THE JOINT US/UK 2000 EPOCH WORLD MAGNETIC MODEL

and the

GEOMAG/MAGVAR ALGORITHM

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1.0 Introduction

The Earth's magnetic field, as measured by a magnetic sensor on or above the Earth's surface, is actually a composite of several magnetic fields generated by a variety of sources. These fields are superimposed on each other and through inductive processes interact with each other. The most important of these geomagnetic sources are:

a. the Earth's conducting, fluid outer core;
b. the Earth's crust/upper mantle;
c. the ionosphere; and
d. the magnetosphere.

More than 95% of the geomagnetic field is generated by the Earth's outer core. It is this portion of the geomagnetic field that is represented by the 2000 Epoch World Magnetic Model (WMM-2000). Those portions of the geomagnetic field not represented by the model are collectively referred to as the anomalous geomagnetic field, which varies both spatially and temporally with respect to the model.

The model itself consists of a degree and order 12 spherical-harmonic Main (i.e., core-generated) field (MF) model comprised of 168 spherical-harmonic Gauss coefficients and a degree and order 8 spherical-harmonic Secular-Variation (SV) (core-generated, slow temporal variation) field model comprised of an additional 80 spherical-harmonic Gauss coefficients. As with previous models (e.g., WMM-95) the SV Gauss coefficients equal to or greater than degree and order 9 were set to zero, due to a lack of data. The primary geomagnetic data set for the 2000 Epoch MF model were the Danish OERSTED satellite's scalar-magnetic-field observations collected from March through May 1999. The SV model data were comprised primarily of vector and scalar observations from Geomagnetic Observatories and Repeat Surveys provided many government and private institutuins in many countries located around the world. The MF Gauss coefficients characterize the geomagnetic field at one instant of time called the *base epoch*, which for WMM-2000 is 2000.0. The *predictive* SV coefficients characterize the slow rate of change of the geomagnetic field for the 5-year period from the base epoch to the *termination epoch*, which for WMM-2000 is 2005.0, at which time, WMM-2000 will be replaced by WMM-2005. The magnetic field components are computed via the magnetic variation algorithm (GEOMAG), which is a FORTRAN subroutine that uses the WMM Gauss coefficients in conjunction with the spherical-harmonic expansions associated with each field component.

The WMM-2000 coefficients were produced jointly by the British Geological Survey (BGS) in Edinburgh, Scotland, and the U. S. Geological Survey in Golden, Colorado, on behalf of the British Hydrographic Office in Taunton, England, and the National Imagery and Mapping Agency (NIMA) [formerly the Defense Mapping Agency (DMA)], Reston, Virginia. In addition to the USGS and BGS, The WMM models are also distributed by the National Oceanic and Atmospheric Administration's National Geophysical Data Center/World Data Center (NOAA's NGDC/WDC-A) on behalf of NIMA in accordance with NIMA Instructions 8000.1 and 8000.2. These models are produced at 5-year intervals, as are NIMA's Declination/Grid-Variation charts, while charts of the other geomagnetic components are published by NIMA every 10 years. The

2000.0 Epoch corresponds to a 5-year interval when only the Declination and Grid Variation components will be published in chart form by NIMA. The military specifications for the WMM are contained in MIL-W-89500 (DMA [1993]). Magnetic model requirements that are more stringent than those set forth in this military specification (e.g., those which must include magnetic effects of the Earth's crust, ionosphere, or magnetosphere and/or require greater spatial or temporal resolution on a regional or local basis) should be addressed to:

Director, National Imagery and Mapping Agency
12310 Sunrise Valley Drive
Reston, VA 20191-3449
ATTN: P33

The magnetic field model and a convenient Window-95 version of the GEOMAG software, as well as page sized charts of various components of the magnetic field from the current WMM can be accessed and downloaded from the following USGS WEB site:

http://geomag.usgs.gov

Wall-Sized Charts published by NIMA are sold to the public for a modest fee through NOAA's international distribution system. To find the nearest authorized distributor go to the following WEB site and click on Sales Agents:

http://chartmaker.ncd.noaa.gov/

These charts are approximately 48 inches long and 36 inches tall. The following charts are available:

a.) World Mercator Projection of Magnetic Declination (D)
NIMA/DMA Stock #WOBZC42

b.) North/South Polar Stereographic Projection of Magnetic Declination (D) and Reverse Side: Grid Variation (GV), same projection; NIMA/DMA Stock #WOBZC43

c.) World Mercator Projection of Magnetic Inclination (I) with Polar Stereographic Projection on reverse side; NIMA/DMA Stock #WOXZC30

d.) World Mercator Projection of the Horizontal Magnetic Intensity (H) with Polar Stereographic Projection on reverse side; NIMA/DMA Stock #WOXZC33 e.) World Mercator Projection of the Vertical Magnetic Component Z) with Polar Stereographic Projection on reverse side; NIMA/DMA Stock #WOXZC36

f.) World Mercator Projection of Total Magnetic Intensity (F) with Polar Stereographic Projection on reverse side; NIMA/DMA Stock #WOXZC39

It is extremely important to recognize that the WMM series of geomagnetic models and the charts produced from these models characterize only that portion of the Earth's magnetic field which is generated by the Earth's fluid outer core. The portions of the geomagnetic field generated by the Earth's crust, upper mantle, ionosphere, and magnetosphere are not represented in these models. Consequently, a magnetic sensor such as a compass or magnetometer may observe spatial and temporal magnetic anomalies when referenced to the appropriate WMM. In particular, certain local, regional, and temporal magnetic declination anomalies can exceed 10 degrees. Anomalies of this magnitude are not common but they do exist. Declination anomalies on the order of 3 or 4 degrees are not uncommon but are of small, spatial extent and are relatively isolated. On land, spatial anomalies are produced by mountain ranges; ore deposits; ground struck by lightning; geological faults; and cultural features such as trains, planes, tanks, railroad tracks, power lines, etc. In ocean areas these anomalies occur most frequently along continental margins; near seamounts; and near ocean ridges, trenches, and fault zones, particularly those of volcanic origin. Ships and submarines are also sources of magnetic anomalies in the ocean.

Temporal anomalies in either ocean or land areas can last from a few minutes to several days and are produced by ionospheric and magnetospheric processes which are driven by the *solar wind*. In particular, *magnetic storms* generated by *solar flare*s and other solar activity can, through modulation of the solar wind, cause severe and persistent magnetic anomalies in the Earth's environment. Even during periods of quiet solar activity, significant spatial and temporal magnetic anomalies are found in the polar and equatorial regions of the Earth, where magnetic fields produced by ionospheric current systems, such as the *auroral electrojets* and the *equatorial electrojet*, are always present. Most sources of magnetic anomalies are comparatively isolated in either space or time. Therefore, from a global perspective, the root-mean-square (RMS) Declination (D), Inclination (I), and Grid Variation (GV) errors of the WMM are estimated to be less than 0.5 degrees in ocean areas and less than 1.0 degree in land areas at the Earth's surface over the entire 5-year life of a particular model. Also, the RMS errors at sea level of the Horizontal Intensity (H), the Vertical component (Z), and the Total Intensity (F) of the WMM over ocean and land areas are estimated to be less than 200 *nanoTeslas* (nT) over the entire 5-year life of a particular model.

1.1 The Mathematical Model

The Earth's core-generated magnetic field has associated with it a geomagnetic potential $V(r,\theta,\varphi,t)$, which can be expressed in spherical coordinates in terms of a spherical-harmonic expansion of the following form:

$$V(r,\theta,\varphi,t) = R_E \sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+1} \sum_{m=0}^{n} \{g_{nm}(t)\cos(m\varphi) + h_{nm}(t)\sin(m\varphi)\} P_n^m(\theta) \tag{1}$$

where the spherical coordinates (r,θ,φ) correspond to the radius from the center of the Earth, the colatitude (i.e., 90^0 - latitude), and the longitude. R_E is the mean radius of the Earth (6371.2 km); $g_{nm}(t)$ and $h_{nm}(t)$ are referred to as the Gauss coefficients at time t, where t is the time in years (e.g., 2002.312). $P_n^m(\theta)$ represents a particular Schmidt-normalized associated Legendre polynomial of spherical-harmonic degree n and order m. These are polynomials in terms of the cosine of the colatitude θ. The Gauss coefficients are slowly varying functions of time and are expressed in the form of a Taylor series expansion, where only terms up to first order in time are retained so that:

$$g_{nm}(t) = g_{nm}(T_{Epoch}) + \dot{g}_{nm}\,(t - T_{Epoch}) \qquad T_{Epoch} \le t \le T_{Epoch} + 5 \tag{2a}$$

$$h_{nm}(t) = h_{nm}(T_{Epoch}) + \dot{h}_{nm}\,(t - T_{Epoch}) \qquad T_{Epoch} \le t \le T_{Epoch} + 5 \tag{2b}$$

where T_{Epoch} is the base epoch of the model, which for WMM-2000 is 2000.0. Thus, $g_{nm}(T_{Epoch})$ and $h_{nm}(T_{Epoch})$ are the Schmidt-normalized Gauss coefficients of the WMM at the model's base epoch, while the Schmidt-normalized SV Gauss coefficients, $\dot{g}_{nm}$ and $\dot{h}_{nm}$ (pronounced g_{nm} dot and h_{nm} dot, where the dot represents differentiation with respect to time: $\frac{d}{dt}$), are the annual rates of change of the MF Gauss coefficients g_{nm} and h_{nm} and are evaluated at the middle of the model's lifespan (i.e., at $T_{Epoch} + 2.5$). The MF Gauss coefficients and SV field Gauss coefficients are collectively referred to as spherical-harmonic coefficients.

Taking the time derivative of eq. (1) yields the spherical-harmonic expression for the Secular-Variation $\dot{V}(r,\theta,\varphi,t)$ of the geomagnetic potential:

$$\dot{V}(r,\theta,\varphi,t) = R_E \sum_{n=1}^{N} \left(\frac{R_E}{r}\right)^{n+1} \sum_{m=0}^{n} \left\{\dot{g}_{nm}(t)\cos(m\varphi) + \dot{h}_{nm}(t)\sin(m\varphi)\right\} P_n^m(\theta) \tag{3}$$

which, due to the assumption that the MF Gauss coefficients vary linearly with time during the course of a 5-year interval, as expressed by eqs. (2a) and (2b), is approximately time independent over this short time span. The maximum degree N of the spherical-harmonic expansions in eqs. (1) and (3) is equal to 12. This value is determined by noting that when the spectral density of the MF Gauss coefficients is plotted as a function of harmonic degree, a distinct break in this density function occurs between degree 12 and degree 15. This is interpreted to mean that the low-degree harmonics corresponding to $N \le 12$ are dominated by core-generated magnetic fields, while those high-degree harmonics for which $N \geq 15$ are dominated by fields generated within the crust and upper mantle. The non-core fields are primarily associated with permanent and induced magnetization. This magnetization is limited to depths for which the ambient temperature does not exceed the Curie temperature. Spherical-harmonic degrees 13 and 14 correspond to a transition region where neither magnetic fields generated within the Earth's fluid core nor those generated within the Earth's crust dominate. We therefore use $N = 12$ as the spherical-harmonic cutoff which permits the best description of the core-generated magnetic field and its slow temporal change. This means that the shortest wavelength contained in the model is:

$$\lambda_{\min} = \frac{2\pi R_E}{N} = 3336\ km \tag{4}$$

Thus, the WMM is a low-resolution model. High-resolution (short wavelength) descriptions of that part of the magnetic field generated by the Earth's upper crust are better characterized via rectangular harmonic modeling of small local areas (Quinn and Shiel [1993]), while intermediate wavelength descriptions of the Earth's magnetic field generated by the lower crust and upper mantle are best characterized via spherical-cap harmonic models of large regional areas (Haines [1985a, 1985b, 1985c, and 1990]). Global geomagnetic data sets currently available do not support high resolution (i.e., $\lambda \leq 500$ km) and only marginally support intermediate-resolution (i.e., 500 km $< \lambda <$ 3336 km) magnetic field modeling. Special local and/or regional magnetic surveys are required to generate intermediate-resolution and high-resolution geomagnetic models. Consequently, there are some applications for which the use of the WMM may be inadequate.

