarters/six months apart, beginning with an upward jump and ending with a downward jump. The requirement that jumps are no more than two quarters apart is dictated by the desire to avoid including in our definition non constrained regions in which sporadic increases/decreases in spreads, which in the model are driven by liquidity shocks, take place. One can think of this heuristic approach as an alternative to estimating a regime switching model where the spread data is divided into high and low volatility regions.

Figure 10: Financial Stability Rate Gap, Spreads, and Financial Constraints, Model



Note: r**-r gap (blue; top panel, left axis), spreads (red; top panel, right axis), and $\bar{\mu}$ (blue; bottom panel), the Lagrange multiplier on the leverage constraint (1), for a section of the simulated data.

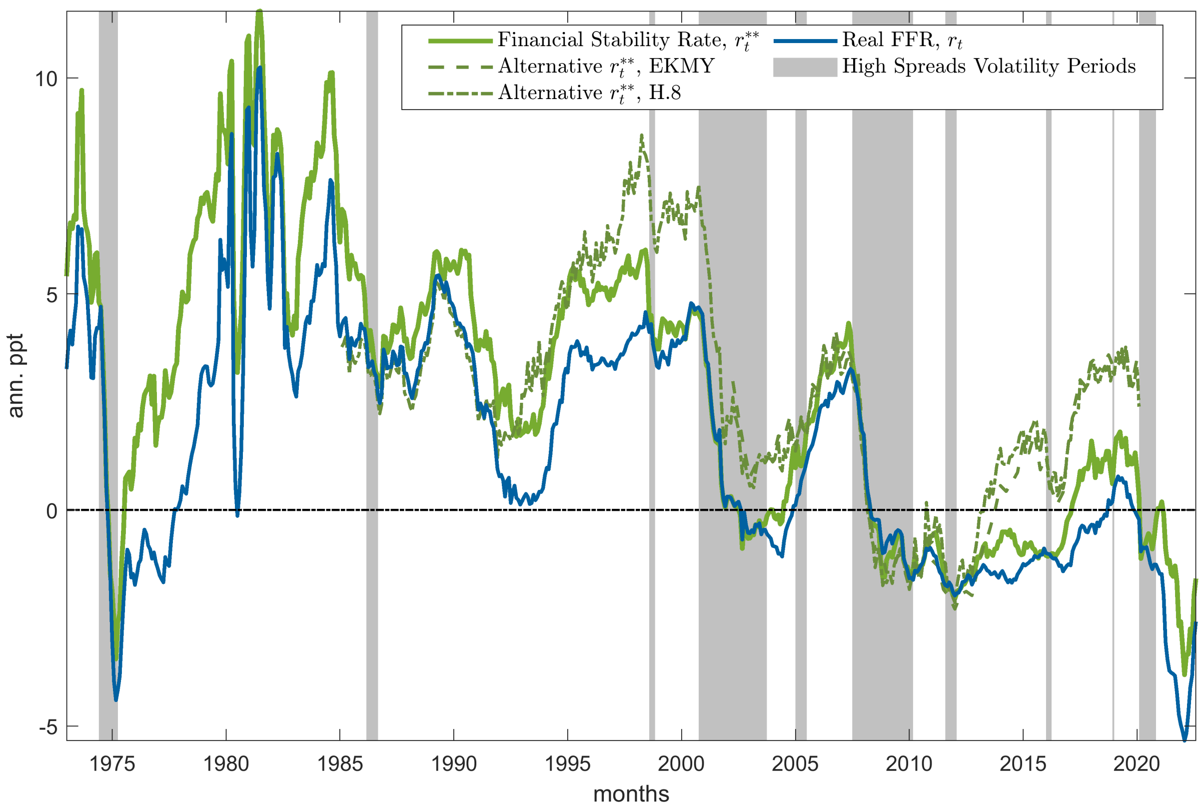
to be that large, as seen also in figure 4. In section 5.3 we will in fact use the heightened sensitivity of spreads to interest rates shocks as an external validation of our r** measure as constructed in the data.

5.2 r** in the data

In the previous section we showed that a non linear function of credit spreads and the level of real interest rates describes very accurately the r**-r gap in the model. In this section we i) use this very same non linear relationship to construct a measure of r**-r for the U.S. data over the past fifty years, and ii) argue that this estimate is sensible, in that it turns close to zero or negative in periods of high credit spreads volatility, consistent with the model results described in the previous section.

The blue line in figure 11 shows the real rate, as measured by the ex-post real federal funds rate.

