THE JOINT US/UK 1995 EPOCH WORLD MAGNETIC MODEL

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1.0 Introduction

The Earth's magnetic field, as measured by a magnetic sensor on or above the Earth's surface, is actually a composite of several magnetic fields generated by a variety of sources. These fields are superimposed on each other and through inductive processes interact with each other. The most important of these geomagnetic sources are:

- a. the Earth's conducting, fluid outer core;
- b. the Earth's crust/upper mantle;
- c. the ionosphere; and
- d. the magnetosphere.

More than 95% of the geomagnetic field is generated by the Earth's outer core. It is this portion of the geomagnetic field that is represented by the 1995 Epoch World Magnetic Model (WMM-95). Those portions of the geomagnetic field not represented by the model are collectively referred to as the anomalous geomagnetic field, which varies both spatially and temporally with respect to the model.

consists of a degree and order 12 spherical-harmonic Main (i.e., The model itself core-generated) field (MF) model comprised of 168 spherical-harmonic Gauss coefficients and a degree and order 12 spherical-harmonic Secular-Variation (SV) (core-generated, slow temporal variation) field model comprised of an additional 168 spherical-harmonic Gauss coefficients. In previous models (e.g., WMM-90) the SV Gauss coefficients equal to or greater than degree and order 9 were set to zero, due to a lack of data. The primary geomagnetic data set for the 1995 Epoch is the Polar Orbiting Geomagnetic Survey (POGS) satellite data set, which provided long-term Total Intensity data from 1991 through 1993. This data set was supplemented by Project MAGNET vector-aeromagnetic data, which spanned the period from 1988 through 1993. These two data sets and the geomagnetic observatory annual means data set also provided sufficient spatial and temporal coverage to permit, for the first time, the computation of the SV Gauss coefficients to the full degree and order 12 for the definitive 1992.5 Epoch. The MF Gauss coefficients characterize the geomagnetic field at one instant of time called the base epoch, which for WMM-95 is 1995.0. The predictive SV coefficients characterize the slow rate of change of the geomagnetic field for the 5-year period from the base epoch to the termination epoch, which for WMM-95 is 2000.0, at which time, WMM-95 will be replaced by WMM-2000. The magnetic field components are computed via the magnetic variation algorithm (GEOMAG), which is a FORTRAN subroutine that uses the WMM Gauss coefficients in conjunction with the spherical-harmonic expansions associated with each field component.

The WMM-95 coefficients were produced jointly by the British Geological Survey (BGS) in Edinburgh, Scotland, and the Naval Oceanographic Office (NAVOCEANO) at Stennis Space Center, Mississippi, on behalf of the British Hydrographic Office in Taunton, England, and the National Imagery and Mapping Agency (NIMA) [formerly the Defense Mapping Agency (DMA)], Washington, D.C. The U. S. portion of the modeling function has been transferred from NAVOCEANO to the U. S. Geological Survey's Geologic Hazard Team, Geomagnetics Group, in Golden, Colorado. The model, associated software, and documentation are now

distributed by USGS. The WMM models are also the National Oceanic and Atmospheric Administration's National Geophysical Data Center/World Data Center (NOAA's NGDC/WDC-A) on behalf of NIMA in accordance with NIMA Instructions 8000.1 and 8000.2. These models are produced at 5-year intervals, as are NIMA's Declination/Grid-Variation charts, while charts of the other geomagnetic components are published by NIMA every 10 years. The 1995.0 Epoch corresponds to a 10-year interval when all of the geomagnetic components will be published in chart form by NIMA. The military specifications for the WMM are contained in MIL-W-89500 (DMA [1993]). Magnetic model requirements that are more stringent than those set forth in this military specification (e.g., those which must include magnetic effects of the Earth's crust, ionosphere, or magnetosphere and/or require greater spatial or temporal resolution on a global, regional, or local basis) should be addressed to:

Director, National Imagery and Mapping Agency 12310 Sunrise Valley Drive Reston, VA 20191-3449 ATTN: P33

The magnetic field model and a convenient Window-95 version of the GEOMAG software, as well as page sized charts of various components of the magnetic field from the current WMM can be accessed and downloaded from the following USGS WEB site:

http://geomag.usgs.gov

Wall-Sized Charts published by NIMA are sold to the public for a modest fee through NOAA's international distribution system. To find the nearest authorized distributor go to the following WEB site:

http://chartmaker.ncd.noaa.gov/ocs/text/agents.html

These charts are approximately 48 inches long and 36 inches tall. The following charts are available:

- a.) World Mercator Projection of Magnetic Declination (D) NIMA/DMA Stock #WOBZC42
- b.) North/South Polar Stereographic Projection of Magnetic Declination (D) and Reverse Side: Grid Variation (GV), same projection; NIMA/DMA Stock #WOBZC43
- c.) World Mercator Projection of Magnetic Inclination (I) with Polar Stereographic Projection on reverse side; NIMA/DMA Stock #WOXZC30
- d.) World Mercator Projection of the Horizontal Magnetic Intensity (H) with Polar Stereographic Projection on reverse side; NIMA/DMA Stock #WOXZC33

- e.) World Mercator Projection of the Vertical Magnetic Component Z) with Polar Stereographic Projection on reverse side; NIMA/DMA Stock #WOXZC36
- f.) World Mercator Projection of Total Magnetic Intensity (F) with Polar Stereographic Projection on reverse side; NIMA/DMA Stock #WOXZC39

It is extremely important to recognize that the WMM series of geomagnetic models and the charts produced from these models characterize only that portion of the Earth's magnetic field which is generated by the Earth's fluid outer core. The portions of the geomagnetic field generated by the Earth's crust, upper mantle, ionosphere, and magnetosphere are not represented in these models. Consequently, a magnetic sensor such as a compass or magnetometer may observe spatial and temporal magnetic anomalies when referenced to the appropriate WMM. In particular, certain local, regional, and temporal magnetic declination anomalies can exceed 10 degrees. Anomalies of this magnitude are not common but they do exist. Declination anomalies on the order of 3 or 4 degrees are not uncommon but are of small, spatial extent and are relatively isolated. On land, spatial anomalies are produced by mountain ranges; ore deposits; ground struck by lightning; geological faults; and cultural features such as trains, planes, tanks, railroad tracks, power lines, etc. In ocean areas these anomalies occur most frequently along continental margins; near seamounts; and near ocean ridges, trenches, and fault zones, particularly those of volcanic origin. Ships and submarines are also sources of magnetic anomalies in the ocean.