The Earth's magnetic field $\mathbf{B}(r,\theta,\varphi,t)$ is a vector quantity having three components which correspond to the projection of the magnetic field vector onto the three coordinate axes. Thus, $B_r(r,\theta,\varphi,t)$ is that portion of the field pointing radially outward from the Earth's center (i.e., perpendicular to the surface of the Earth); $B_\theta(r,\theta,\varphi,t)$ is that portion of the field pointing locally due south; and $B_\varphi(r,\theta,\varphi,t)$ is that portion of the field pointing locally due east. The magnetic field vector can be computed from the geomagnetic potential by taking its negative gradient, thus:

$$\boldsymbol{B}(r,\theta,\varphi,t) = -\nabla V(r,\theta,\varphi,t) \tag{5}$$

Consequently, the magnetic field components are related to the geomagnetic potential as follows:

$$B_r(r,\theta,\varphi,t) = -\frac{\partial V(r,\theta,\varphi,t)}{\partial r} \tag{6a}$$

$$B_\theta(r,\theta,\varphi,t) = -\frac{1}{r}\frac{\partial V(r,\theta,\varphi,t)}{\partial\theta} \tag{6b}$$

$$B_\varphi(r,\theta,\varphi,t) = -\frac{1}{r\sin\theta}\frac{\partial V(r,\theta,\varphi,t)}{\partial\varphi} \tag{6c}$$

which yield the following spherical-harmonic expansions:

$$B_r(r,\theta,\varphi,t) \;=\; \sum_{n=1}^{N}(n+1)\Big(\tfrac{R_E}{r}\Big)^{n+2}\sum_{m=0}^{n}\{g_{nm}(t)\cos(m\varphi) \;+\; h_{nm}(t)\sin(m\varphi)\}P_n^m(\theta) \tag{7a}$$

$$B_\theta(r,\theta,\varphi,t) \;=\; -\sum_{n=1}^{N}\Big(\tfrac{R_E}{r}\Big)^{n+2}\sum_{m=0}^{n}\{g_{nm}(t)\cos(m\varphi) \;+\; h_{nm}(t)\sin(m\varphi)\}\;\frac{dP_n^m(\theta)}{d\theta} \tag{7b}$$

$$B_\varphi(r,\theta,\varphi,t) \;=\; \frac{1}{\sin\theta}\sum_{n=1}^{N}\Big(\tfrac{R_E}{r}\Big)^{n+2}\sum_{m=0}^{n} m\,\{g_{nm}(t)\sin(m\varphi) \;-\; h_{nm}(t)\cos(m\varphi)\}P_n^m(\theta) \tag{7c}$$

These expressions are solutions of the Laplace equation, which in turn is derived from Maxwell's famous electromagnetic field equations under the assumptions that the magnetic field exists in a source-free region (i.e., no charges or currents are present), and the fields are slowly varying.

Similarly, for the geomagnetic SV field we have:

$$\boldsymbol{B}(r,\theta,\varphi,t) \;=\; -\,\nabla\,\dot{V}(r,\theta,\varphi,t) \tag{8}$$

so that:

$$\dot{B}_r\,(r,\theta,\varphi,t) \;=\; -\frac{\partial\dot{V}(r,\theta,\varphi,t)}{\partial r} \tag{9a}$$

$$\dot{B}_\theta\,(r,\theta,\varphi,t) \;=\; -\frac{1}{r}\,\frac{\partial\dot{V}(r,\theta,\varphi,t)}{\partial\theta} \tag{9b}$$

$$\dot{B}_\varphi\,(r,\theta,\varphi,t) \;=\; -\frac{1}{r\sin\theta}\,\frac{\partial\dot{V}(r,\theta,\varphi,t)}{\partial\varphi} \tag{9c}$$

which yield the following spherical-harmonic expressions:

$$\dot{B}_r\,(r,\theta,\varphi,t) \;=\; \sum_{n=1}^{N}(n\,+\,1)\Big(\tfrac{R_E}{r}\Big)^{n+2}\sum_{m=0}^{n}\Big\{\dot{g}_{nm}\,(t)\cos(m\varphi) \;+\; \dot{h}_{nm}\,(t)\sin(m\varphi)\Big\}P_n^m(\theta) \tag{10a}$$

$$\dot{B}_\theta\,(r,\theta,\varphi,t) \;=\; -\sum_{n=1}^{N}\Big(\tfrac{R_E}{r}\Big)^{n+2}\sum_{m=0}^{n}\Big\{\dot{g}_{nm}\,(t)\cos(m\varphi) \;+\; \dot{h}_{nm}\,(t)\sin(m\varphi)\Big\}\;\frac{dP_n^m(\theta)}{d\theta} \tag{10b}$$

$$\dot{B}_{\varphi}(r,\theta,\varphi,t) = \frac{1}{\sin\theta}\sum_{n=1}^{N}\left(\frac{R_E}{r}\right)^{n+2}\sum_{m=0}^{n} m\left\{\dot{g}_{nm}(t)\sin(m\varphi) - \dot{h}_{nm}(t)\cos(m\varphi)\right\}P_n^m(\theta) \tag{10c}$$

1.2 Spherical-Harmonic Normalization

The Gauss coefficients $g_{nm}(t)$, $h_{nm}(t)$, $\dot{g}_{nm}(t)$, and $\dot{h}_{nm}(t)$, as well as the associated Legendre polynomials and their derivatives, are Schmidt normalized by international agreement (circa 1930) of the International Union of Geodesy and Geophysics. This particular normalization allows one to determine which spherical-harmonic terms of a particular model are the most significant simply by a cursory inspection of the model coefficients' relative magnitudes. The Schmidt-normalized associated Legendre polynomials $P_n^m(\theta)$ are related to the unnormalized associated Legendre polynomials $P^{nm}(\theta)$ (note position of indices) by the following relation:

$$P_n^m(\theta) = S^{nm}P^{nm}(\theta) \tag{11}$$

The Schmidt normalization factors S^{nm} and the unnormalized associated Legendre polynomials $P^{nm}(\theta)$ are computed via recurrence relations as follows (Cain et al., 1967):

$$P^{00}(\theta) = 1 \tag{12a}$$

$$P^{nm}(\theta) = \sin\theta\, P^{n-1,m-1}(\theta) \qquad m = n \neq 0 \tag{12b}$$

$$P^{nm}(\theta) = \cos\theta\, P^{n-1,m}(\theta) - \kappa^{nm}P^{n-2,m}(\theta) \qquad m \neq n,\ n \geq 1 \tag{12c}$$

$$\frac{dP^{00}(\theta)}{d\theta} = 0 \tag{12d}$$

$$\frac{dP^{nm}(\theta)}{d\theta} = \sin\theta\frac{dP^{n-1,m-1}(\theta)}{d\theta} + \cos\theta\, P^{n-1,m-1}(\theta) \qquad m = n \neq 0 \tag{12e}$$

$$\frac{dP^{nm}(\theta)}{d\theta} = \cos\theta\frac{dP^{n-1,m}(\theta)}{d\theta} - \sin\theta\, P^{n-1,m}(\theta) - \kappa^{nm}\frac{dP^{n-2,m}(\theta)}{d\theta} \qquad m \neq n,\ n \geq 1 \tag{12f}$$

where:

$$\kappa^{nm} = \frac{(n-1)^2 - m^2}{(2n-1)(2n-3)} \tag{13}$$

and where it is understood that the undefined polynomial $P^{-1,0}(\theta)$ and its derivatives are set equal to zero. Similarly:

$$S^{0\,0} = 1 \tag{14a}$$

$$S^{n\,0} = \left(\frac{2n-1}{n}\right) S^{n-1,\,0} \qquad\qquad n > 0 \tag{14b}$$

$$S^{n\,m} = \sqrt{\frac{(n-m+1)J}{n+m}}\; S^{n,\,m-1} \qquad\qquad \left\{ \begin{array}{l} J = 2 \quad for \quad m = 1 \\ J = 1 \quad for \quad m > 1 \end{array} \right\} \tag{14c}$$

Also computed via recursion relations are the longitudinally dependent functions *cos(mφ)* and *sin(mφ)*. They are computed as follows:

$$\sin(m\,\varphi) = 0 \qquad\qquad m = 0 \tag{15a}$$

$$\cos(m\,\varphi) = 1 \qquad\qquad m = 0 \tag{15b}$$

$$\sin(m\,\varphi) = \sin(\varphi)\,\cos[(m-1)\,\varphi] + \cos(\varphi)\,\sin[(m-1)\,\varphi] \qquad m > 0 \tag{15c}$$

$$\cos(m\varphi) = \cos(\varphi)\,\cos[(m-1)\,\varphi] - \sin(\varphi)\,\sin[(m-1)\,\varphi] \qquad m > 0 \tag{15d}$$

1.3 Coordinate Transformations

Although the magnetic field model is defined in terms of *spherical* coordinates, the intended application is in *geodetic* coordinates. So, a coordinate transformation is necessary (Cain et al., 1967). The 1984 World Geodetic System (WGS-84) (DMA [1987]) and its corresponding ellipsoid are used as the reference datum for this purpose. Computing the magnetic field components at a given location expressed in geodetic coordinates using the WMM-2000 model is a three-step procedure:

a. Convert the geodetic latitude, longitude, and altitude (λ,φ,h) to spherical coordinates (r,θ,φ).
b. Compute the magnetic field components B_r(r,θ,φ,t), B_θ(r,θ,φ,t), and B_φ(r,θ,φ,t).
c. Rotate the magnetic field components from spherical coordinates back to geodetic coordinates.