Figure 11: The financial stability rate r_t^{**} and the real rate r_t in the data



Note: The figure shows: the real rate r (blue line) computed as the (ex-post) real federal funds rate; the financial stability real interest rate, r** (solid green line) computed from the ML regressions discussed in section 5.1 using the GZ credit spreads and the real federal funds rate as observables; alternative measures of r** computed from a different ML regressions using as observables the Hubrich and Waggoner (2022) measure of financial system leverage and, as measure of the safe asset ratio x, either the time series based on (Eisenbach et al., 2014, EKMY, dashed green line) or that based on the H.8 Federal Reserve report (dash-and-dotted green line). Shaded areas are high credit spreads volatility periods identified using the algorithm described in footnote 17.

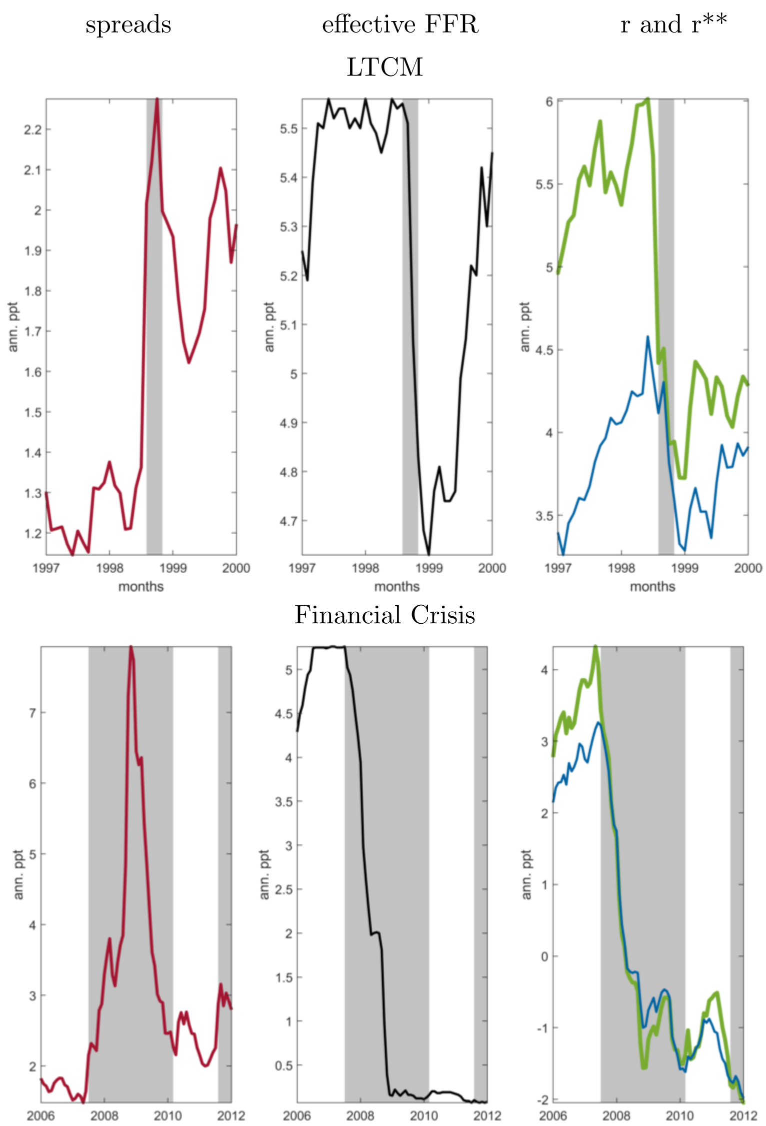
The green line shows the point estimate of r** implied by the non linear function of spreads and the level of real interest rates described in the previous section, where we use Gilchrist and Zakrajsek (2012)'s GZ spread as a counterpart for credit spreads in the model. Vertical shaded gray areas denote high volatility periods in spreads, identified using the heuristic approach described in the previous section. In the data, as in the model, periods where r**-r is close to zero or negative are also periods of elevated volatility in spreads. *Vice versa*, times of low credit spreads volatility are periods where r** is generally well above r.[18]

In broad terms, it appears that during the first part of the Great Moderation period, in the mid to late 80s and the 90s, r** is well above r except for short-lived episodes of stress such as the LTCM crisis. In the 2000s and right after the Great Recession the gap between r** and r is close to zero, meaning that the constraint is close to being binding. In the mid to late 2010s r** is generally above r, except again for a couple of very short-lived periods of stress, until the Covid pandemic hits the