Temporal anomalies in either ocean or land areas can last from a few minutes to several days and are produced by ionospheric and magnetospheric processes which are driven by the *solar wind*. In particular, *magnetic storms* generated by *solar flares* and other solar activity can, through modulation of the solar wind, cause severe and persistent magnetic anomalies in the Earth's environment. Even during periods of quiet solar activity, significant spatial and temporal magnetic anomalies are found in the polar and equatorial regions of the Earth, where magnetic fields produced by ionospheric current systems, such as the *auroral electrojets* and the *equatorial electrojet*, are always present. Most sources of magnetic anomalies are comparatively isolated in either space or time. Therefore, from a global perspective, the root-mean-square (RMS) Declination (D), Inclination (I), and Grid Variation (GV) errors of the WMM are estimated to be less than 0.5 degrees in ocean areas and less than 1.0 degree in land areas at the Earth's surface over the entire 5-year life of a particular model. Also, the RMS errors at sea level of the Horizontal Intensity (H), the Vertical component (Z), and the Total Intensity (F) of the WMM over ocean and land areas are estimated to be less than 200 *nanoTeslas* (nT) over the entire 5-year life of a particular model.

1.1 The Mathematical Model

The Earth's core-generated magnetic field has associated with it a geomagnetic potential $V(r, \theta, \varphi, t)$, which can be expressed in spherical coordinates in terms of a spherical-harmonic expansion of the following form:

$$V(r,\theta,\varphi,t) = R_E \sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+1} \sum_{m=0}^{n} \{g_{nm}(t) \cos(m\varphi) + h_{nm}(t) \sin(m\varphi)\} P_n^m(\theta)$$
 (1)

where the spherical coordinates (r, θ, φ) correspond to the radius from the center of the Earth, the colatitude (i.e., 90° - latitude), and the longitude. R_E is the mean radius of the Earth (6371.2 km); $g_{nm}(t)$ and $h_{nm}(t)$ are referred to as the Gauss coefficients at time t, where t is the time in years (e.g., 1997.312). $P_n^m(\theta)$ represents a particular Schmidt-normalized associated Legendre polynomial of spherical-harmonic degree n and order m. These are polynomials in terms of the cosine of the colatitude θ . The Gauss coefficients are slowly varying functions of time and are expressed in the form of a Taylor series expansion, where only terms up to first order in time are retained so that:

$$g_{nm}(t) = g_{nm}(T_{Epoch}) + \dot{g}_{nm}(t - T_{Epoch}) \qquad T_{Epoch} \le t \le T_{Epoch} + 5$$
 (2a)

$$h_{nm}(t) = h_{nm}(T_{Epoch}) + h_{nm}(t - T_{Epoch}) \qquad T_{Epoch} \le t \le T_{Epoch} + 5$$
 (2b)

where T_{Epoch} is the base epoch of the model, which for WMM-95 is 1995.0. Thus, $g_{nm}(T_{Epoch})$ and $h_{nm}(T_{Epoch})$ are the Schmidt-normalized Gauss coefficients of the WMM at the model's base epoch, while the Schmidt-normalized SV Gauss coefficients, \dot{g}_m and \dot{h}_m (pronounced g_{nm} dot and h_{nm} dot, where the dot represents differentiation with respect to time: $\frac{d}{dt}$), are the annual rates of change of the MF Gauss coefficients g_{nm} and h_{nm} and are evaluated at the middle of the model's lifespan (i.e., at $T_{Epoch} + 2.5$). The MF Gauss coefficients and SV field Gauss coefficients are collectively referred to as spherical-harmonic coefficients.

Taking the time derivative of eq. (1) yields the spherical-harmonic expression for the Secular-Variation $\dot{V}(r, \theta, \varphi, t)$ of the geomagnetic potential:

$$\dot{V}(r,\theta,\varphi,t) = R_E \sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+1} \sum_{m=0}^{n} \left\{ \dot{g}_{nm}(t) \cos(m\varphi) + \dot{h}_{nm}(t) \sin(m\varphi) \right\} P_n^m(\theta)$$
 (3)

which, due to the assumption that the MF Gauss coefficients vary linearly with time during the course of a 5-year interval, as expressed by eqs. (2a) and (2b), is approximately time independent over this short time span. The maximum degree N of the spherical-harmonic expansions in eqs. (1) and (3) is equal to 12. This value is determined by noting that when the spectral density of the MF Gauss coefficients is plotted as a function of harmonic degree, a distinct break in this density function occurs between degree 12 and degree 15. This is

interpreted to mean that the low-degree harmonics corresponding to $N \le 12$ are dominated by core-generated magnetic fields, while those high-degree harmonics for which $N \ge 15$ are dominated by fields generated within the crust and upper mantle. These fields are primarily associated with permanent and induced magnetization. This magnetization is limited to depths for which the ambient temperature does not exceed the Curie temperature. Spherical-harmonic degrees 13 and 14 correspond to a transition region where neither magnetic fields generated within the Earth's fluid core nor those generated within the Earth's crust dominate. We therefore use N=12 as the spherical-harmonic cutoff which permits the best description of the core-generated magnetic field and its slow temporal change. This means that the shortest wavelength contained in the model is:

$$\lambda_{\min} = \frac{2\pi R_E}{N} = 3336 \, km \tag{4}$$

Thus, the WMM is a low-resolution model. High-resolution (short wavelength) descriptions of that part of the magnetic field generated by the Earth's upper crust are better characterized via rectangular harmonic modeling of small local areas (Quinn and Shiel [1993]), while intermediate wavelength descriptions of the Earth's magnetic field generated by the lower crust and upper mantle are best characterized via spherical-cap harmonic models of large regional areas (Haines [1985a, 1985b, 1985c, and 1990]). Global geomagnetic data sets currently available do not support high resolution (i.e., $\lambda \leq 500$ km) and only marginally support intermediate-resolution (i.e., 500 km < $\lambda < 3336$ km) magnetic field modeling. Special local and/or regional magnetic surveys are required to generate intermediate-resolution and high-resolution geomagnetic models. Consequently, there are some applications for which the use of the WMM may be inadequate.