This yields the magnetic field components B_X(λ,φ,h,t), B_Y(λ,φ,h,t), and B_Z(λ,φ,h,t), which are projections of the magnetic field vector **B**(λ,φ,h,t) onto the **X**-north, **Y**-east, and **Z**-vertically down coordinate axes of the local rectangular coordinate system defined by the tangent plane to the ellipsoid, which is concentric about the WGS-84 ellipsoid and which encompasses the point (λ,φ,h).

The transformations corresponding to *step a* are as follows:

$$\cos\theta = \frac{\sin\lambda}{\sqrt{Q^2 \cos^2\lambda + \sin^2\lambda}} \tag{16a}$$

$$\sin\theta = \sqrt{1-\cos^2\theta} \tag{16b}$$

where, if a and b are respectively the semi-major and semi-minor axes of the WGS-84 ellipsoid, then:

$$Q = \frac{h\sqrt{a^2-(a^2-b^2)\sin^2\lambda} + a^2}{h\sqrt{a^2-(a^2-b^2)\sin^2\lambda} + b^2} \tag{17}$$

Furthermore:

$$r^2 = h^2 + 2h\sqrt{a^2 - (a^2 - b^2)\sin^2\lambda} + \frac{a^4 - (a^4-b^4)\sin^2\lambda}{a^2 - (a^2-b^2)\sin^2\lambda} \tag{18}$$

The transformations corresponding to *step c* depend on the angle α through which the magnetic field vector must be rotated while transforming from spherical to geodetic coordinates. This rotation angle is defined by the following transformation equations:

$$\cos\alpha = \frac{h + \sqrt{a^2 \cos^2\lambda + b^2 \sin^2\lambda}}{r} \tag{19a}$$

$$\sin\alpha = \frac{(a^2-b^2)\cos\lambda \sin\lambda}{r\sqrt{a^2 \cos^2 + b^2 \sin^2\lambda}} \tag{19b}$$

$$\alpha = \lambda + \theta - \frac{\pi}{2} \tag{19c}$$

Consequently, the components of the magnetic field vector in geodetic coordinates may be computed as follows:

$$B_X(\lambda,\varphi,h,t) = -\cos\alpha\, B_\theta(r,\theta,\varphi,t) - \sin\alpha\, B_r(r,\theta,\varphi,t) \tag{20a}$$

$$B_Y(\lambda,\varphi,h,t) = B_\varphi(r,\theta,\varphi,t) \tag{20b}$$

$$B_Z(\lambda,\varphi,h,t) = \sin\alpha\, B_\theta(r,\theta,\varphi,t) - \cos\alpha\, B_r(r,\theta,\varphi,t) \tag{20c}$$

From these three rectangular geomagnetic field components, it is possible to compute all others. In particular, the following magnetic components can be computed:

$$B_H(\lambda,\varphi,h,t) \;=\; \sqrt{B_X^2(\lambda,\varphi,h,t) \;+\; B_Y^2(\lambda,\varphi,h,t)} \qquad \text{(Horizontal Intensity)} \tag{21a}$$

$$B_F(\lambda,\varphi,h,t) \;=\; \sqrt{B_H^2(\lambda,\varphi,h,t) \;+\; B_Z^2(\lambda,\varphi,h,t)} \qquad \text{(Total Intensity)} \tag{21b}$$

$$B_D(\lambda,\varphi,h,t) \;=\; \tan^{-1}\left\{\frac{B_Y(\lambda,\varphi,h,t)}{B_X(\lambda,\varphi,h,t)}\right\} \qquad \text{(Declination)} \tag{21c}$$

$$B_I(\lambda,\varphi,h,t) \;=\; \tan^{-1}\left\{\frac{B_Z(\lambda,\varphi,h,t)}{B_H(\lambda,\varphi,h,t)}\right\} \qquad \text{(Inclination)} \tag{21d}$$

$$B_G(\lambda,\varphi,h,t) \;=\; \left\{\begin{array}{lr} B_D \;-\; \varphi & \lambda \;\geq\; 0 \\ B_D \;+\; \varphi & \lambda \;<\; 0 \end{array}\right\} \qquad \text{(Grid Variation)} \tag{21e}$$

Inclination is often referred to as the *Dip* angle, while the magnetic declination is sometimes referred to as the *magnetic variation (magvar)*. Frequently, the magnetic field components in eqs. (21a) through (21e) are simply referred to in terms of their subscripts: X, Y, Z, H, F, D, I, and G. The Total Magnetic Intensity is sometimes referred to as *TI*, while the Grid Variation is sometimes referred to as *GV*. Some additional and quite useful relationships among these magnetic field components are:

$$H(\lambda,\varphi,h,t) \;=\; F(\lambda,\varphi,h,t)\,\cos[I(\lambda,\varphi,h,t)] \tag{22a}$$

$$X(\lambda,\varphi,h,t) \;=\; H(\lambda,\varphi,h,t)\,\cos[D(\lambda,\varphi,h,t)] \tag{22b}$$

$$Y(\lambda,\varphi,h,t) \;=\; H(\lambda,\varphi,h,t)\,\sin[D(\lambda,\varphi,h,t)] \tag{22c}$$

$$Z(\lambda,\varphi,h,t) \;=\; F(\lambda,\varphi,h,t)\,\sin[I(\lambda,\varphi,h,t)] \tag{22d}$$

2.0 The GEOMAG or MAGVAR Algorithm

The Main-Field Gauss coefficients at the base epoch, T_{Epoch}, are stored in array C of the GEOMAG algorithm (sometimes referred to as the MAGVAR algorithm), which is listed in the appendix, such that the lower half of array C is occupied by those Gauss coefficients $g_{nm}(T_{Epoch})$ corresponding to the *cosine* terms in the potential function of eq. (1), while the upper half of array C is occupied by those Gauss coefficients $h_{nm}(T_{Epoch})$ corresponding to the *sine* terms in eq. (1). Table 1 illustrates the details of this storage scheme, which is equivalent to the following mathematical assignments:

$$C_{nm} = \left\{ \begin{array}{ll} g_{nm} & m \geq n \\ h_{m,n+1} & m > n \end{array} \right\} \tag{23}$$

which implies that:

$$g_{nm} = C_{nm} \qquad m \leq n \tag{24a}$$

$$h_{nm} = C_{m-1,n} \qquad m \leq n, \quad m \neq 0 \tag{24b}$$

The Secular-Variation Gauss coefficients which describe the Main field's slow annual change are stored in array CD (which stands for $\dot{C}$ [pronounced C dot]) such that the lower half of array CD is occupied by the Gauss coefficients $\dot{g}_{nm}$, which correspond to the *cosine* terms in eq. (3), while the upper half of the array is occupied by the Gauss coefficients $\dot{h}_{nm}$, corresponding to the *sine* terms in eq. (3). Table 2 illustrates the details of this storage scheme for array CD. It takes essentially the same form as table 1 for array C and corresponds to the following mathematical assignments:

$$\dot{C}_{nm} = \left\{ \begin{array}{ll} \dot{g}_{nm} & m \leq n \\ \dot{h}_{m,n+1} & m > n \end{array} \right\} \tag{25}$$

which implies that:

$$\dot{g}_{nm} = \dot{C}_{nm} \qquad m \leq n \tag{26a}$$

$$\dot{h}_{nm} = \dot{C}_{m-1,n} \qquad m \leq n, \quad m \neq 0 \tag{26b}$$

Table 3 lists the numerical values of the Gauss coefficients at the base epoch and their corresponding predictive annual rates of change for the WMM-2000 geomagnetic model. These numerical values are inserted into arrays C and CD through data statements in the GEOMAG algorithm in the Appendix. Replacing the Gauss coefficients in these data statements and the date of their base epoch are the only changes that need to be made to update the algorithm from an older model to the new model. In all other respects the GEOMAG routine remains unaltered.

Other versions of the GEOMAG (MAGVAR) routine exist for which the coefficients can be read in from an external file. Then, only the external coefficient data file needs to be updated, while the algorithm remains unchanged.

TABLE 1. ARRANGEMENT OF MAIN FIELD COEFFICIENTS IN ARRAY $\mathbf{C}_{nm}$

n\m	0	1	2	3	4	5	6	7	8	9	10	11	12
0	g_{00}	h_{11}	h_{21}	h_{31}	h_{41}	h_{51}	h_{61}	h_{71}	h_{81}	h_{91}	$h_{10,1}$	$h_{11,1}$	$h_{12,1}$
1	g_{10}	g_{11}	h_{22}	h_{32}	h_{42}	h_{52}	h_{62}	h_{72}	h_{82}	h_{92}	$h_{10,2}$	$h_{11,2}$	$h_{12,2}$
2	g_{20}	g_{21}	g_{22}	h_{33}	h_{43}	h_{53}	h_{63}	h_{73}	h_{83}	h_{93}	$h_{10,3}$	$h_{11,3}$	$h_{12,3}$
3	g_{30}	g_{31}	g_{32}	g_{33}	h_{44}	h_{54}	h_{64}	h_{74}	h_{84}	h_{94}	$h_{10,4}$	$h_{11,4}$	$h_{12,4}$
4	g_{40}	g_{41}	g_{42}	g_{43}	g_{44}	h_{55}	h_{65}	h_{75}	h_{85}	h_{95}	$h_{10,5}$	$h_{11,5}$	$h_{12,5}$
5	g_{50}	g_{51}	g_{52}	g_{53}	g_{54}	g_{55}	h_{66}	h_{76}	h_{86}	h_{96}	$h_{10,6}$	$h_{11,6}$	$h_{12,6}$
6	g_{60}	g_{61}	g_{62}	g_{63}	g_{64}	g_{65}	g_{66}	h_{77}	h_{87}	h_{97}	$h_{10,7}$	$h_{11,7}$	$h_{12,7}$
7	g_{70}	g_{71}	g_{72}	g_{73}	g_{74}	g_{75}	g_{76}	g_{77}	h_{88}	h_{98}	$h_{10,8}$	$h_{11,8}$	$h_{12,8}$
8	g_{80}	g_{81}	g_{82}	g_{83}	g_{84}	g_{85}	g_{86}	g_{87}	g_{88}	h_{99}	$h_{10,9}$	$h_{11,9}$	$h_{12,9}$
9	g_{90}	g_{91}	g_{92}	g_{93}	g_{94}	g_{95}	g_{96}	g_{97}	g_{98}	g_{99}	$h_{10,10}$	$h_{11,10}$	$h_{12,10}$
10	$g_{10,0}$	$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{10,4}$	$g_{10,5}$	$g_{10,6}$	$g_{10,7}$	$g_{10,8}$	$g_{10,9}$	$g_{10,10}$	$h_{11,11}$	$h_{12,11}$
11	$g_{11,0}$	$g_{11,1}$	$g_{11,2}$	$g_{11,3}$	$g_{11,4}$	$g_{11,5}$	$g_{11,6}$	$g_{11,7}$	$g_{11,8}$	$g_{11,9}$	$g_{11,10}$	$g_{11,11}$	$h_{12,12}$
12	$g_{12,0}$	$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{12,4}$	$g_{12,5}$	$g_{12,6}$	$g_{12,7}$	$g_{12,8}$	$g_{12,9}$	$g_{12,10}$	$g_{12,11}$	$g_{12,12}$