[18]Figure A4 shows the time series of the GZ spreads together with identified high volatility periods.

economy in March 2020. In most financial stress episodes r** is rarely if ever significantly below r for extended periods of time, with the Great Recession being the only exception, when monetary policy was constrained by the zero lower bound. In interpreting r** in the data, we should also recall that in the model the r**-r is only a measure of *current* financial vulnerabilities and *not* a predictor of future vulnerabilities, especially a few years down the road. So for instance the r**-r gap becomes positive as the real rate declines around 2004, in line with the results in figure 4, but financial instability erupts a few years later, also in line with those results.

Figure 12: Spreads, r, and r** in specific episodes



Note: Left panels: GZ credit spreads; middle panels: nominal federal funds rate (annualized); right panels: r and r** constructed as described in the notes to figure 11. Shaded areas are high credit spreads volatility periods identified using the algorithm described in footnote 17.

The bird's eye view on r** afforded by figure 11 makes it difficult to disentangle what happens during specific episodes. For this reason in the remainder of the section we will zoom into two such episodes. The first, shown in the top row of figure 12, is the LTCM financial stress period in the late 1990s. Because of the currency crisis in Russia and related turmoil in emerging markets in the summer of 1998, the hedge fund LTCM ran into liquidity and solvency problems and had to be bailed out. As LTCM had large trades with a number of important counterparties, the events of 1998 put the US financial system under considerable stress. The upper left panel shows that credit spreads jumped by about 100 basis points within two months. The right panel shows that r** (green line) fell by about 75 bps from the beginning to the end of the financial stress episode. That is exactly by how much Greenspan cut interest rates during this period, thereby quelling the financial distress (middle panel). In other words, when financial constraints become binding and r** falls toward or below r, the real rate soon follows it down. This finding provides circumstantial evidence in favor of the existence of the "Greenspan put"—the notion that the central bank cuts rates whenever financial intermediaries become constrained (see Bornstein and Lorenzoni, 2018, and Caballero and Simsek, 2022, for theoretical discussion of the idea of the "put").

During the first part of the Great Recession (bottom row of figure 12) the story is quite similar. Spreads increase and, as a consequence, r** falls. Initially the real rate r follows r** downward, thereby limiting the effects of the financial turmoil and keeping the r**-r gap close to zero. However, in mid 2008 the nominal rate hit the zero lower bound, and as a consequence r could not fall any longer. When the Lehman crisis hit the economy, spreads increased further, r** fell and the gap between r** and r became negative until late 2009 and early 2010.

5.3 Validating r**

We have seen in the previous section that our time series of r** in the data, which is constructed using model-based formulas, appears to be broadly consistent with the narrative of previous episodes of financial turmoil. In this section we subject this measure to two more stringent tests.

The first test consists in constructing r** using an alternative ML regression based on completely different observables than those used in our baseline estimates: leverage and the safe asset ratio x. We try to use measures for these two quantities that are as close as possible to the model concepts. For leverage, we therefore use the broadest measure we could find—a micro-data based measure constructed by Hubrich and Waggoner (2022) from a CRSP/Compustat merged database that covers a broad range of publicly listed depository and nondepository institutions, bank holding companies, and nonbanks. These authors measure leverage as book assets over market equity and their time series is available from 1985 to the end of 2019. We obtain a time series of x, the safe assets ratio in the intermediaries' portfolio, using the measure of "liquid assets"—which includes cash, Fed funds, and Treasuries—as constructed by Eisenbach et al. (2014, EKMY) for the 50 largest bank holding companies, and divided by total book assets. Their measure was updated to 2022 but only starts in 2002. For this reason we also use an alternative time series constructed using the same definitions but based on the H.8 Federal Reserve report data set, which covers commercial banks and is available since 1973.[19] Eisenbach et al. (2014)'s definition of liquid assets coincides with our notion of safe assets, except that it also includes long-term Treasuries, which are arguably safe from a credit perspective but not from an interest rate risk perspective, as evidenced by the failure of Silicon Valley Bank.[20]

These alternative r** measures are also displayed in figure 11 (green dashed and dash-and-dotted lines corresponding to the EKMY and H.8 time series for x, respectively). Both are very correlated with our baseline r** measure, which is based on credit spreads and the real rate. Both measures see the Great Recession and its aftermath as a period when intermediaries' ability to lend was constrained, in that r** is close to or below r during this period. The alternative r** measures however rise more in the mid to late-2010s compared to the baseline measure. The baseline and alternative measures differ especially from the mid-1980s to the early 1990s, which is considered as a continuous period of financial distress according to the alternative measure.

