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UNDERSTANDING THE FISHER EQUATION

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SUMMARY

It is argued that univariate long memory estimates based on *ex post* data tend to underestimate the persistence of *ex ante* variables (and, hence, that of the *ex post* variables themselves) because of the presence of unanticipated shocks whose short-run volatility masks the degree of long-range dependence in the data. Empirical estimates of long-range dependence in the Fisher equation are shown to manifest this problem and lead to an apparent imbalance in the memory characteristics of the variables in the Fisher equation. Evidence in support of this typical underestimation is provided by results obtained with inflation forecast survey data and by direct calculation of the finite sample biases.

To address the problem of bias, the paper introduces a bivariate exact Whittle (BEW) estimator that explicitly allows for the presence of short memory noise in the data. The new procedure enhances the empirical capacity to separate low-frequency behaviour from high-frequency fluctuations, and it produces estimates of long-range dependence that are much less biased when there is noise contaminated data. Empirical estimates from the BEW method suggest that the three Fisher variables are integrated of the same order, with memory parameter in the range (0.75, 1). Since the integration orders are balanced, the *ex ante* real rate has the same degree of persistence as expected inflation, thereby furnishing evidence against the existence of a (fractional) cointegrating relation among the Fisher variables and, correspondingly, showing little support for a long-run form of Fisher hypothesis.

1. INTRODUCTION

This study investigates the long-run properties of three *ex ante* Fisher variables including the *ex ante* real rate, expected inflation and the nominal interest rate. The properties are of intrinsic interest because these variables play a crucial role in determining investment, savings, and indeed virtually all intertemporal decisions. Since both the *ex ante* real interest rate and expected inflation are not directly observable, it is not a straightforward matter to study their long-run behaviour. To circumvent the difficulty, most empirical studies use *ex post* variables as proxies for the *ex ante* variables. In particular, actual inflation observed *ex post* is used as a proxy for expected inflation, and the implied *ex post* real rate, defined as the difference between the nominal interest rate and actual inflation according to the *ex post* Fisher equation, as a proxy for the *ex ante* real rate. This practice often leads to controversial results. For example, Rose (1988) concluded that the *ex ante* real rate is unit root nonstationary by showing that the nominal rate is a unit root process while inflation and inflation forecasting errors are $I(0)$ stationary. In contrast, Mishkin (1992, 1995) found support for an $I(0)$ *ex ante* real rate by rejecting the null of a unit root in the *ex post* real rate. Recently, Phillips (1998) showed that the three *ex post* Fisher components are fractionally

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integrated, and that the nominal interest rate is more persistent than both the real interest rate and inflation, an outcome that is strikingly at odds with the *ex post* Fisher equation. According to the *ex post* Fisher equation $i_t = \pi_{t+1} + r_{t+1}$, where π_{t+1} and r_{t+1} are the realized inflation rate and the *ex post* real interest rate, respectively, the degree of persistence of i_t is necessarily the same as that of the dominant component of π_{t+1} and r_{t+1}.

This paper attempts to reconcile the findings of Rose (1988) and Mishkin (1992, 1995), and resolve the empirical incompatibility found in Phillips (1998). All three studies estimated or inferred the integration orders of the Fisher variables based on the *ex post* Fisher equation. We argue here that empirical results obtained in this way can be misleading because the *ex post* Fisher equation appears unbalanced for the reasons explained below.

First, the timing of the three components is different. The nominal interest rate can be regarded as being set in advance. For example, the widely used three-month Treasury Bill rates are set every Monday and are 'expected' to be relevant over the next three months. Put this way, the nominal interest rate can be regarded as an observable *ex ante* variable. Therefore, when the Fisher equation is written in the form $i_t = r_{t+1} + \pi_{t+1}$, it expresses an *ex ante* variable as the sum of two *ex post* variables. More formally, if $\mathcal{F}_t$ is a filtration representing information at time t, i_t is adapted to the filtration $\mathcal{F}_t$ while π_{t+1} and, in consequence, r_{t+1} are adapted to the filtration $\mathcal{F}_{t+1}$. Interpreted in this way, the Fisher equation implies that the sum of two $\mathcal{F}_{t+1}$-measurable random variables is $\mathcal{F}_t$-measurable, which at first seems puzzling. But the Fisher equation is actually an accounting identity that defines the *ex post* real rate r_{t+1}. The forces that really determine the nominal interest rate i_t are the expected real rate and expected inflation formed at time t, i.e. $i_t = E_t r_{t+1} + E_t \pi_{t+1}$, where E_t is the expectation operator conditional on the information $\mathcal{F}_t$. By adding and subtracting the $\mathcal{F}_{t+1}$-measurable forecasting errors e_{t+1}, we get $i_t = (E_t r_{t+1} - e_{t+1}) + (E_t \pi_{t+1} + e_{t+1}) = r_{t+1} + \pi_{t+1}$.

Second, the short-run dynamics of the three components are different. The nominal interest rate is often less volatile than inflation and the *ex post* real rate in the short run. The nominal rate is a rate that is *expected* to prevail during some period and is not affected, by definition, by the unexpected shocks that arrive during that period. On the other hand, the inflation rate and *ex post* real rate are rates that are realized during that period and thus carry the effects of the unexpected shocks over that period.