TABLE 2. ARRANGEMENT OF SECULAR VARIATION COEFFICIENTS IN ARRAY $\dot{C}_{nm}$

n\m	0	1	2	3	4	5	6	7	8	9	10	11	12
0	$\dot{g}_{00}$	$\dot{h}_{11}$	$\dot{h}_{21}$	$\dot{h}_{31}$	$\dot{h}_{41}$	$\dot{h}_{51}$	$\dot{h}_{61}$	$\dot{h}_{71}$	$\dot{h}_{81}$	$\dot{h}_{91}$	$\dot{h}_{10,1}$	$\dot{h}_{11,1}$	$\dot{h}_{12,1}$
1	$\dot{g}_{10}$	$\dot{g}_{11}$	$\dot{h}_{22}$	$\dot{h}_{32}$	$\dot{h}_{42}$	$\dot{h}_{52}$	$\dot{h}_{62}$	$\dot{h}_{72}$	$\dot{h}_{82}$	$\dot{h}_{92}$	$\dot{h}_{10,2}$	$\dot{h}_{11,2}$	$\dot{h}_{12,2}$
2	$\dot{g}_{20}$	$\dot{g}_{21}$	$\dot{g}_{22}$	$\dot{h}_{33}$	$\dot{h}_{43}$	$\dot{h}_{53}$	$\dot{h}_{63}$	$\dot{h}_{73}$	$\dot{h}_{83}$	$\dot{h}_{93}$	$\dot{h}_{10,3}$	$\dot{h}_{11,3}$	$\dot{h}_{12,3}$
3	$\dot{g}_{30}$	$\dot{g}_{31}$	$\dot{g}_{32}$	$\dot{g}_{33}$	$\dot{h}_{44}$	$\dot{h}_{54}$	$\dot{h}_{64}$	$\dot{h}_{74}$	$\dot{h}_{84}$	$\dot{h}_{94}$	$\dot{h}_{10,4}$	$\dot{h}_{11,4}$	$\dot{h}_{12,4}$
4	$\dot{g}_{40}$	$\dot{g}_{41}$	$\dot{g}_{42}$	$\dot{g}_{43}$	$\dot{g}_{44}$	$\dot{h}_{55}$	$\dot{h}_{65}$	$\dot{h}_{75}$	$\dot{h}_{85}$	$\dot{h}_{95}$	$\dot{h}_{10,5}$	$\dot{h}_{11,5}$	$\dot{h}_{12,5}$
5	$\dot{g}_{50}$	$\dot{g}_{51}$	$\dot{g}_{52}$	$\dot{g}_{53}$	$\dot{g}_{54}$	$\dot{g}_{55}$	$\dot{h}_{66}$	$\dot{h}_{76}$	$\dot{h}_{86}$	$\dot{h}_{96}$	$\dot{h}_{10,6}$	$\dot{h}_{11,6}$	$\dot{h}_{12,6}$
6	$\dot{g}_{60}$	$\dot{g}_{61}$	$\dot{g}_{62}$	$\dot{g}_{63}$	$\dot{g}_{64}$	$\dot{g}_{65}$	$\dot{g}_{66}$	$\dot{h}_{77}$	$\dot{h}_{87}$	$\dot{h}_{97}$	$\dot{h}_{10,7}$	$\dot{h}_{11,7}$	$\dot{h}_{12,7}$
7	$\dot{g}_{70}$	$\dot{g}_{71}$	$\dot{g}_{72}$	$\dot{g}_{73}$	$\dot{g}_{74}$	$\dot{g}_{75}$	$\dot{g}_{76}$	$\dot{g}_{77}$	$\dot{h}_{88}$	$\dot{h}_{98}$	$\dot{h}_{10,8}$	$\dot{h}_{11,8}$	$\dot{h}_{12,8}$
8	$\dot{g}_{80}$	$\dot{g}_{81}$	$\dot{g}_{82}$	$\dot{g}_{83}$	$\dot{g}_{84}$	$\dot{g}_{85}$	$\dot{g}_{86}$	$\dot{g}_{87}$	$\dot{g}_{88}$	$\dot{h}_{99}$	$\dot{h}_{10,9}$	$\dot{h}_{11,9}$	$\dot{h}_{12,9}$
9	$\dot{g}_{90}$	$\dot{g}_{91}$	$\dot{g}_{92}$	$\dot{g}_{93}$	$\dot{g}_{94}$	$\dot{g}_{95}$	$\dot{g}_{96}$	$\dot{g}_{97}$	$\dot{g}_{98}$	$\dot{g}_{99}$	$\dot{h}_{10,10}$	$\dot{h}_{11,10}$	$\dot{h}_{12,10}$
10	$\dot{g}_{10,0}$	$\dot{g}_{10,1}$	$\dot{g}_{10,2}$	$\dot{g}_{10,3}$	$\dot{g}_{10,4}$	$\dot{g}_{10,5}$	$\dot{g}_{10,6}$	$\dot{g}_{10,7}$	$\dot{g}_{10,8}$	$\dot{g}_{10,9}$	$\dot{g}_{10,10}$	$\dot{h}_{11,11}$	$\dot{h}_{12,11}$
11	$\dot{g}_{11,0}$	$\dot{g}_{11,1}$	$\dot{g}_{11,2}$	$\dot{g}_{11,3}$	$\dot{g}_{11,4}$	$\dot{g}_{11,5}$	$\dot{g}_{11,6}$	$\dot{g}_{11,7}$	$\dot{g}_{11,8}$	$\dot{g}_{11,9}$	$\dot{g}_{11,10}$	$\dot{g}_{11,11}$	$\dot{h}_{12,12}$
12	$\dot{g}_{12,0}$	$\dot{g}_{12,1}$	$\dot{g}_{12,2}$	$\dot{g}_{12,3}$	$\dot{g}_{12,4}$	$\dot{g}_{12,5}$	$\dot{g}_{12,6}$	$\dot{g}_{12,7}$	$\dot{g}_{12,8}$	$\dot{g}_{12,9}$	$\dot{g}_{12,10}$	$\dot{g}_{12,11}$	$\dot{g}_{12,12}$

Table 3. WMM-2000 Model Coefficients

n	m	g_n^m	h_n^m	$\dot{g}_n^m$	$\dot{h}_n^m$
1	0	-29,616	0	14.7	0
1	1	-1,722.7	5,194.5	11.1	-20.4
2	0	-2,266.7	0	-13.6	0
2	1	3,070.2	-2,484.8	-0.7	-21.5
2	2	1,677.6	-467.9	-1.8	-9.6
3	0	1,322.4	0	0.3	0
3	1	-2,291.5	-224.7	-4.3	6.4
3	2	1,255.9	293	0.9	-1.3
3	3	724.8	-486.5	-8.4	-13.3
4	0	932.1	0	-1.6	0
4	1	786.3	273.3	0.9	2.3
4	2	250.6	-227.9	-7.6	0.7
4	3	-401.5	120.9	2.2	3.7
4	4	106.2	-302.7	-3.2	-0.5
5	0	-211.9	0	-0.9	0
5	1	351.6	42	-0.2	0
5	2	220.8	173.8	-2.5	2.1
5	3	-134.5	-135	-2.7	2.3
5	4	-168.8	-38.6	-0.9	3.1
5	5	-13.3	105.2	1.7	0
6	0	73.8	0	1.2	0
6	1	68.2	-17.4	0.2	-0.3
6	2	74.1	61.2	1.7	-1.7
6	3	-163.5	63.2	1.6	-0.9
6	4	-3.8	-62.9	-0.1	-1
6	5	17.1	0.2	-0.3	-0.1
6	6	-85.1	43	0.8	1.9
7	0	77.4	0	-0.4	0
7	1	-73.9	-62.3	-0.8	1.4
7	2	2.2	-24.5	-0.2	0.2

Table 3. WMM-2000 Model Coefficients (Con.)

n	m	g_n^m	h_n^m	$\dot{g}_n^m$	$\dot{h}_n^m$
7	3	35.7	8.9	1.1	0.7
7	4	7.3	23.4	0.4	0.4
7	5	5.2	15	0	-0.3
7	6	8.4	-27.6	-0.2	-0.8
7	7	-1.5	-7.8	-0.2	-0.1
8	0	23.3	0	-0.3	0
8	1	7.3	12.4	0.6	-0.5
8	2	-8.5	-20.8	-0.8	0.1
8	3	-6.6	8.4	0.3	-0.2
8	4	-16.9	-21.2	-0.2	0
8	5	8.6	15.5	0.5	0.1
8	6	4.9	9.1	0	-0.1
8	7	-7.8	-15.5	-0.6	0.3
8	8	-7.6	-5.4	0.1	0.2
9	0	5.7	0	0	0
9	1	8.5	-20.4	0	0
9	2	2	13.9	0	0
9	3	-9.8	12	0	0
9	4	7.6	-6.2	0	0
9	5	-7	-8.6	0	0
9	6	-2	9.4	0	0
9	7	9.2	5	0	0
9	8	-2.2	-8.4	0	0
9	9	-6.6	3.2	0	0
10	0	-2.2	0	0	0
10	1	-5.7	0.9	0	0
10	2	1.6	-0.7	0	0
10	3	-3.7	3.9	0	0
10	4	-0.6	4.8	0	0
10	5	4.1	-5.3	0	0

Table 3. WMM-2000 Model Coefficients (Con.)

n	m	g_n^m	h_n^m	$\dot{g}_n^m$	$\dot{h}_n^m$
10	6	2.2	-1	0	0
10	7	2.2	-2.4	0	0
10	8	4.6	1.3	0	0
10	9	2.3	-2.3	0	0
10	10	0.1	-6.4	0	0
11	0	3.3	0	0	0
11	1	-1.1	-1.5	0	0
11	2	-2.4	0.7	0	0
11	3	2.6	-1.1	0	0
11	4	-1.3	-2.3	0	0
11	5	-1.7	1.3	0	0
11	6	-0.6	-0.6	0	0
11	7	0.4	-2.8	0	0
11	8	0.7	-1.6	0	0
11	9	-0.3	-0.1	0	0
11	10	2.3	-1.9	0	0
11	11	4.2	1.4	0	0
12	0	-1.5	0	0	0
12	1	-0.2	-1	0	0
12	2	-0.3	0.7	0	0
12	3	0.5	2.2	0	0
12	4	0.2	-2.5	0	0
12	5	0.9	-0.2	0	0
12	6	-‘1.4	0	0	0
12	7	0.6	-0.2	0	0
12	8	-0.6	0	0	0
12	9	-1	0.2	0	0
12	10	-0.3	-0.9	0	0
12	11	0.3	-0.2	0	0
12	12	0.4	1	0	0

Important parameters in the GEOMAG (MAGVAR) routine and their mathematical correspondences are:

A	~	a = 6378.137 km
B	~	b = 6356.7523142 km
RE	~	R_E = 6371.2 km
TIME	~	t
EPOCH	~	T_{Epoch}
DT	~	$\mathrm{t}\text{-}\mathrm{T}_{\mathrm{Epoch}}$
ALT	~	h
SNORM(N,M)	~	$S^{n\,m}$
K(N,M)	~	$\kappa^{n\,m}$
GLAT	~	λ
GLON	~	φ
SP(M)	~	$sin(m\varphi)$
CP(M)	~	$cos(m\varphi)$
ST	~	$sin(\theta)$
CT	~	$cos(\theta)$
CA	~	$cos(\alpha)$
SA	~	$sin(\alpha)$
BR	~	B_r
BT	~	B_θ
BP	~	B_φ
BX	~	B_X
BY	~	B_Y
BZ	~	B_Z
DEC	~	B_D
DIP	~	B_I
TI	~	B_F
MAXDEG	~	N
MAXORD	~	$M = N$
P(N,M)	~	$P^{n\,m}(\theta)$
DP(N,M)	~	$\frac{dP^{n\,m}(\theta)}{d\theta}$
TC	~	$C\ +\ (t - T_{Epoch})\dot{C}$
CD	~	$\dot{C}$
Q2	~	Q^2

Note that R_E is not intended to be the mean radius of the WGS-84 ellipsoid. By international convention established by the International Association of Geomagnetism and Aeronomy circa 1968, it is the mean radius of a modified ellipsoid established by the International Astronomical Union (IAU) in 1966. This ellipsoid is referred to as the modified IAU-66 ellipsoid (Zmuda [1971]). Due to the fact that the World Magnetic Model is of low degree and order, the differences in computed magnetic field valuses is barely noticeable from one datum to another.