The Earth's magnetic field $\mathbf{B}(r,\theta,\varphi,t)$ is a vector quantity having three components which correspond to the projection of the magnetic field vector onto the three coordinate axes. Thus, $B_r(r,\theta,\varphi,t)$ is that portion of the field pointing radially outward from the Earth's center (i.e., perpendicular to the surface of the Earth); $B_{\theta}(r,\theta,\varphi,t)$ is that portion of the field pointing locally due south; and $B_{\varphi}(r,\theta,\varphi,t)$ is that portion of the field pointing locally due east. The magnetic field vector can be computed from the geomagnetic potential by taking its negative gradient, thus:

$$\mathbf{B}(r,\theta,\varphi,t) = -\nabla V(r,\theta,\varphi,t) \tag{5}$$

Consequently, the magnetic field components are related to the geomagnetic potential as follows:

$$B_r(r,\theta,\varphi,t) = -\frac{\partial V(r,\theta,\varphi,t)}{\partial r}$$
 (6a)

$$B_{\theta}(r,\theta,\varphi,t) = -\frac{1}{r} \frac{\partial V(r,\theta,\varphi,t)}{\partial \theta}$$
 (6b)

$$B_{\varphi}(r,\theta,\varphi,t) = -\frac{1}{r\sin\theta} \frac{\partial V(r,\theta,\varphi,t)}{\partial \varphi}$$
 (6c)

which yield the following spherical-harmonic expansions:

$$B_r(r,\theta,\varphi,t) = \sum_{n=1}^{N} (n+1) \left(\frac{R_E}{r}\right)^{n+2} \sum_{m=0}^{n} \{g_{nm}(t)\cos(m\,\varphi) + h_{nm}(t)\sin(m\,\varphi)\} P_n^m(\theta)$$
 (7a)

$$B_{\theta}(r,\theta,\varphi,t) = -\sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+2} \sum_{m=0}^{n} \left\{g_{nm}(t)\cos(m\varphi) + h_{nm}(t)\sin(m\varphi)\right\} \frac{dP_n^m(\theta)}{d\theta}$$
 (7b)

$$B_{\varphi}(r,\theta,\varphi,t) = \frac{1}{\sin\theta} \sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+2} \sum_{m=0}^{n} m \left\{g_{nm}(t)\sin(m\varphi) - h_{nm}(t)\cos(m\varphi)\right\} P_n^m(\theta)$$
 (7c)

These expressions are solutions of the Laplace equation, which in turn is derived from Maxwell's famous electromagnetic field equations under the assumptions that the magnetic field exists in a source-free region (i.e., no charges or currents are present), and the fields are slowly varying.

Similarly, for the geomagnetic SV field we have:

$$\mathbf{B}(r,\theta,\varphi,t) = -\nabla \dot{V}(r,\theta,\varphi,t) \tag{8}$$

so that:

$$\dot{B}_r(r,\theta,\varphi,t) = -\frac{\partial \dot{V}(r,\theta,\varphi,t)}{\partial r}$$
(9a)

$$\dot{B}_{\theta}(r,\theta,\varphi,t) = -\frac{1}{r} \frac{\partial \dot{V}(r,\theta,\varphi,t)}{\partial \theta}$$
(9b)

$$\dot{B}_{\varphi}(r,\theta,\varphi,t) = -\frac{1}{r\sin\theta} \frac{\partial \dot{V}(r,\theta,\varphi,t)}{\partial \varphi}$$
(9c)

which yield the following spherical-harmonic expressions:

$$\dot{B}_{r}(r,\theta,\varphi,t) = \sum_{n=1}^{N} (n+1) \left(\frac{R_{E}}{r}\right)^{n+2} \sum_{m=0}^{n} \left\{ \dot{g}_{nm}(t) \cos(m\varphi) + \dot{h}_{nm}(t) \sin(m\varphi) \right\} P_{n}^{m}(\theta)$$
 (10a)

$$\dot{B}_{\theta}(r,\theta,\varphi,t) = -\sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+2} \sum_{m=0}^{n} \left\{ \dot{g}_{nm}(t) \cos(m\varphi) + \dot{h}_{nm}(t) \sin(m\varphi) \right\} \frac{dP_n^m(\theta)}{d\theta}$$
 (10b)

$$\dot{B}_{\varphi}(r,\theta,\varphi,t) = \frac{1}{\sin\theta} \sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+2} \sum_{m=0}^{n} m \left\{\dot{g}_{nm}(t)\sin(m\varphi) - \dot{h}_{nm}(t)\cos(m\varphi)\right\} P_n^m(\theta)$$
 (10c)