Next, we turn to the second test. The definition of r** given in section 2 is tightly connected to the effects of an interest rate shock: in fact, the r**-r gap measures how large such a shock needs to be in order to enter the financial instability region when the economy is currently out of it, or exit such region when the economy is in it. A complementary approach to validating our measure of r**-r, more akin to the idea of stress testing, is therefore to subject the economy to the *same* size interest rate shock at different points in time, and measure its effects on the economy: when the economy is far away from the financial instability region and r**-r is large, such shocks should have little effects on the economy. *Vice versa*, when the economy is close to, or in, the financial instability region and the r**-r gap is nearly zero or negative, such shocks should have much larger effects. The left panels of figure 13 verify this hypothesis on model-generated data by showing the coefficient β_t of the time-varying parameters regression

$$\Delta spread_t = \alpha_t + \beta_t \epsilon_t + u_t, \tag{32}$$

where the change in spreads $\Delta spread_t = spread_t - spread_{t-1}$ measures the effect on the economy, and in particular on financial conditions, ϵ_t is the real interest rate innovation in period t, and the time-varying estimates are obtained using kernel-based estimation (see Giraitis et al., 2014; Petrova, 2019). The top row uses a flat kernel (standard rolling window estimation, with window of size $2H + 1$) and the bottom row uses a Gaussian kernel.[21]

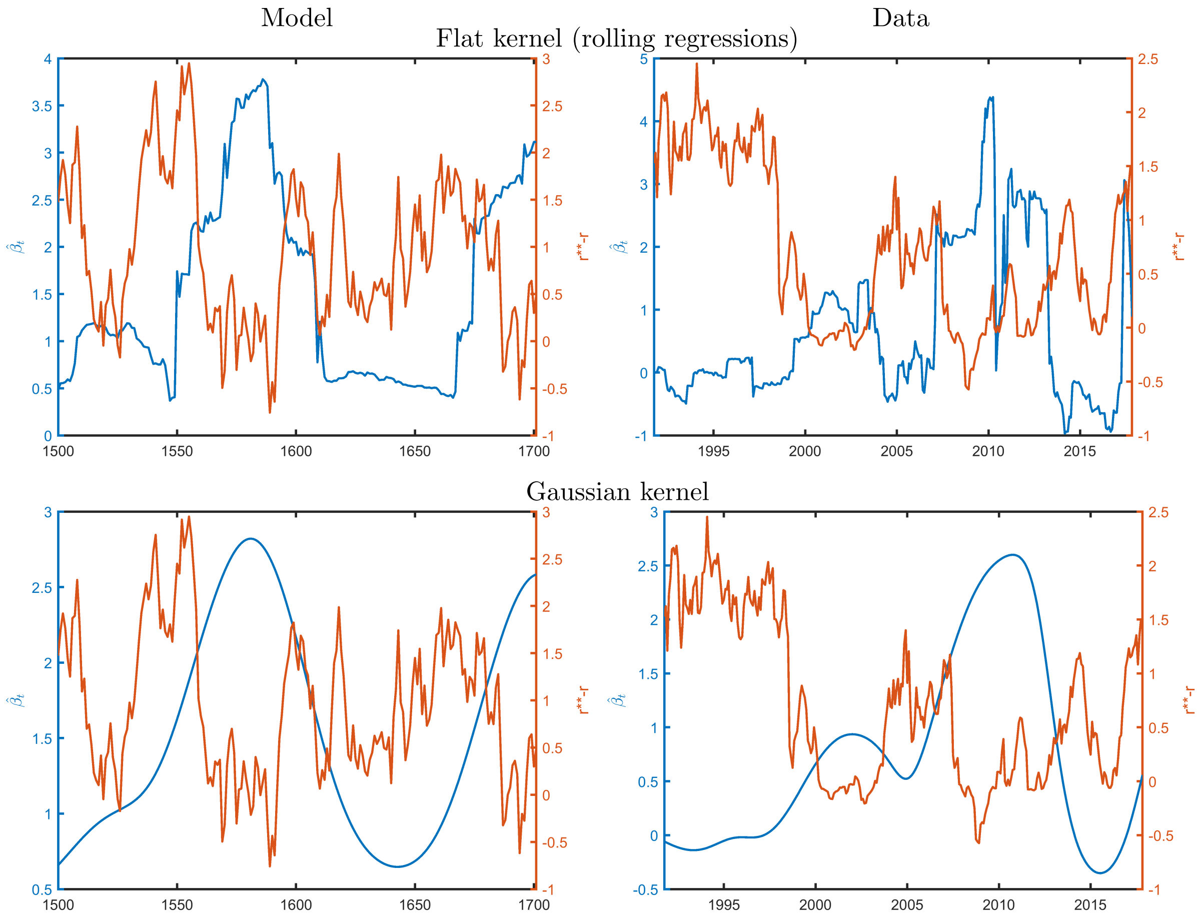
For model-generated data the estimate of β_t (blue line; left axis) indeed rises sharply as the gap r**-r (orange line; right axis) nears 0, and falls equally sharply as the gap rises. The panel in the right column show the results of this very same regression in the data. Unlike in the model, the real rate in the data is obviously endogenous: as shown above the central bank in the past often cut rates when the economy entered a financially unstable period. We therefore need to use exogenous shocks to interest rates as a measure for ϵ_t in the data, and for this purpose we employ Jarociński