The GEOMAG (MAGVAR) algorithm is written in Fortran 77. A *C* version is also availale. The algorithm is organized into two modules, each with its own entry point. The first entry point is an *Initialization Module*. Its purpose is to compute all constants such as the recursion relation factors for the associated Legendre polynomials κ^{nm}, the Schmidt normalization factors S^{nm}, and any other parameters that do not depend on position or time. The entry point for this module is:

GEOMAG(MAXDEG) or MAGVAR(MAXDEG)

The parameter MAXDEG determines the maximum degree and order of the magnetic model to be used in the computations. Normally, MAXDEG=12, which is the maximum degree and order of the WMM series of geomagnetic models. Depending on the version of GEOMAG (MAGVAR) the Gauss coefficients may be read in from an external file or fixed within these routines as data statements. In the later case MAXDEG is always fixed to 12 in the callingor *driver* program (e.g., GEOMDR or MAGVDR). In the latter case, MAXDEG is set by the degree and order of the last coefficient read. Normally this will also be 12. However, in order to reduce computation time, MAXDEG may be set to a number less that 12 (e.g., 8 or 10) by just truncating the coefficient file being read. However, the accuracy of the computed magnetic parameters is correspondingly reduced. GEOMDR and MAGVDR are *driver* programs for their respective computational subroutines. They are supplied on the accompanying diskette as examples of how to use the computational subroutines in the event that the user wishes to incorporate them into his own software applications. These drivers will allow a grid of values to be generated for various magnetic field components at a specific time, for a specified chart area, at a specified grid interval. The GEOMDR routine was used to generate the 1^0 x 1^0 grids used to create the charts in section 3.

The second module is the *Processing Module*, which has the following entry point:

GEOMG1(ALT,GLAT,GLON,TIME,DEC,DIP,TI,GV)

or

MAGVAR(ALT,GLAT,GLON,TIME,DEC,DIP,TI,GV)

The purpose of this module is to compute the magnetic *Declination*, *Inclination*, *Total Intensity*, and the *Grid Variation* at each *geodetic* position and time supplied to it. The units of the parameters in the argument list of the GEOMG1 entry point are as follows:

ALT	~	kilometers	(e.g., 5.314)	(IN)
GLAT	~	degrees	(e.g., 33.716)	(IN)
GLON	~	degrees	(e.g., -163.315)	(IN)
TIME	~	years	(e.g., 1997.427)	(IN)
DEC	~	degrees	(e.g., -121.734)	(OUT)
DIP	~	degrees	(e.g., 48.387)	(OUT)
TI	~	nanoTeslas	(e.g., 35781.7)	(OUT)
GV	~	degrees	(e.g., 51.768)	(OUT)

The computed magnetic field parameters are referenced to the WGS-84 ellipsoid. The last parameter, GV, is the Grid Variation which is computed only for the polar regions (i.e., above $+55^0$ latitude or below -55^0 latitude). Outside of these regions, a default value of -999.0 is dummied in. The Grid Variation is referenced to *Grid North* of a polar stereographic projection. The model is considered to be a valid representation of the Earth's core magnetic field at geodetic altitudes ranging from the *ocean bottom* to +1000 *km* for all geodetic latitudes and longitudes.

The SV computation of a geomagnetic component at a fixed time $t = \tau$ is accomplished by making two calls to the entry point GEOMG1, one at time $t_1 = \tau - 0.5$ and one at $t_2 = \tau + 0.5$, where t is expressed in years. This yields the Declination, Inclination, and the Total Intensity at two different times spaced one year apart. Using these three magnetic components, any other magnetic component can be calculated at these same two times via eqs. (21a) through (21e) and eqs. (22a) through (22d). The SV is then determined by differencing the two MF values of a particular component. For example, the Horizontal component's SV is computed by inserting eq. (22a) into the following:

$$\dot{H}\,(\lambda,\varphi,h,\tau) = [H(\lambda,\varphi,h,t_2) - H(\lambda,\varphi,h,t_1)]\,/\,\Delta t \tag{27}$$

where

$$\Delta t = t_2 - t_1 = 1\,year \tag{28}$$

3.0 Geomagnetic Coordinates

Geomagnetic coordinates are computed from the dipole terms (g_{00}, g_{11}, and h_{11}) of WMM-2000. This coordinate system has Dipole Magnetic Poles located at:

North	South
Latitude = 79.5305^0	Latitude = -79.5305^0
Longitude = -71.6525^0	Longitude = 108.348^0

A graphic representation of the geomagnetic coordinate system is presented with the charts discussed in the of section 4.0.

4.0 Magnetic Charts

At the end of this report a series of contour maps is presented for all of the geomagnetic components and their annual rates of change, first in a Mercator projection of the world between

+/- 70^0 latitude, followed by a Polar Stereographic projection of the North Pole to + 55^0 latitude, and finally in a Polar Sereographic projection of the South Pole to -55^0 latitude. These charts are similar to the wall-size charts of these geomagnetic components produced by NIMA.

5.0 Magnetic Pole Positions

Through the International Geomagnetic Reference Field (IGRF) committee of the International Association of Geomagnetism and Aeronomy (IAGA), a series of magnetic field models have been developed covering the entire 20'th century. The US/UK models are always submitted to the IGRF committee as candidates for the IGRF models. The North and South magnetic pole positions can be computed from these models by noting where the Horizontal Magnetic Field Component is zero. The year-by-year locations of these poles have been computed using the IGRF series of models from 1900 through 1994, the revised WMM-95 model from 1995 through 1999, and the WMM-2000 model from 2000 through 2005. The results are presented in table 4. These results are also presented as North and South Polar Stereographic Charts. At Epoch 2000, the North Magnetic Pole is located at: 80.81 degrees North, 109.37 degrees South, while the South Magnetic Pole is located at: 64.67 degrees South, 138.36 degrees East.

TABLE 4. MAGNETIC POLE POSITIONS (1900 - 2005)

	NORTH		SOUTH			NORTH		SOUTH	
YEAR	LAT	LON	LAT	LON	YEAR	LAT	LON	LAT	LON
1900	70.49	-96.20	-71.73	148.33	1940	73.35	-99.92	-68.57	144.60
1901	70.52	-96.25	-71.67	148.38	1941	73.46	-99.97	-68.48	144.57
1902	70.56	-96.30	-71.62	248.42	1942	73.57	-100.02	-6840.	144.53
1903	70.59	-96.34	-71.57	148.46	1943	73.68	-100.06	-68.32	144.51
1904	70.62	-96.39	-71.51	148.51	1944	73.80	-100.14	-68.23	144.48
1905	70.65	-96.43	-71.46	148.55	1945	73.93	-100.24	-68.15	144.44
1906	70.68	-96.49	-71.40	148.57	1946	74.07	-100.36	-68.10	144.26
1907	70.72	-96.54	-71.34	148.59	1947	74.21	-100.48	-68.05	144.09
1908	70.75	-96.60	-71.27	148.61	1948	74.35	-100.60	-67.99	143.91
1909	70.78	-96.66	-71.21	148.63	1949	74.49	-100.73	-67.94	143.73
1910	70.80	-96.70	-71.15	148.65	1950	74.64	-100.86	-67.89	143.54
1911	70.85	-96.75	-71.08	148.63	1951	74.74	-100.96	-67.75	143.13
1912	70.90	-96.81	-71.01	148.61	1952	74.85	-101.07	-67.61	142.72
1913	70.94	-96.87	-70.94	148.59	1953	74.96	-101.18	-67.47	142.32
1914	70.99	-96.93	-70.87	148.58	1954	75.07	-101.30	-67.34	141.91
1915	71.03	-96.98	-70.80	148.56	1955	75.18	-101.42	-67.19	141.50
1916	71.09	-97.05	-70.73	148.49	1956	75.21	-101.34	-67.10	141.26
1917	71.16	-97.13	-70.65	148.42	1957	75.23	-101.26	-67.00	141.00
1918	71.22	-97.20	-70.57	148.35	1958	75.26	-101.19	-66.90	140.75
1919	71.29	-97.27	-70.50	148.28	1959	75.28	-101.11	-66.80	140.49
1920	71.35	-97.36	-70.42	148.21	1960	75.30	-101.03	-66.70	140.23
1921	71.44	-97.48	-70.33	148.09	1961	75.36	-101.09	-66.63	140.09
1922	71.53	-97.60	-70.25	147.97	1962	75.43	-101.15	-66.55	139.95
1923	71.62	-97.72	-70.16	147.85	1963	75.49	-101.21	-66.48	139.81
1924	71.71	-97.85	-70.08	147.74	1964	75.56	-101.28	-66.40	139.67
1925	71.79	-97.97	-69.99	147.62	1965	75.63	-101.34	-66.33	139.53
1926	71.89	-98.11	-69.90	147.45	1966	75.67	-101.27	-66.27	139.49
1927	71.99	-98.25	-69.80	147.28	1967	75.72	-101.20	-66.21	139.45
1928	72.09	-98.40	-69.71	147.11	1968	75.77	-101.14	-66.14	139.41
1929	72.18	-98.54	-69.62	146.94	1969	75.82	-101.07	-66.08	139.37
1930	72.28	-98.69	-69.52	146.77	1970	75.88	-100.97	-66.02	139.40
1931	72.39	-98.82	-69.43	146.57	1971	75.94	-100.89	-65.96	139.42
1932	72.49	-98.95	-69.34	146.36	1972	75.99	-100.80	-65.90	139.43
1933	72.59	-99.09	-69.24	146.16	1973	76.05	-100.71	-65.83	139.45
1934	72.70	-99.22	-69.15	145.95	1974	76.11	-100.62	-65.77	139.47
1935	72.81	-99.36	-69.05	145.75	1975	76.15	-100.64	-65.74	139.52
1936	72.91	-99.47	-68.96	145.51	1976	76.28	-100.84	-65.68	139.51
1937	73.02	-99.57	-68.86	145.28	1977	76.41	-101.05	-65.63	139.50
1938	73.13	-99.18	-68.76	145.05	1978	76.53	-101.25	-65.57	139.49
1939	73.24	-99.80	-68.66	144.82	1979	76.66	-101.46	-65.52	139.48

TABLE 4. MAGNETIC POLE POSITIONS (1900 - 2005) (Con.)