1.2 Spherical-Harmonic Normalization

The Gauss coefficients $g_{nm}(t)$, $h_{nm}(t)$, $\dot{g}_{nm}(t)$, and $\dot{h}_{nm}(t)$, as well as the associated Legendre polynomials and their derivatives, are Schmidt normalized by international agreement (circa 1930) of the International Union of Geodesy and Geophysics. This particular normalization allows one to determine which spherical-harmonic terms of a particular model are the most significant simply by a cursory inspection of the model coefficients' relative magnitudes. The Schmidt-normalized associated Legendre polynomials $P_n^m(\theta)$ are related to the unnormalized associated Legendre polynomials $P^{nm}(\theta)$ (note position of indices) by the following relation:

$$P_n^m(\theta) = S^{nm} P^{nm}(\theta) \tag{11}$$

The Schmidt normalization factors S^{nm} and the unnormalized associated Legendre polynomials $P^{nm}(\theta)$ are computed via recurrence relations as follows (Cain et al., 1967):

$$P^{00}(\theta) = 1 \tag{12a}$$

$$P^{nm}(\theta) = \sin \theta P^{n-1, m-1}(\theta) \qquad m = n \neq 0 \tag{12b}$$

$$P^{nm}(\theta) = \cos \theta P^{n-1,m}(\theta) - \kappa^{nm} P^{n-2,m}(\theta) \qquad m \neq n, n \ge 1$$
 (12c)

$$\frac{dP^{00}(\theta)}{d\theta} = 0 \tag{12d}$$

$$\frac{dP^{nm}(\theta)}{d\theta} = \sin\theta \frac{dP^{n-1,m-1}(\theta)}{d\theta} + \cos\theta P^{n-1,m-1}(\theta) \qquad m = n \neq 0$$
 (12e)

$$\frac{dP^{nm}(\theta)}{d\theta} = \cos\theta \frac{dP^{n-1,m}(\theta)}{d\theta} - \sin\theta P^{n-1,m}(\theta) - \kappa^{nm} \frac{dP^{n-2,m}(\theta)}{d\theta} \qquad m \neq n, n \geq 1$$
 (12f)

where:

$$\kappa^{nm} = \frac{(n-1)^2 - m^2}{(2n-1)(2n-3)} \tag{13}$$

and where it is understood that the undefined polynomial $P^{-1,0}(\theta)$ and its derivatives are set equal to zero. Similarly:

$$S^{00} = 1 ag{14a}$$

$$S^{n0} = \left(\frac{2n-1}{n}\right) S^{n-1,0} \qquad n > 0 \tag{14b}$$

$$S^{nm} = \sqrt{\frac{(n-m+1)J}{n+m}} S^{n,m-1}$$

$$\begin{cases} J = 2 & for \ m = 1 \\ J = 1 & for \ m > 1 \end{cases}$$
 (14c)

Also computed via recursion relations are the longitudinally dependent functions $cos(m\varphi)$ and $sin(m\varphi)$. They are computed as follows:

$$\sin(m\,\varphi) = 0 \qquad m = 0 \tag{15a}$$

$$\cos(m\,\varphi) = 1 \qquad m = 0 \tag{15b}$$

$$\sin(m\,\varphi) = \sin(\varphi)\,\cos[(m-1)\,\varphi] + \cos(\varphi)\,\sin[(m-1)\,\varphi] \quad m > 0 \tag{15c}$$

$$\cos(m\varphi) = \cos(\varphi)\cos[(m-1)\varphi] - \sin(\varphi)\sin[(m-1)\varphi] \quad m > 0$$
 (15d)

1.3 Coordinate Transformations

Although the magnetic field model is defined in terms of *spherical* coordinates, the intended application is in *geodetic* coordinates. So, a coordinate transformation is necessary (Cain et al., 1967). The 1984 World Geodetic System (WGS-84) (DMA [1987]) and its corresponding ellipsoid are used as the reference datum for this purpose. Computing the magnetic field components at a given location expressed in geodetic coordinates using the WMM-95 model is a three-step procedure:

- a. Convert the geodetic latitude, longitude, and altitude (λ, ϕ, h) to spherical coordinates (r, θ, ϕ) .
- b. Compute the magnetic field components $B_r(r,\theta,\phi,t)$, $B_{\theta}(r,\theta,\phi,t)$, and $B_{\phi}(r,\theta,\phi,t)$.
- c. Rotate the magnetic field components from spherical coordinates back to geodetic coordinates.

This yields the magnetic field components $B_X(\lambda, \phi, h, t)$, $B_Y(\lambda, \phi, h, t)$, and $B_Z(\lambda, \phi, h, t)$, which are projections of the magnetic field vector $\mathbf{B}(\lambda, \phi, h, t)$ onto the \mathbf{X} -north, \mathbf{Y} -east, and \mathbf{Z} -vertically down coordinate axes of the local rectangular coordinate system defined by the tangent plane to the ellipsoid, which is concentric about the WGS-84 ellipsoid and which encompasses the point (λ, ϕ, h) .

The transformations corresponding to *step a* are as follows:

$$\cos \theta = \frac{\sin \lambda}{\sqrt{Q^2 \cos^2 \lambda + \sin^2 \lambda}} \tag{16a}$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta} \tag{16b}$$

where, if a and b are respectively the semi-major and semi-minor axes of the WGS-84 ellipsoid, then:

$$Q = \frac{h\sqrt{a^2 - (a^2 - b^2)\sin^2\lambda} + a^2}{h\sqrt{a^2 - (a^2 - b^2)\sin^2\lambda} + b^2}$$
(17)

Furthermore:

$$r^{2} = h^{2} + 2h\sqrt{a^{2} - (a^{2} - b^{2})\sin^{2}\lambda} + \frac{a^{4} - (a^{4} - b^{4})\sin^{2}\lambda}{a^{2} - (a^{2} - b^{2})\sin^{2}\lambda}$$
(18)

The transformations corresponding to *step c* depend on the angle α through which the magnetic field vector must be rotated while transforming from spherical to geodetic coordinates. This rotation angle is defined by the following transformation equations:

$$\cos \alpha = \frac{h + \sqrt{a^2 \cos^2 \lambda + b^2 \sin^2 \lambda}}{r} \tag{19a}$$

$$\sin \alpha = \frac{(a^2 - b^2)\cos \lambda \sin \lambda}{r\sqrt{a^2 \cos^2 + b^2 \sin^2 \lambda}}$$
(19b)

$$a = \lambda + \theta - \frac{\pi}{2} \tag{19c}$$

Consequently, the components of the magnetic field vector in geodetic coordinates may be computed as follows:

$$B_X(\lambda, \varphi, h, t) = -\cos \alpha B_{\theta}(r, \theta, \varphi, t) - \sin \alpha B_r(r, \theta, \varphi, t)$$
(20a)

$$B_{Y}(\lambda, \varphi, h, t) = B_{\varphi}(r, \theta, \varphi, t) \tag{20b}$$

$$B_Z(\lambda, \varphi, h, t) = \sin \alpha B_{\theta}(r, \theta, \varphi, t) - \cos \alpha B_r(r, \theta, \varphi, t)$$
 (20c)