[19] The H.8 measure excludes Fed funds, as these are not available before the 2000s, but this makes little difference.

[20] Figure A5 in the appendix displays time series of leverage, the safe assets ratio x, and the r**-r gap in the data.

[21] In constructing the kernel $K(\frac{t-j}{H})$ we use standard parameters in the literature. In particular, given that the size T of our actual time series of spreads is 353 we use $H = 353^{\cdot 5} \sim 19$. We use the very same parameters for both the model simulated and the actual data.

Figure 13: The sensitivity of financial conditions to interest rate shocks



Note: The figure plots the r**-r gap (orange) as well as the kernel-based estimates of β_t from the time-varying regression (32) (blue) using model-generated (left panels) and actual (right panels) data. The top panels use a flat kernel and the bottom panels a Gaussian kernel. The kernel regression on the data is run from December 1991 to November 2017.

and Karadi (2020)'s series of policy surprises, which are available from middle of 1990 to the end of 2019.[22] Not only also in the data the estimated β_t rises sharply as the gap r**-r falls to zero, but also the magnitude of β_t during periods of financial distress is roughly comparable between model and data.

6 Conclusions

In this paper we introduce the concept of the financial stability real interest rate, r**. As a vehicle to illustrate our idea, we use a macroeconomic banking model based on Gertler and Kiyotaki (2010) where intermediaries face a constraint in terms of a limit on the amount of funds that they

[22] Jarociński and Karadi (2020)'s measure has the advantage, relative to alternative measures in the literature, of being purged of information effects, which would contaminate the regression.

can raise. When the constraint binds the economy experiences financial instability with increasing credit spreads, declining asset prices and a contraction in economy activity.

We show that the gap between r** and the real rate serves as a useful quantitative summary statistic for financial vulnerabilities in the model, and, after constructing such a measure for the US economy, argue that the same holds for the actual data. We also show that in the model real interest rates have opposite effects on current and future financial vulnerability, generating a potential intertemporal trade-off for policy makers.

In order to keep the exposition simple and the message clear, our analysis is conducted within a simple real model where the natural real interest rate is exogenous. In future work we plan to build a framework with nominal rigidities where we can discuss the aforementioned intertemporal trade-off, as well as the the potential trade-off between macroeconomic and financial stability.

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Online Appendix

Equilibrium

Equilibrium is characterized by the following system of equations.

$$Y_t = C_t + I_t + T_t \tag{A1}$$

$$T_t = B_t - R_{t-1}B_{t-1} \tag{A2}$$

$$K_t = (a_1 I_t^{1-\eta} + a_2) + (1-\delta)e^{\psi_t}K_{t-1} \tag{A3}$$

$$Q_t = \left[a_1(1-\eta)I_t^{-\eta}\right]^{-1} \tag{A4}$$

$$\mathbb{E}_t(\Lambda_{t+1})R_t = 1 \tag{A5}$$

$$\Lambda_t = \beta\frac{U_{Ct}}{U_{Ct-1}} \tag{A6}$$

$$U_{C,t} = \left(C_t - \frac{\chi}{1+\epsilon}L_t^{1+\epsilon}\right)^{-\gamma} \tag{A7}$$

$$R_{K,t} = \frac{\alpha\frac{Y_t}{e^{\psi_t}K_{t-1}} + (1-\delta)Q_t}{Q_{t-1}} \tag{A8}$$