	NORTH		SOUTH			NORTH		SOUTH	
YEAR	LAT	LON	LAT	LON	YEAR	LAT	LON	LAT	LON
1980	76.91	-101.68	-65.42	139.35	1993	78.48	-104.33	-64.75	138.72
1981	77.00	-101.88	-65.37	139.34	1994	78.61	-104.55	-64.70	138.66
1982	77.10	-102.09	-65.31	139.34	1995	79.16	-105.37	-64.74	138.58
1983	77.19	-102.30	-65.25	139.33	1996	79.49	-106.00	-64.73	138.52
1984	77.29	-102.51	-65.20	139.32	1997	79.81	-106.81	-64.71	138.47
1985	77.40	-102.61	-65.13	139.18	1998	80.13	-107.60	-64.70	138.41
1986	77.53	-102.83	-65.07	139.12	1999	80.59	-108.89	-64.68	138.36
1987	77.66	-103.04	-65.02	139.06	2000	80.81	-109.37	-64.67	138.31
1988	77.79	-103.27	-64.97	139.01	2001	81.17	-110.35	-64.65	138.25
1989	77.92	-103.50	-64.92	138.95	2002	81.65	-111.89	-64.63	138.19
1990	78.10	-103.69	-64.91	138.90	2003	81.90	-112.61	-64.61	138.13
1991	78.22	-103.90	-64.86	138.84	2004	82.26	-113.89	-64.59	138.08
1992	78.35	-104.11	-64.80	138.78	2005	82.74	-115.89	-64.58	138.02

ACKNOWLEDGEMENTS

Overall coordination and production of the joint US/UK World Magnetic Model for the 2000 Epoch (WMM-2000) were the responsibilities of Susan Macmillan of the British Geological Survey (BGS) and John M. Quinn of the U. S. Geological Survey.

The OERSTED satellite's Overhauser Pre-Cal scalar-magnetic-data, which served as the primary source for Main Field modeling, were kindly provided by the Danish Meteorological Institute, Copenhagen, Denmark.

This model could not have been produced without the tireless efforts of those many unnamed individuals from many countries, private institutions, and government agencies around the world who collect and process magnetic field data on a day-to-day basis at the geomagnetic observatories or at reomote Repeat Survey sites. These data served as the primary sources for the Secular Variation Model.

The Preliminary Disturbance-storm-time (Dst) indices were provided by M. Sugiura of the University of Kyoto, Japan, while the Preliminary Kp indices were computed by the Adolf-Schmidt-Observatorium fur Geomagnetismus, GeoForshumgs Zentrum, Potsdam, Germany. Both indices are based on geomagnetic observatory data. They allow the selection of magnetically quiet data for modeling. Both indices were distributed through the National Oceanic and Atmospheric Administration's National Geophysical Data Center, World Data Center A (NOAA/NGDC/WDCA) in Boulder, Colorado.