From these three rectangular geomagnetic field components, it is possible to compute all others. In particular, the following magnetic components can be computed:

$$B_H(\lambda, \varphi, h, t) = \sqrt{B_X^2(\lambda, \varphi, h, t) + B_Y^2(\lambda, \varphi, h, t)}$$
 (Horizontal Intensity) (21a)

$$B_F(\lambda, \varphi, h, t) = \sqrt{B_H^2(\lambda, \varphi, h, t) + B_Z^2(\lambda, \varphi, h, t)}$$
 (Total Intensity) (21b)

$$B_D(\lambda, \varphi, h, t) = \tan^{-1} \left\{ \frac{B_Y(\lambda, \varphi, h, t)}{B_X(\lambda, \varphi, h, t)} \right\}$$
 (Declination) (21c)

$$B_I(\lambda, \varphi, h, t) = \tan^{-1} \left\{ \frac{B_Z(\lambda, \varphi, h, t)}{B_H(\lambda, \varphi, h, t)} \right\}$$
 (Inclination) (21d)

$$B_G(\lambda, \varphi, h, t) = \begin{cases} B_D - \varphi & \lambda \ge 0 \\ B_D + \varphi & \lambda < 0 \end{cases}$$
 (Grid Variation) (21e)

Inclination is often referred to as the *Dip* angle, while the magnetic declination is sometimes referred to as the *magnetic variation*. Frequently, the magnetic field components in eqs. (21a) through (21e) are simply referred to in terms of their subscripts: X, Y, Z, H, F, D, I, and G. The Total Magnetic Intensity is sometimes referred to as *TI*, while the Grid Variation is sometimes referred to as *GV*. Some additional and quite useful relationships among these magnetic field components are:

$$H(\lambda, \varphi, h, t) = F(\lambda, \varphi, h, t) \cos[I(\lambda, \varphi, h, t)]$$
(22a)

$$X(\lambda, \varphi, h, t) = H(\lambda, \varphi, h, t) \cos[D(\lambda, \varphi, h, t)]$$
(22b)

$$Y(\lambda, \varphi, h, t) = H(\lambda, \varphi, h, t) \sin[D(\lambda, \varphi, h, t)]$$
(22c)

$$Z(\lambda, \varphi, h, t) = F(\lambda, \varphi, h, t) \sin[I(\lambda, \varphi, h, t)]$$
(22d)

1.4 The GEOMAG Algorithm

The Main-Field Gauss coefficients at the base epoch, T_{Epoch} , are stored in array C of the GEOMAG algorithm (sometimes referred to as the MAGVAR algorithm), which is listed in the appendix, such that the lower half of array C is occupied by those Gauss coefficients $g_{nm}(T_{Epoch})$ corresponding to the *cosine* terms in the potential function of eq. (1), while the upper half of

array C is occupied by those Gauss coefficients $h_{nm}(T_{Epoch})$ corresponding to the *sine* terms in eq. (1). Table 1 illustrates the details of this storage scheme, which is equivalent to the following mathematical assignments:

$$C_{nm} = \left\{ \begin{array}{ll} g_{nm} & m \ge n \\ h_{m,n+1} & m > n \end{array} \right\} \tag{23}$$

which implies that:

$$g_{nm} = C_{nm} \qquad m \le n \tag{24a}$$

$$h_{nm} = C_{m-1,n} \qquad m \le n, \quad m \ne 0$$
 (24b)

The Secular-Variation Gauss coefficients which describe the Main field's slow annual change are stored in array CD (which stands for \dot{C} [pronounced C dot]) such that the lower half of array CD is occupied by the Gauss coefficients $\dot{g}_{\rm im}$, which correspond to the *cosine* terms in eq. (3), while the upper half of the array is occupied by the Gauss coefficients $\dot{h}_{\rm im}$, corresponding to the *sine* terms in eq. (3). Table 2 illustrates the details of this storage scheme for array CD. It takes essentially the same form as table 1 for array C and corresponds to the following mathematical assignments:

$$\dot{C}_{nm} = \begin{cases} \dot{g}_{nm} & m \leq n \\ \dot{h}_{m,n+1} & m > n \end{cases}$$
(25)

which implies that:

$$\dot{g}_{nm} = \dot{C}_{nm} \qquad m \le n \tag{26a}$$

$$\dot{h}_{nm} = \dot{C}_{m-1,n} \qquad m \le n, \qquad m \ne 0 \tag{26b}$$

The numerical values of the Gauss coefficients at the base epoch and their corresponding predictive annual rates of change for the WMM-95 geomagnetic model are listed in table 3. These numerical values are inserted into arrays C and CD through data statements in the GEOMAG algorithm. Replacing the Gauss coefficients in these data statements and the date of their base epoch are the only changes that need to be made to update the algorithm from an older model to the new model. In all other respects the GEOMAG routine remains unaltered.

Other versions of the GEOMAG routine exist for which the coefficients can be read in from an external file. Then, only the external coefficient data file needs to be updated, while the algorithm remains unchanged.