$$Y_t = K_{t-1}^{\alpha}L_t^{1-\alpha} \tag{A9}$$

$$(1-\alpha)\frac{Y_t}{L_t} = \chi L_t^{\epsilon}\left[1 + \Upsilon\left(R_t^d + \frac{\mu_t}{\mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}]} - 1\right)\right] \tag{A10}$$

$$\mu_t = \mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}(R_{K,t+1} - R_t^d)] \tag{A11}$$

$$\mu_{K,t} = \mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}(R_{K,t+1} - R_t)] \tag{A12}$$

$$\nu_t = \mathbb{E}_t[\Lambda_{t+1}\Omega_{t+1}]R_t^d \tag{A13}$$

$$\overline{\mu}_t = \mu_t + (\zeta_t - \mu_{K,t})x_t \tag{A14}$$

$$\Omega_t = 1 - \sigma + \sigma(\nu_t + \overline{\mu}_t\phi_t) \tag{A15}$$

$$N_t = \sigma[(R_{K,t} - R_{t-1})Q_{t-1}K_{t-1} + (R_{B,t-1} - R_{t-1}^d)B_{t-1} + R_{t-1}^d N_{t-1}] + (1-\sigma)\xi Q_{t-1}K_{t-1} \tag{A16}$$

$$\overline{\phi}_t = \frac{\nu_t}{\Theta_t - \overline{\mu}_t} \tag{A17}$$

$$\Theta_t = \theta(1 - \frac{\lambda}{\kappa}x_t^{\kappa}) \tag{A18}$$

$$x_t = \frac{B_t}{B_t + Q_tK_t} \tag{A19}$$

$$Q_tK_t + B_t = \phi_tN_t \tag{A20}$$

$$\mu_{K,t} = \zeta_t + \overline{\mu}_t\frac{\lambda x_t^{\kappa-1}}{(1 - \frac{\lambda}{\kappa}x_t^{\kappa})} \tag{A21}$$

$$\overline{\mu}_t \times (\overline{\phi}_t - \phi_t) = 0 \tag{A22}$$

$$R_t = \overline{R} - \varphi(e^{x_t - \overline{x}} - 1) + e^{R_t^* - 1} - 1 \tag{A23}$$

$$\log(R_t^*) = \rho_r\log(R_{t-1}^*) + \sigma_r\varepsilon_{r,t} \tag{A24}$$

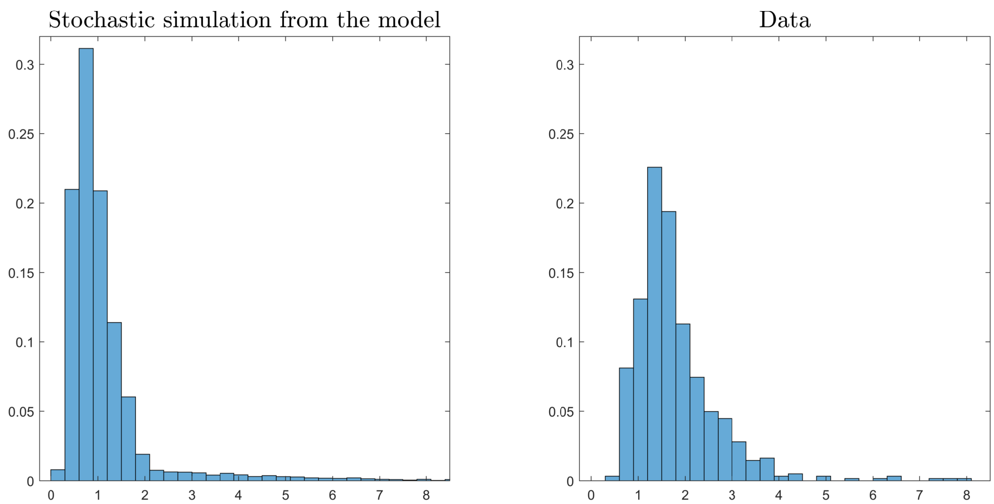
$$\zeta_t = \overline{\zeta} + \varepsilon_{\zeta,t} \tag{A25}$$

We have variables $Y_t, C_t, NX_t, B_t, K_t, I_t, Q_t, \Lambda_t, R_t, R_{B,t}, U_{C,t}, L_t, R_{K,t}, \mu_t, \mu_{K,t}, \overline{\mu}_t, \nu_t, \Omega_t, N_t, \phi_t, \overline{\phi}_t$, $x_t, \Theta_t, R_t^*, \zeta_t$ (25 variables for 25 equations). We have $\overline{\mu}_t \geq 0$ and $(\overline{\phi}_t - \phi_t) \geq 0$. Equation (A22) indicates that if $\overline{\mu}_t = 0$, we must have $\phi_t < \overline{\phi}_t$ (banks' leverage constraint does not bind); conversely, if $\overline{\mu}_t > 0$, we have $\phi_t = \overline{\phi}_t$ (the leverage constraint binds).