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Appendix: The GEOMAG Algorithm

```
C*********************************************************************
C
C
C    SUBROUTINE GEOMAG (GEOMAGNETIC FIELD COMPUTATION)
C
C
C*********************************************************************
C
C
C    WMM-2000 is a National Imagery and Mapping Agency (NIMA) standard
C    product. It is covered under NIMA Military Specification:
C    MIL-W-89500 (1993).
C
C    For information on the use and applicability of this product contact:
C
C              DIRECTOR
C              NATIONAL IMAGERY AND MAPPING AGENCY/HEADQUARTERS
C              ATTN: CODE P33
C              12310 SUNRISE VALLEY DRIVE
C              RESTON, VA 20191-3449
C              PHONE:  (703) 264-3002
C
C
C*********************************************************************
C
C    GEOMAG PROGRAMMED BY:
C
C              JOHN M. QUINN  7/19/90
C              FLEET PRODUCTS DIVISION, CODE N342
C              NAVAL OCEANOGRAPHIC OFFICE (NAVOCEANO)
C              STENNIS SPACE CENTER (SSC), MS 39522-5001
C              USA
C              PHONE:   COM:  (601) 688-5828
C                         AV:        485-5828
C                        FAX:  (601) 688-5521
C
C    NOW AT:
C
C              GEOMAGNETICS GROUP
C              U. S. GEOLOGICAL SURVEY   MS 966
C              FEDERAL CENTER
C              DENVER, CO   80225-0046
C              USA
C              PHONE:   COM: (303) 273-8475
C                       FAX: (303) 273-8600
```

```
C                           EMAIL:   quinn@ghtmail.cr.usgs.gov
C
C
C*********************************************************************
C
C
C    PURPOSE:  THIS ROUTINE COMPUTES THE DECLINATION (DEC),
C              INCLINATION (DIP), TOTAL INTENSITY (TI) AND
C              GRID VARIATION (GV - POLAR REGIONS ONLY, REFERENCED
C              TO GRID NORTH OF POLAR STEREOGRAPHIC PROJECTION) OF
C              THE EARTH'S MAGNETIC FIELD IN GEODETIC COORDINATES
C              FROM THE COEFFICIENTS OF THE CURRENT OFFICIAL
C              DEPARTMENT OF DEFENSE (DOD) SPHERICAL HARMONIC WORLD
C              MAGNETIC MODEL (WMM-2000).  THE WMM SERIES OF MODELS IS
C              UPDATED EVERY 5 YEARS ON JANUARY 1'ST OF THOSE YEARS
C              WHICH ARE DIVISIBLE BY 5 (I.E. 1980, 1985, 1990 ETC.)
C              BY THE NAVAL OCEANOGRAPHIC OFFICE IN COOPERATION
C              WITH THE BRITISH GEOLOGICAL SURVEY (BGS).  THE MODEL
C              IS BASED ON GEOMAGNETIC SURVEY MEASUREMENTS FROM
C              AIRCRAFT, SATELLITE AND GEOMAGNETIC OBSERVATORIES.
C
C
C*********************************************************************
C
C
C    MODEL:  THE WMM SERIES GEOMAGNETIC MODELS ARE COMPOSED
C            OF TWO PARTS:  THE MAIN FIELD MODEL, WHICH IS
C            VALID AT THE BASE EPOCH OF THE CURRENT MODEL AND
C            A SECULAR VARIATION MODEL, WHICH ACCOUNTS FOR SLOW
C            TEMPORAL VARIATIONS IN THE MAIN GEOMAGNETIC FIELD
C            FROM THE BASE EPOCH TO A MAXIMUM OF 5 YEARS BEYOND
C            THE BASE EPOCH.  FOR EXAMPLE, THE BASE EPOCH OF
C            THE WMM-2000 MODEL IS 2000.0.  THIS MODEL IS THEREFORE
C            CONSIDERED VALID BETWEEN 2000.0 AND 2005.0. THE
C            COMPUTED MAGNETIC PARAMETERS ARE REFERENCED TO THE
C            WGS-84 ELLIPSOID.
C
C
C*********************************************************************
C
C
C    ACCURACY:  IN OCEAN AREAS AT THE EARTH'S SURFACE OVER THE
C               ENTIRE 5 YEAR LIFE OF A DEGREE AND ORDER 12
C               SPHERICAL HARMONIC MODEL SUCH AS WMM-2000, THE ESTIMATED
C               RMS ERRORS FOR THE VARIOUS MAGENTIC COMPONENTS ARE:
```

```
C
C           DEC  -    0.5 Degrees
C           DIP   -    0.5 Degrees
C           TI     - 280.0 nanoTeslas (nT)
C           GV   -    0.5 Degrees
C
C           OTHER MAGNETIC COMPONENTS THAT CAN BE DERIVED FROM
C           THESE FOUR BY SIMPLE TRIGONOMETRIC RELATIONS WILL
C           HAVE THE FOLLOWING APPROXIMATE ERRORS OVER OCEAN AREAS:
C
C           X    - 140 nT (North)
C           Y    - 140 nT (East)
C           Z    - 200 nT (Vertical)  Positive is down
C           H    - 200 nT (Horizontal)
C
C           OVER LAND THE RMS ERRORS ARE EXPECTED TO BE SOMEWHAT
C           HIGHER, ALTHOUGH THE RMS ERRORS FOR DEC, DIP AND GV
C           ARE STILL ESTIMATED TO BE LESS THAN 0.5 DEGREE, FOR
C           THE ENTIRE 5-YEAR LIFE OF THE MODEL AT THE EARTH's
C           SURFACE.  THE OTHER COMPONENT ERRORS OVER LAND ARE
C           MORE DIFFICULT TO ESTIMATE AND SO ARE NOT GIVEN.
C
C           THE ACCURACY AT ANY GIVEN TIME OF ALL FOUR
C           GEOMAGNETIC PARAMETERS DEPENDS ON THE GEOMAGNETIC
C           LATITUDE.  THE ERRORS ARE LEAST AT THE EQUATOR AND
C           GREATEST AT THE MAGNETIC POLES.
C
C           IT IS VERY IMPORTANT TO NOTE THAT A DEGREE AND
C           ORDER 12 MODEL, SUCH AS WMM-2000 DESCRIBES ONLY
C           THE LONG WAVELENGTH SPATIAL MAGNETIC FLUCTUATIONS
C           DUE TO EARTH'S CORE.  NOT INCLUDED IN THEWMM SERIES
C           MODELS ARE INTERMEDIATE AND SHORT WAVELENGTH
C           SPATIAL FLUCTUATIONS OF THE GEOMAGNETIC FIELD
C           WHICH ORIGINATE IN THE EARTH'S MANTLE AND CRUST.
C           CONSEQUENTLY, ISOLATED ANGULAR ERRORS AT VARIOUS
C           POSITIONS ON THE SURFACE (PRIMARILY OVER LAND, IN
C           CONTINENTAL MARGINS AND OVER OCEANIC SEAMOUNTS,
C           RIDGES AND TRENCHES) OF SEVERAL DEGREES MAY BE
C           EXPECTED. ALSO NOT INCLUDED IN THE MODEL ARE
C           NONSECULAR TEMPORAL FLUCTUATIONS OF THE GEOMAGNETIC
C           FIELD OF MAGNETOSPHERIC AND IONOSPHERIC ORIGIN.
C           DURING MAGNETIC STORMS, TEMPORAL FLUCTUATIONS CAN
C           CAUSE SUBSTANTIAL DEVIATIONS OF THE GEOMAGNETIC
C           FIELD FROM MODEL VALUES.  IN ARCTIC AND ANTARCTIC
C           REGIONS, AS WELL AS IN EQUATORIAL REGIONS, DEVIATIONS
```

```
C            FROM MODEL VALUES ARE BOTH FREQUENT AND PERSISTENT.
C
C            IF THE REQUIRED DECLINATION ACCURACY IS MORE
C            STRINGENT THAN THE WMM SERIES OF MODELS PROVIDE, THEN
C            THE USER IS ADVISED TO REQUEST SPECIAL (REGIONAL OR
C            LOCAL) SURVEYS BE PERFORMED AND MODELS PREPARED BY
C            THE USGS, WHICH OPERATES THE US GEOMAGNETIC
C            OBSERVATORIES.  REQUESTS OF THIS NATURE SHOULD
C            BE MADE THROUGH NIMA AT THE ADDRESS ABOVE.
C
C
C*********************************************************************
C
C
C    USAGE:  THIS ROUTINE IS BROKEN UP INTO TWO PARTS:
C
C          A) AN INITIALIZATION MODULE, WHICH IS CALLED ONLY
C            ONCE AT THE BEGINNING OF THE MAIN (CALLING)
C            PROGRAM
C          B) A PROCESSING MODULE, WHICH COMPUTES THE MAGNETIC
C            FIELD PARAMETERS FOR EACH SPECIFIED GEODETIC
C            POSITION (ALTITUDE, LATITUDE, LONGITUDE) AND TIME
C
C          INITIALIZATION IS MADE VIA A SINGLE CALL TO THE MAIN
C          ENTRY POINT (GEOMAG), WHILE SUBSEQUENT PROCESSING
C          CALLS ARE MADE THROUGH THE SECOND ENTRY POINT (GEOMG1).
C          ONE CALL TO THE PROCESSING MODULE IS REQUIRED FOR EACH
C          POSITION AND TIME.
C
C          THE VARIABLE MAXDEG IN THE INITIALIZATION CALL IS THE
C          MAXIMUM DEGREE TO WHICH THE SPHERICAL HARMONIC MODEL
C          IS TO BE COMPUTED.  IT MUST BE SPECIFIED BY THE USER
C          IN THE CALLING ROUTINE.  NORMALLY IT IS 12 BUT IT MAY
C          BE SET LESS THAN 12 TO INCREASE COMPUTATIONAL SPEED AT
C          THE EXPENSE OF REDUCED ACCURACY.
C
C          THE PC VERSION OF THIS SUBROUTINE MUST BE COMPILED
C          WITH A FORTRAN 77 COMPATIBLE COMPILER SUCH AS THE
C          MICROSOFT OPTIMIZING FORTRAN COMPILER VERSION 4.1
C          OR LATER.
C
C
C*********************************************************************
C
C
```

```
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C
C
C*********************************************************************
C
C
C    PARAMETER DESCRIPTIONS:
C
C      A      - SEMIMAJOR AXIS OF WGS-84 ELLIPSOID (KM)
C      B      - SEMIMINOR AXIS OF WGS-84 ELLIPSOID (KM)
C      RE     - MEAN RADIUS OF IAU-66 ELLIPSOID (KM)
C      SNORM  - SCHMIDT NORMALIZATION FACTORS
C      C      - GAUSS COEFFICIENTS OF MAIN GEOMAGNETIC MODEL (NT)
C      CD     - GAUSS COEFFICIENTS OF SECULAR GEOMAGNETIC MODEL (NT/YR)
C      TC     - TIME ADJUSTED GEOMAGNETIC GAUSS COEFFICIENTS (NT)
C      OTIME  - TIME ON PREVIOUS CALL TO GEOMAG (YRS)
```

```
C      OALT    - GEODETIC ALTITUDE ON PREVIOUS CALL TO GEOMAG (YRS)
C      OLAT    - GEODETIC LATITUDE ON PREVIOUS CALL TO GEOMAG (DEG.)
C      OLON    - GEODETIC LONGITUDE ON PREVIOUS CALL TO GEOMAG (DEG.)
C      TIME    - COMPUTATION TIME (YRS)                    (INPUT)
C                     (EG. 1 JULY 2000 = 2000.500)
C      ALT     - GEODETIC ALTITUDE (KM)                    (INPUT)
C      GLAT    - GEODETIC LATITUDE (DEG.)                  (INPUT)
C      GLON    - GEODETIC LONGITUDE (DEG.)                 (INPUT)
C      EPOCH - BASE TIME OF GEOMAGNETIC MODEL (YRS)
C      DTR     - DEGREE TO RADIAN CONVERSION
C      SP(M)   - SINE OF (M*SPHERICAL COORD. LONGITUDE)
C      CP(M)   - COSINE OF (M*SPHERICAL COORD. LONGITUDE)
C      ST      - SINE OF (SPHERICAL COORD. LATITUDE)
C      CT      - COSINE OF (SPHERICAL COORD. LATITUDE)
C      R       - SPHERICAL COORDINATE RADIAL POSITION (KM)
C      CA      - COSINE OF SPHERICAL TO GEODETIC VECTOR ROTATION ANGLE
C      SA      - SINE OF SPHERICAL TO GEODETIC VECTOR ROTATION ANGLE
C      BR      - RADIAL COMPONENT OF GEOMAGNETIC FIELD (NT)
C      BT      - THETA COMPONENT OF GEOMAGNETIC FIELD (NT)
C      BP      - PHI COMPONENT OF GEOMAGNETIC FIELD (NT)
C      P(N,M)  - ASSOCIATED LEGENDRE POLYNOMIALS (UNNORMALIZED)
C      PP(N)   - ASSOCIATED LEGENDRE POLYNOMIALS FOR M=1 (UNNORMALIZED)
C      DP(N,M)- THETA DERIVATIVE OF P(N,M) (UNNORMALIZED)
C      BX      - NORTH GEOMAGNETIC COMPONENT (NT)
C      BY      - EAST GEOMAGNETIC COMPONENT (NT)
C      BZ      - VERTICALLY DOWN GEOMAGNETIC COMPONENT (NT)
C      BH      - HORIZONTAL GEOMAGNETIC COMPONENT (NT)
C      DEC     - GEOMAGNETIC DECLINATION (DEG.)            (OUTPUT)
C                     EAST=POSITIVE ANGLES