TABLE 1. ARRANGEMENT OF MAIN FIELD COEFFICIENTS IN ARRA' \mathbf{C}_{nm}

n\m	0	1	2	3	4	5	6	7	8	9	10	11	12
0	800	h_{11}	h_{21}	h_{31}	h_{41}	h ₅₁	h_{61}	h_{71}	h_{81}	h_{91}	$h_{10,1}$	$h_{11,1}$	$h_{12,1}$
1	g 10	<i>g</i> ₁₁	h_{22}	h_{32}	h_{42}	h_{52}	h_{62}	h_{72}	h ₈₂	h_{92}	$h_{10,2}$	$h_{11,2}$	$h_{12,2}$
2	g ₂₀	g ₂₁	g 22	h_{33}	h_{43}	h ₅₃	h ₆₃	h_{73}	h ₈₃	h_{93}	$h_{10,3}$	$h_{11,3}$	$h_{12,3}$
3	g 30	<i>g</i> ₃₁	g 32	g 33	h_{44}	h ₅₄	h ₆₄	h_{74}	h ₈₄	h ₉₄	$h_{10,4}$	$h_{11,4}$	$h_{12,4}$
4	g 40	<i>g</i> ₄₁	g 42	<i>g</i> 43	<i>g</i> 44	h ₅₅	h ₆₅	h ₇₅	h ₈₅	h ₉₅	$h_{10,5}$	$h_{11,5}$	$h_{12,5}$
5	g 50	g ₅₁	g 52	g 53	g 54	g 55	h ₆₆	h ₇₆	h ₈₆	h ₉₆	$h_{10,6}$	$h_{11,6}$	$h_{12,6}$
6	g 60	g ₆₁	8 62	g 63	g 64	g 65	g 66	h_{77}	h ₈₇	h_{97}	$h_{10,7}$	$h_{11,7}$	$h_{12,7}$
7	g 70	<i>g</i> ₇₁	<i>g</i> 72	<i>g</i> 73	<i>g</i> 74	<i>g</i> 75	8 76	<i>g</i> 77	h_{88}	h_{98}	$h_{10,8}$	$h_{11,8}$	$h_{12,8}$
8	g 80	g ₈₁	g 82	g 83	g 84	g 85	g 86	g 87	g 88	h ₉₉	$h_{10,9}$	h _{11,9}	$h_{12,9}$
9	g 90	g 91	g ₉₂	g 93	g 94	g 95	g 96	g 97	g 98	g 99	$h_{10,10}$	$h_{11,10}$	$h_{12,10}$
10	g 10,0	g 10,1	g 10,2	<i>g</i> 10,3	<i>g</i> 10,4	g 10,5	<i>g</i> 10,6	g 10,7	<i>g</i> 10,8	g 10,9	<i>g</i> 10,10	$h_{11,11}$	$h_{12,11}$
11	g 11,0	g _{11,1}	<i>g</i> 11,2	<i>g</i> 11,3	<i>g</i> 11,4	g 11,5	g 11,6	<i>g</i> 11,7	<i>g</i> 11,8	g 11,9	<i>g</i> 11,10	<i>g</i> 11,11	$h_{12,12}$
12	g 12,0	<i>g</i> _{12,1}	<i>g</i> 12,2	<i>g</i> 12,3	<i>g</i> 12,4	g 12,5	<i>g</i> 12,6	g 12,7	<i>g</i> 12,8	g 12,9	<i>g</i> 12,10	<i>g</i> 12,11	<i>g</i> 12,12

TABLE 2. ARRANGEMENT OF SECULAR VARIATION COEFFICIENTS IN ARRAY $\dot{\mathbf{C}}_{nm}$

n\m	0	1	2	3	4	5	6	7	8	9	10	11	12
n	•	•	•	•	•	•	•	•	•	•	•	•	•
0	g_{00}	h_{11}	h_{21}	h_{31}	h_{41}	h_{51}	h_{61}	h_{71}	h_{81}	h_{91}	$h_{10,1}$	$h_{11,1}$	$h_{12,1}$
1	<i>g</i> 10	ġ ₁₁	\dot{h}_{22}	\dot{h}_{32}	\dot{h}_{42}	\dot{h}_{52}	\dot{h}_{62}	\dot{h}_{72}	\dot{h}_{82}	\dot{h}_{92}	$\dot{h}_{10,2}$	$\dot{h}_{11,2}$	$\dot{h}_{12,2}$
2	ġ ₂₀	\dot{g}_{21}	\dot{g}_{22}	\dot{h}_{33}	\dot{h}_{43}	\dot{h}_{53}	\dot{h}_{63}	\dot{h}_{73}	\dot{h}_{83}	\dot{h}_{93}	$\dot{h}_{10,3}$	$\dot{h}_{11,3}$	$\dot{h}_{12,3}$
3	ġ ₃₀	ġ ₃₁	ġ ₃₂	ġ ₃₃	\dot{h}_{44}	\dot{h}_{54}	\dot{h}_{64}	\dot{h}_{74}	\dot{h}_{84}	\dot{h}_{94}	$h_{10,4}$	$h_{11,4}$	$h_{12,4}$
4	$\overset{\bullet}{g}_{40}$	g ₄₁	ġ ₄₂	ġ ₄₃	ġ ₄₄	\dot{h}_{55}	\dot{h}_{65}	\dot{h}_{75}	\dot{h}_{85}	$\overset{ullet}{h}_{95}$	$h_{10,5}$	$h_{11,5}$	$h_{12,5}$
5	ġ ₅₀	ġ ₅₁	ġ ₅₂	ġ ₅₃	ġ ₅₄	ġ ₅₅	\dot{h}_{66}	\dot{h}_{76}	\dot{h}_{86}	$\overset{ullet}{h}_{96}$	$h_{10,6}$	$h_{11,6}$	$h_{12,6}$
6	ġ ₆₀	ġ ₆₁	ġ ₆₂	ġ ₆₃	ġ ₆₄	ġ ₆₅	ġ ₆₆	\dot{h}_{77}	\dot{h}_{87}	$\overset{ullet}{h}_{97}$	$h_{10,7}$	$h_{11,7}$	$h_{12,7}$
7	ġ ₇₀	ġ ₇₁	ġ ₇₂	ġ ₇₃	ġ ₇₄	ġ ₇₅	ġ ₇₆	ġ ₇₇	\dot{h}_{88}	\dot{h}_{98}	$\dot{h}_{10,8}$	$\dot{h}_{11,8}$	$h_{12,8}$
8	ġ ₈₀	ġ ₈₁	ġ ₈₂	ġ ₈₃	ġ ₈₄	ġ ₈₅	ġ ₈₆	ġ ₈₇	ġ ₈₈	$\overset{ullet}{h}_{99}$	$\dot{h}_{10,9}$	$\dot{h}_{11,9}$	$\dot{h}_{12,9}$
9	ġ ₉₀	\dot{g}_{91}	ġ ₉₂	ġ ₉₃	ġ ₉₄	ġ ₉₅	ġ ₉₆	ġ ₉₇	ġ ₉₈	ġ ₉₉	$h_{10,10}$	$h_{11,10}$	$h_{12,10}$
10	ġ _{10,0}	<i>g</i> 10,1	<i>g</i> 10,2	ġ _{10,3}	ġ _{10,4}	ġ _{10,5}	<i>g</i> 10,6	<i>g</i> 10,7	ġ _{10,8}	<i>g</i> 10,9	$\dot{g}_{10,10}$	$\dot{h}_{11,11}$	$\dot{h}_{12,11}$
11	$\dot{g}_{11,0}$	<i>g</i> 11,1	<i>g</i> 11,2	ġ _{11,3}	ġ _{11,4}	ġ _{11,5}	<i>g</i> 11,6	<i>g</i> 11,7	ġ _{11,8}	<i>g</i> 11,9	$\dot{g}_{11,10}$	$\dot{g}_{11,11}$	$h_{12,12}$
12	<i>g</i> 12,0	<i>g</i> 12,1	<i>g</i> 12,2	<i>g</i> 12,3	<i>g</i> 12,4	<i>g</i> 12,5	<i>g</i> 12,6	<i>g</i> 12,7	ġ _{12,8}	<i>g</i> 12,9	<i>g</i> 12,10	<i>g</i> 12,11	<i>g</i> 12,12