Our computational strategy is based on Judd et al. (2011), but relies on approximating one-step-ahead expectations rather than policy functions, as in the "parameterized expectations" approach (Marcet and Lorenzoni, 1998). We use Hermite polynomials to approximate the expectations in (A1)-(A25) and use stochastic simulations to iterate until convergence. Given knowledge of the expectations and of the states, it is possible to solve system (A1)-(A25) in closed form in the unconstrained regime, and to collapse it to just one non-linear equation in the constrained regime.

A Additional figures

Figure A1: Histogram of credit spreads



Note: Credit spreads stand for corporate bond spreads for non-financial firms. Spreads are calculated as the average spreads between the yield of private-sector bonds in the US relative to US government securities, of matched maturities. Data sources: Gilchrist and Zakrajsek (2012).

Figure A2: Response to decline in real interest rates for different initial levels of R

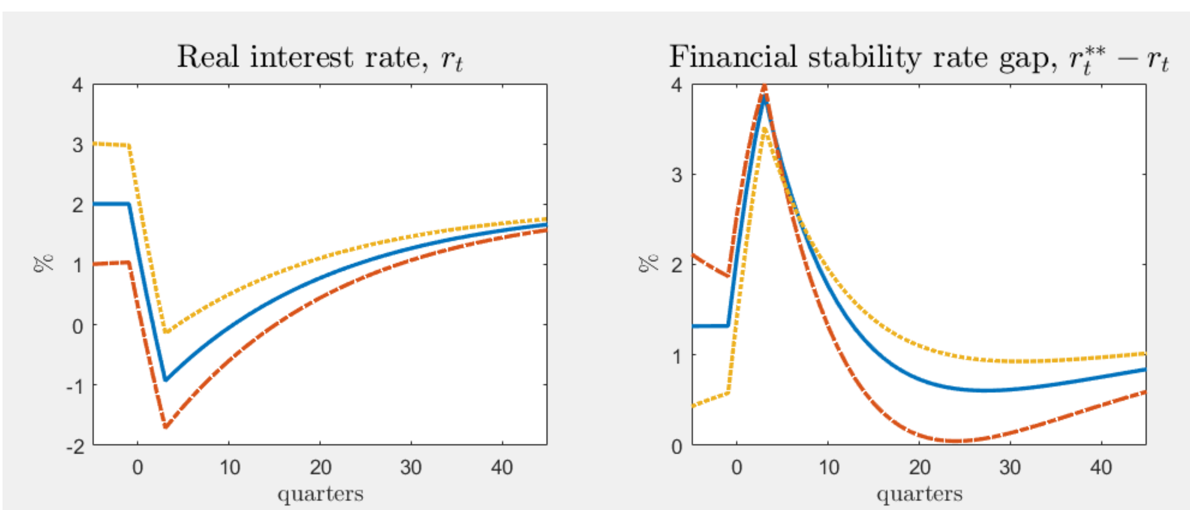


Figure A3: Real rate, share of safe assets, and the financial stability rate gap: lead-lag correlations