C                     WEST=NEGATIVE ANGLES
C      DIP     - GEOMAGNETIC INCLINATION (DEG.)            (OUTPUT)
C                     DOWN=POSITIVE ANGLES
C                       UP=NEGATIVE ANGLES
C      TI      - GEOMAGNETIC TOTAL INTENSITY (NT)          (OUTPUT)
C      GV      - GEOMAGNETIC GRID VARIATION (DEG.)         (OUTPUT)
C                     REFERENCED TO GRID NORTH
C                     GRID NORTH REFERENCED TO 0 MERIDIAN
C                     OF A POLAR STEREOGRAPHIC PROJECTION
C                     (ARCTIC/ANTARCTIC ONLY)
C      MAXDEG - MAXIMUM DEGREE OF SPHERICAL HARMONIC MODEL    (INPUT)
C      MOXORD - MAXIMUM ORDER OF SPHERICAL HARMONIC MODEL
C
C
C*****************************************************************
C
```

```
C
C    NOTE:  THIS VERSION OF GEOMAG USES THE WMM-2000 GEOMAGNETIC
C           MODEL REFERENCED TO THE WGS-84 GRAVITY MODEL ELLIPSOID
C
C
C*******************************************************************
C
C
C
C
C
C
C
C
C
C*******************************************************************
C
C
C                    INITIALIZATION MODULE
C
C
C*******************************************************************
C
C
      SUBROUTINE GEOMAG(MAXDEG)
C
C
      DIMENSION C(0:12,0:12),CD(0:12,0:12),TC(0:12,0:12)
      DIMENSION P(0:12,0:12),DP(0:12,0:12),SNORM(0:12,0:12)
      DIMENSION SP(0:12),CP(0:12),FN(0:12),FM(0:12),PP(0:12)
      REAL K(0:12,0:12)
      EQUIVALENCE (SNORM,P)
C
C
      DATA EPOCH/2000.0/
C
C
      DATA C/      0.0, -29616.0, -2266.7,  1322.4,    932.1,  -211.9,
     *            73.8,     77.4,    23.3,     5.7,     -2.2,     3.3,
     *            -1.5,   5194.5, -1722.7,  3070.2,  -2291.5,   786.3,
     *           351.6,     68.2,   -73.9,     7.3,      8.5,    -5.7,
     *            -1.1,     -0.2, -2484.8,  -467.9,   1677.6,  1255.9,
     *           250.6,    220.8,    74.1,     2.2,     -8.5,     2.0,
     *             1.6,     -2.4,    -0.3,  -224.7,    293.0,  -486.5,
     *           724.8,   -401.5,  -134.5,  -163.5,     35.7,    -6.6,
```

```
     *          -9.8,     -3.7,      2.6,     0.5,  273.3, -227.9,
     *         120.9,   -302.7,    106.2,  -168.8,   -3.8,    7.3,
     *         -16.9,      7.6,     -0.6,    -1.3,    0.2,   42.0,
     *         173.8,   -135.0,    -38.6,   105.2,  -13.3,   17.1,
     *           5.2,      8.6,     -7.0,     4.1,   -1.7,    0.9,
     *         -17.4,     61.2,     63.2,   -62.9,    0.2,   43.0,
     *         -85.1,      8.4,      4.9,    -2.0,    2.2,   -0.6,
     *          -1.4,    -62.3,    -24.5,     8.9,   23.4,   15.0,
     *         -27.6,     -7.8,     -1.5,    -7.8,    9.2,    2.2,
     *           0.4,      0.6,     12.4,   -20.8,    8.4,  -21.2,
     *          15.5,      9.1,    -15.5,    -5.4,   -7.6,   -2.2,
     *           4.6,      0.7,     -0.6,   -20.4,   13.9,   12.0,
     *          -6.2,     -8.6,      9.4,     5.0,   -8.4,    3.2,
     *          -6.6,      2.3,     -0.3,    -1.0,    0.9,   -0.7,
     *           3.9,      4.8,     -5.3,    -1.0,   -2.4,    1.3,
     *          -2.3,     -6.4,      0.1,     2.3,   -0.3,   -1.5,
     *           0.7,     -1.1,     -2.3,     1.3,   -0.6,   -2.8,
     *          -1.6,     -0.1,     -1.9,     1.4,    4.2,    0.3,
     *          -1.0,      0.7,      2.2,    -2.5,   -0.2,    0.0,
     *          -0.2,      0.0,      0.2,    -0.9,   -0.2,    1.0,
     *           0.4/
C
C
      DATA CD/   0.0,  14.7, -13.6,   0.3,  -1.6,  -0.9,   1.2,  -0.4,
     *          -0.3,   0.0,   0.0,   0.0,   0.0, -20.4,  11.1,  -0.7,
     *          -4.3,   0.9,  -0.2,   0.2,  -0.8,   0.6,   0.0,   0.0,
     *           0.0,   0.0, -21.5,  -9.6,  -1.8,   0.9,  -7.6,  -2.5,
     *           1.7,  -0.2,  -0.8,   0.0,   0.0,   0.0,   0.0,   6.4,
     *          -1.3, -13.3,  -8.4,   2.2,  -2.7,   1.6,   1.1,   0.3,
     *           0.0,   0.0,   0.0,   0.0,   2.3,   0.7,   3.7,  -0.5,
     *          -3.2,  -0.9,  -0.1,   0.4,  -0.2,   0.0,   0.0,   0.0,
     *           0.0,   0.0,   2.1,   2.3,   3.1,   0.0,   1.7,  -0.3,
     *           0.0,   0.5,   0.0,   0.0,   0.0,   0.0,  -0.3,  -1.7,
     *          -0.9,  -1.0,  -0.1,   1.9,   0.8,  -0.2,   0.0,   0.0,
     *           0.0,   0.0,   0.0,   1.4,   0.2,   0.7,   0.4,  -0.3,
     *          -0.8,  -0.1,  -0.2,  -0.6,   0.0,   0.0,   0.0,   0.0,
     *          -0.5,   0.1,  -0.2,   0.0,   0.1,  -0.1,   0.3,   0.2,
     *           0.1, 56*0.0/
C
C
C
C
C
C     INITIALIZE CONSTANTS
C
```

```
C
      IF (MAXDEG .GT. 12) MAXDEG=12
      MAXORD=MAXDEG
      PI=3.14159265359
      DTR=PI/180.0
      SP(0)=0.
      CP(0)=1.
      P(0,0)=1.
      PP(0)=1.
      DP(0,0)=0.
      A=6378.137
      B=6356.7523142
      RE=6371.2
      A2=A**2
      B2=B**2
      C2=A2-B2
      A4=A2**2
      B4=B2**2
      C4=A4-B4
C
C      CONVERT SCHMIDT NORMALIZED GAUSS COEFFICIENTS TO UNNORMALIZED
C
      SNORM(0,0)=1.
      DO 20 N=1,MAXORD
      SNORM(N,0)=SNORM(N-1,0)*FLOAT(2*N-1)/FLOAT(N)
      J=2
      DO 10 M=0,N
      K(N,M)=FLOAT((N-1)**2-M**2)/FLOAT((2*N-1)*(2*N-3))
      IF (M .GT. 0) THEN
      FLNMJ=FLOAT((N-M+1)*J)/FLOAT(N+M)
      SNORM(N,M)=SNORM(N,M-1)*SQRT(FLNMJ)
      J=1
      C(M-1,N)=SNORM(N,M)*C(M-1,N)
      CD(M-1,N)=SNORM(N,M)*CD(M-1,N)
      ENDIF
      C(N,M)=SNORM(N,M)*C(N,M)
      CD(N,M)=SNORM(N,M)*CD(N,M)
 10   CONTINUE
      FN(N)=FLOAT(N+1)
      FM(N)=FLOAT(N)
 20   CONTINUE
      K(1,1)=0.
C
C
      OTIME=-1000.
```

```
      OALT=-1000.
      OLAT=-1000.
      OLON=-1000.
C
C
      RETURN
C
C
C*****************************************************************************
C
C
C                       PROCESSING MODULE
C
C
C*****************************************************************************
C
C
      ENTRY GEOMG1(ALT,GLAT,GLON,TIME,DEC,DIP,TI,GV)
C
C
      DT=TIME-EPOCH
      IF (OTIME .LT. 0. .AND. (DT .LT. 0. .OR. DT .GT. 5.)) THEN
      PRINT *, '      '
      PRINT *, ' WARNING - TIME EXTENDS BEYOND MODEL 5-YEAR LIFE SPAN'
      PRINT *, ' CONTACTNIMA FOR PRODUCT UPDATES:'
      PRINT *, '      '
      PRINT *, '         NATIONALIMAAGERY AND MAPPING AGENCY'
      PRINT *, '         ATTN: Code P33'
      PRINT *, '         12310 SUNRISEVALEY DRIVE'
      PRINT *, '         RESTON, VA 20191-3449'
      PRINT *, '         USA'
      PRINT *, '      '
      PRINT *, '         PHONE:  (703) 264-3002'
      PRINT *, '      '
      PRINT *, ' EPOCH  = ',EPOCH
      PRINT *, ' TIME   = ',TIME
      ENDIF
C
C
      RLON=GLON*DTR
      RLAT=GLAT*DTR
      SRLON=SIN(RLON)
      SRLAT=SIN(RLAT)
      CRLON=COS(RLON)
      CRLAT=COS(RLAT)
```

```
      SRLON2=SRLON**2
      SRLAT2=SRLAT**2
      CRLON2=CRLON**2
      CRLAT2=CRLAT**2
      SP(1)=SRLON
      CP(1)=CRLON
C
C        CONVERT FROM GEODETIC COORDS. TO SPHERICAL COORDS.
C
      IF (ALT .NE. OALT .OR. GLAT .NE. OLAT) THEN
      Q=SQRT(A2-C2*SRLAT2)
      Q1=ALT*Q
      Q2=((Q1+A2)/(Q1+B2))**2
      CT=SRLAT/SQRT(Q2*CRLAT2+SRLAT2)
      ST=SQRT(1.0-CT**2)
      R2=ALT**2+2.0*Q1+(A4-C4*SRLAT2)/Q**2
      R=SQRT(R2)
      D=SQRT(A2*CRLAT2+B2*SRLAT2)
      CA=(ALT+D)/R
      SA=C2*CRLAT*SRLAT/(R*D)
      ENDIF
C
C
      IF (GLON .NE. OLON) THEN
      DO 40 M=2,MAXORD
      SP(M)=SP(1)*CP(M-1)+CP(1)*SP(M-1)
      CP(M)=CP(1)*CP(M-1)-SP(1)*SP(M-1)
 40   CONTINUE
      ENDIF
C
C
      AOR=RE/R
      AR=AOR**2
C
C
      BR=0.
      BT=0.
      BP=0.
      BPP=0.
C
C
      DO 70 N=1,MAXORD
      AR=AR*AOR
      DO 60 M=0,N
C
```

```
C       COMPUTE UNNORMALIZED ASSOCIATED LEGENDRE POLYNOMIALS
C       AND DERIVATIVES VIA RECURSION RELATIONS
C
      IF (ALT .NE. OALT .OR. GLAT .NE. OLAT) THEN
      IF (N .EQ. M) THEN
      P(N,M)=ST*P(N-1,M-1)
      DP(N,M)=ST*DP(N-1,M-1)+CT*P(N-1,M-1)
      GO TO 50
      ENDIF
      IF (N .EQ. 1 .AND. M .EQ. 0) THEN
      P(N,M)=CT*P(N-1,M)
      DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)
      GO TO 50
      ENDIF
      IF (N .GT. 1 .AND. N .NE. M) THEN
      IF (M .GT. N-2) P(N-2,M)=0.0
      IF (M .GT. N-2) DP(N-2,M)=0.0
      P(N,M)=CT*P(N-1,M)-K(N,M)*P(N-2,M)
      DP(N,M)=CT*DP(N-1,M)-ST*P(N-1,M)-K(N,M)*DP(N-2,M)
      ENDIF
      ENDIF
 50   CONTINUE
C
C       TIME ADJUST THE GAUSS COEFFICIENTS
C
      IF (TIME .NE. OTIME) THEN
      TC(N,M)=C(N,M)+DT*CD(N,M)
      IF (M .NE. 0) THEN
      TC(M-1,N)=C(M-1,N)+DT*CD(M-1,N)
      ENDIF
      ENDIF
C
C       ACCUMULATE TERMS OF THE SPHERICAL HARMONIC EXPANSIONS
C
      PAR=AR*P(N,M)
      IF (M .EQ. 0) THEN
      TEMP1=TC(N,M)*CP(M)
      TEMP2=TC(N,M)*SP(M)
      ELSE
      TEMP1=TC(N,M)*CP(M)+TC(M-1,N)*SP(M)
      TEMP2=TC(N,M)*SP(M)-TC(M-1,N)*CP(M)
      ENDIF
      BT=BT-AR*TEMP1*DP(N,M)
      BP=BP+FM(M)*TEMP2*PAR
      BR=BR+FN(N)*TEMP1*PAR
```

```
C
C        SPECIAL CASE:  NORTH/SOUTH GEOGRAPHIC POLES
C
      IF (ST .EQ. 0.0 .AND. M .EQ. 1) THEN
      IF (N .EQ. 1) THEN
      PP(N)=PP(N-1)
      ELSE
      PP(N)=CT*PP(N-1)-K(N,M)*PP(N-2)
      ENDIF
      PARP=AR*PP(N)
      BPP=BPP+FM(M)*TEMP2*PARP
      ENDIF
C
C
   60 CONTINUE
   70 CONTINUE
C
C
      IF (ST .EQ. 0.0) THEN
      BP=BPP
      ELSE
      BP=BP/ST
      ENDIF
C
C       ROTATE MAGNETIC VECTOR COMPONENTS FROM SPHERICAL TO
C       GEODETIC COORDINATES
C
      BX=-BT*CA-BR*SA
      BY=BP
      BZ=BT*SA-BR*CA
C
C       COMPUTE DECLINATION (DEC), INCLINATION (DIP) AND
C       TOTAL INTENSITY (TI)
C
      BH=SQRT(BX**2+BY**2)
      TI=SQRT(BH**2+BZ**2)
      DEC=ATAN2(BY,BX)/DTR
      DIP=ATAN2(BZ,BH)/DTR
C
C       COMPUTE MAGNETIC GRID VARIATION IF THE CURRENT
C       GEODETIC POSITION IS IN THE ARCTIC OR ANTARCTIC
C       (I.E. GLAT > +55 DEGREES OR GLAT < -55 DEGREES)
C
C       OTHERWISE, SET MAGNETIC GRID VARIATION TO -999.0
C
```

```
      GV=-999.0
      IF (ABS(GLAT) .GE. 55.) THEN
      IF (GLAT .GT. 0. .AND. GLON .GE. 0.) GV=DEC-GLON
      IF (GLAT .GT. 0. .AND. GLON .LT. 0.) GV=DEC+ABS(GLON)
      IF (GLAT .LT. 0. .AND. GLON .GE. 0.) GV=DEC+GLON
      IF (GLAT .LT. 0. .AND. GLON .LT. 0.) GV=DEC-ABS(GLON)
      IF (GV .GT. +180.) GV=GV-360.
      IF (GV .LT. -180.) GV=GV+360.
      ENDIF
C
C
      OTIME=TIME
      OALT=ALT
      OLAT=GLAT
      OLON=GLON
C
C
      RETURN
C
C
      END
```

Magnetic Charts for Epoch 2000