Table 3. WMM-95 Model Coefficients

n	m	g_n^m	h_n^m	g_n^m	\dot{h}_n^m
1	0	-29,682.1	0	17.6	0
1	1	-1,782.2	5,315.6	13.2	-18
2	0	-2,194.7	0	-13.7	0
2	1	3,078.6	-2,359.1	4	-14.6
2	2	1,685.7	-418.6	-0.3	-7.2
3	0	1,318.8	0	0.8	0
3	1	-2,273.6	-261.1	-6.6	4
3	2	1,246.9	301	-0.5	2.2
3	3	766.3	-416.5	-8.5	-12.6
4	0	940	0	1.2	0
4	1	782.9	259.4	1.1	1.3
4	2	290.9	-230.9	-6.8	1
4	3	-418.9	99.8	0.3	2.5
4	4	113.8	-306.1	-4.5	-1.2
5	0	-209.5	0	0.9	0
5	1	354	43.7	0.5	0.5
5	2	238.2	157.6	-1.4	1.5
5	3	-122.1	-150.1	-1.7	0.6
5	4	-162.8	-59.2	0	1.7
5	5	-23.3	104.4	2.1	0.6
6	0	68.5	0	0.4	0
6	1	65.6	-15.2	-0.3	0.7
6	2	64.1	74.3	0.3	-1.5
6	3	-169.1	69.4	2.1	-0.5
6	4	-0.5	-55.3	0	-0.7
6	5	16.5	3	-0.4	1.1
6	6	-91	33.3	-0.4	2.6
7	0	78	0	-0.3	0
7	1	-68.1	-76.1	-1.1	0.3
7	2	0.1	-24.5	-0.5	0

Table 3. WMM-95 Model Coefficients (Con.)

n	m	g_n^m	h_n^m	$\overset{ullet}{g}_{n}^{m}$	$\dot{\boldsymbol{h}}_{n}^{m}$
7	3	29.6	1.6	0.5	0.7
7	4	6	20	1.3	-0.6
7	5	8.7	16.5	0.1	0.1
7	6	9.2	-23.6	0	-0.6
7	7	-2.4	-6.8	-0.9	-0.4
8	0	24.7	0	0.1	0
8	1	3.4	14.9	0	0.4
8	2	-1.5	-19.5	0.4	-0.3
8	3	-9.6	6.3	0.3	0.1
8	4	-16.5	-20.4	-1.3	0.8
8	5	2.6	12.2	0.5	-0.1
8	6	3.6	7	0.4	-1.3
8	7	-4.9	-19	-0.9	-0.9
8	8	-8.5	-8.8	0.1	-1.1
9	0	2.9	0	0	0
9	1	7.5	-19.8	0	0
9	2	0.4	14.6	0	0
9	3	-10.3	10.9	0	0
9	4	9.7	-7.5	0	0
9	5	-2.3	-6.8	0	0
9	6	-2.4	9.3	0	0
9	7	6.8	7.7	0	0
9	8	-0.5	-8.1	0	0
9	9	-6.5	2.6	0	0
10	0	-2.9	0	0	0
10	1	-3.3	3.2	0	0
10	2	2.8	1.7	0	0
10	3	-4.3	2.9	0	0
10	4	-3.1	5.6	0	0
10	5	2.4	-3.4	0	0

Table 3. WMM-95 Model Coefficients (Con.)