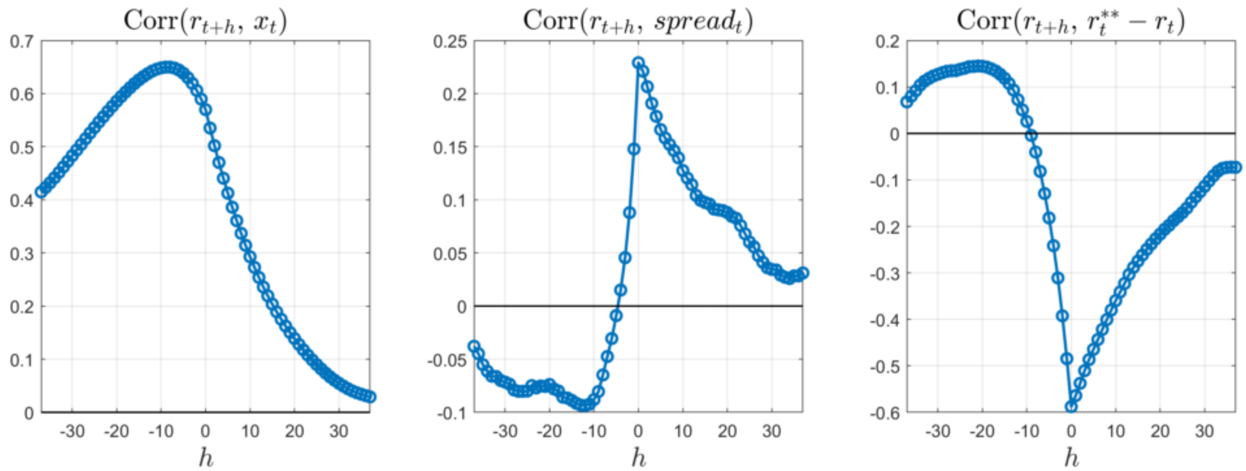


Figure A4: GZ spreads and identified high spreads' volatility periods

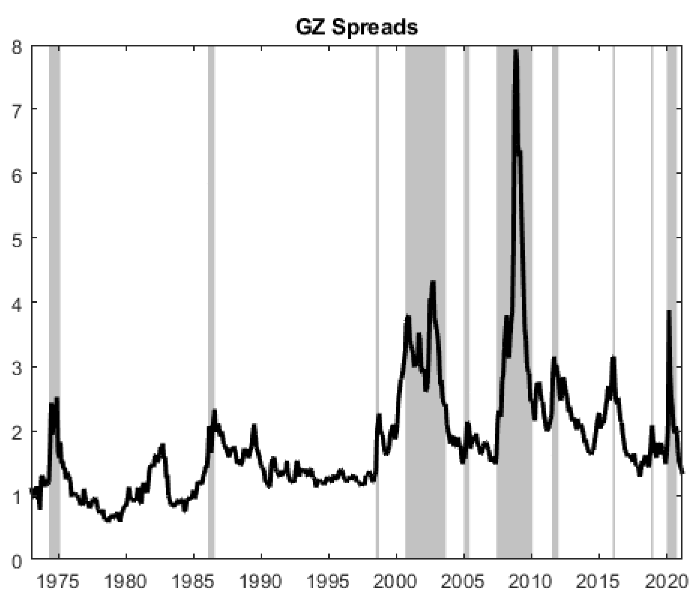
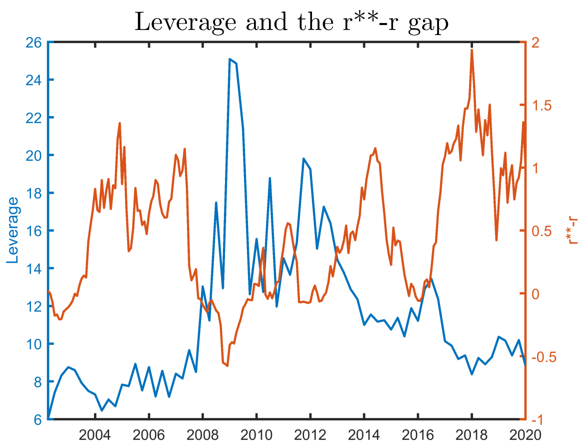
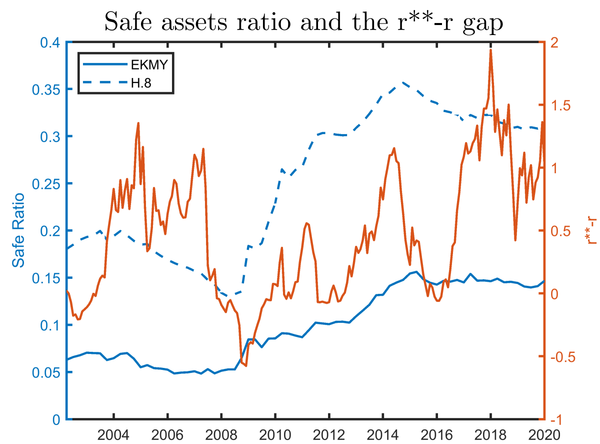


Figure A5: Leverage, the safe assets ratio, and the r**-r gap





Note: Left panel: Leverage (blue) and the r**-r gap (orange). Right panel: Safe assets ratio (blue) and the r**-r gap (orange)