n	m	g_n^m	h_n^m	$\overset{\bullet}{g}_{n}^{m}$	$\overset{ullet}{h}_{n}^{m}$
10	6	2.8	-0.7	0	0
10	7	0.7	-2.9	0	0
10	8	4.1	2.3	0	0
10	9	3.6	-1.6	0	0
10	10	0.6	-6.6	0	0
11	0	1.7	0	0	0
11	1	-1.6	0.3	0	0
11	2	-3.6	1	0	0
11	3	1.2	-3.6	0	0
11	4	-0.6	-1.4	0	0
11	5	0.1	1.9	0	0
11	6	-0.7	0.2	0	0
11	7	-0.8	-1.3	0	0
11	8	1.3	-2.4	0	0
11	9	-0.3	-0.6	0	0
11	10	2.2	-2.2	0	0
11	11	4.2	1.3	0	0
12	0	-1.8	0	0	0
12	1	0.9	0.3	0	0
12	2	-0.1	1.4	0	0
12	3	-0.5	0.8	0	0
12	4	0.8	-3	0	0
12	5	0.2	0.7	0	0
12	6	0.5	0.5	0	0
12	7	0.4	-0.8	0	0
12	8	-0.4	0.6	0	0
12	9	0.3	0.1	0	0
12	10	0.2	-1.3	0	0
12	11	0.4	-0.4	0	0
12	12	0.6	0.9	0	0

Important parameters in the GEOMAG routine and their mathematical correspondences are:

		(250 125 1
A	~	a = 6378.137 km
В	~	b = 6356.7523142 km
RE	~	$R_E = 6371.2 \ km$
TIME	~	t
EPOCH	~	T_{Epoch}
DT	~	t - T_{Epoch}
ALT	~	h
SNORM(N,M)	~	S^{nm}
K(N,M)	~	κ^{nm}
GLAT	~	λ
GLON	~	φ
SP(M)	~	$sin(m\varphi)$
CP(M)	~	$cos(m\varphi)$
ST	~	$sin(\theta)$
CT	~	$cos(\theta)$
CA	~	$cos(\alpha)$
SA	~	$sin(\alpha)$
BR	~	B_r
BT	~	$B_{ heta}$
BP	~	B_{arphi}
BX	~	B_X
BY	~	B_Y
BZ	~	B_Z
DEC	~	B_D
DIP	~	B_I
TI	~	B_F
MAXDEG	~	N
MAXORD	~	M = N
P(N,M)	~	$P^{nm}(\theta)$
DP(N,M)	~	$\frac{dP^{n\hat{m}}(\hat{\theta})}{d\theta}$
TC	~	$C + (t - T_{Enoch})\dot{C}$
CD	~	$C + (t - T_{Epoch})\dot{C}$ \dot{C}
Q2	~	Q^2
~-		×

Note that R_E is not intended to be the mean radius of the WGS-84 ellipsoid. By international convention established by the International Association of Geomagnetism and Aeronomy circa 1968, it is the mean radius of a modified ellipsoid established by the International Astronomical Union (IAU) in 1966. This ellipsoid is referred to as the modified IAU-66 ellipsoid (Zmuda [1971]).

The GEOMAG algorithm is organized into two modules, each with its own entry point. The first is an *Initialization Module*. Its purpose is to compute all constants such as the recursion relation

factors for the associated Legendre polynomials κ^{nm} , the Schmidt normalization factors S^{nm} , and any other parameters that do not depend on position or time. The entry point for this module is:

GEOMAG(MAXDEG)

The parameter MAXDEG determines the maximum degree and order of the magnetic model to be used in the computations. Normally, MAXDEG=12, which is the maximum degree and order of the WMM series of geomagnetic models. In order to reduce computation time, MAXDEG may be set to a number less that 12 (e.g., 8 or 10). However, the accuracy of the computed magnetic parameters is correspondingly reduced. MAXDEG must be set in the calling program.

The second module is the *Processing Module*, which has the following entry point: GEOMG1(ALT,GLAT,GLON,TIME,DEC,DIP,TI,GV)

The purpose of this module is to compute the magnetic *Declination*, *Inclination*, *Total Intensity*, and the *Grid Variation* at each *geodetic* position and time supplied to it. The units of the parameters in the argument list of the GEOMG1 entry point are as follows:

ALT	~	kilometers	(e.g., 5.314)	(IN)
GLAT	~	degrees	(e.g., 33.716)	(IN)
GLON	~	degrees	(e.g., -163.315)	(IN)
TIME	~	years	(e.g., 1997.427)	(IN)
DEC	~	degrees	(e.g., -121.734)	(OUT)
DIP	~	degrees	(e.g., 48.387)	(OUT)
TI	~	nanoTeslas	(e.g., 35781.7)(OUT))
GV	~	degrees	(e.g., 51.768)	(OUT)

The computed magnetic field parameters are referenced to the WGS-84 ellipsoid. The last parameter, GV, is the Grid Variation which is computed only for the polar regions (i.e., above +55° latitude or below -55° latitude). Outside of these regions, a default value of -999.0 is dummied in. The Grid Variation is referenced to *Grid North* of a polar stereographic projection. The model is considered to be a valid representation of the Earth's core magnetic field at geodetic altitudes ranging from the *ocean bottom* to +1000 km for all geodetic latitudes and longitudes.

The SV computation of a geomagnetic component at a fixed time $t=\tau$ is accomplished by making two calls to the entry point GEOMG1, one at time $t_1=\tau$ - 0.5 and one at $t_2=\tau+0.5$, where t is expressed in years. This yields the Declination, Inclination, and the Total Intensity at two different times spaced one year apart. Using these three magnetic components, any other magnetic component can be calculated at these same two times via eqs. (21a) through (21e) and eqs. (22a) through (22d). The SV is then determined by differencing the two MF values of a particular component. For example, the Horizontal component's SV is computed by inserting eq. (22a) into the following:

$$\dot{H}(\lambda, \varphi, h, \tau) = [H(\lambda, \varphi, h, t_2) - H(\lambda, \varphi, h, t_1)]/\Delta t$$
(27)

where

$$\Delta t = t_2 - t_1 = 1 \text{ year} \tag{28}$$

Charts 1, 2, and 3 are Mercator projections of the MF magnetic Declination, Inclination, and Total Intensity at the 1995.0 Epoch. Charts 4, 5, and 6 are Mercator projections of the SV for these same three magnetic components at the 1997.5 Epoch. These charts are similar to the wall-size charts of these and other geomagnetic components produced by NIMA.

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