Third, it can be misleading to infer the integrating order of the real rate in small samples from the *ex post* Fisher equation as is done in Rose (1988). Due to the presence of possibly large forecasting errors, unit root tests may falsely reject the null that expected inflation contains a unit root, if *ex post* inflation is used as a proxy for expected inflation. The false rejection, coupled with evidence that the nominal rate contains a unit root, can lead to the false conclusion that the *ex ante* real rate is an $I(1)$ process. Again, because of forecasting errors, unit root tests are likely to reject the null of a unit root in the *ex ante* real rate, if the *ex post* real rate is used as a proxy for the *ex ante* real rate. This leads to the possibly false conclusion reached by some earlier researchers (e.g., Mishkin, 1992, 1995) that the *ex ante* real rate is an $I(0)$ process.

The empirical incompatibility found in Phillips (1998) is direct evidence of the apparent imbalance of the *ex post* Fisher equation. Suppose the forecasting errors are stationary and weakly dependent, and expected inflation $E_t \pi_{t+1}$ follows a fractional process. Then actual inflation follows a perturbed fractional process in the sense that it is the sum of a fractional process and weakly dependent noise. From a statistical perspective, a perturbed fractional process is a long memory process with the same degree of persistence as the original fractional process. However, it can be difficult to estimate the fractional integration parameter even in large samples, especially when the perturbation is volatile because the long memory component gets buried in a lot of short memory noise. In these circumstances, the widely used log-periodogram (LP) estimator (Geweke and Porter-Hudak, 1983) and local Whittle estimator (Robinson, 1995) suffer substantial downward bias. This bias is large enough to account for the empirical incompatibility that Phillips discovered in the *ex post* Fisher relation.

Using a new approach, we find evidence that the three Fisher variables are indeed integrated of the same order and are fractionally nonstationary. The evidence presented here takes three forms. First, one cause of the bias is that *ex post* rather than *ex ante* variables are observed. If good proxies for the *ex ante* variables were available, we could perform estimation with these proxies and presumably avoid or at least reduce bias. Of course, the *ex post* variables can themselves be regarded as proxies for the *ex ante* variables. However, unexpected subsequent shocks make the *ex post* variables more volatile than their *ex ante* counterparts, so these variables may not be such good proxies because of their contamination with short memory effects. This point is especially important because it is the long-run properties of the variables that are the focus of interest in the Fisher relation. In search of better proxies than *ex post* realizations, we employ the inflation forecast from the *Survey of Professional Forecasters* (for details of the survey, see Croushore, 1993) as a proxy for expected inflation, and we use the implied real rate forecast as a proxy for the *ex ante* real rate. Using these variables, we find that the estimated orders of integration are larger than those based on the realized *ex post* series. This finding corroborates the bias argument and indicates that the true *ex ante* variables are more persistent than they appear to be from *ex post* realizations.

Second, we calculate the bias effects explicitly using asymptotic expressions. Asymptotic theory shows that the asymptotic bias of the LP estimator depends on the ratio of the long-run variance of the forecasting errors and that of the innovations that drive the inflation forecasts. Using inflation and the inflation forecasts, we evaluate the ratio and calculate the asymptotic bias of the LP estimator. The evidence points to a substantial asymptotic bias and these findings are supported by simulation evidence from finite samples. Furthermore, we investigate the bias of the exact Whittle (EW) estimator of Shimotsu and Phillips (2002a), which is more efficient and more widely applicable (to stationary and nonstationary fractional series) than the LP estimator. The EW estimator produces the same empirical incompatibility as the LP estimator. Simulations show that the EW estimator also has a substantial downward bias.

Third, we introduce a bivariate exact Whittle (BEW) estimator that accounts for the possible presence of additive perturbations in the data. The estimator resembles the bivariate Whittle estimator of Lobato (1999) but involves an additional term in the approximation of the spectral density matrix and uses an exact version of the local Whittle likelihood (see Phillips, 1999; Shimotsu and Phillips, 2002a). Simulations show that the BEW estimator has a significantly smaller bias than the LP and EW estimators. Applying the BEW estimator to the *ex post* data, we find the BEW estimates are significantly higher than LP and EW estimates, which again lends strong support for the small sample bias argument. Moreover, the empirical estimates suggest that the three Fisher variables are integrated of the same order, with memory parameter in the range (0.75,1).

The BEW estimator also provides a framework for testing the equality of the integration orders of the three Fisher components. Applying this approach, we find that we cannot reject the null that inflation and the real rate are integrated of the same order. Since the integration orders are balanced, the *ex ante* real rate has the same degree of persistence as expected inflation, thereby furnishing evidence against the existence of a (fractional) cointegrating relation among the Fisher variables and correspondingly, showing little support for a long-run form of Fisher hypothesis.

The rest of the paper is organized as follows. Section 2 gives LP and EW estimates of the fractional integration parameters using *ex post* data and confirms the empirical incompatibility described above. Section 3 presents evidence from the *Survey of Professional Forecasters* to show that the *ex post* variables are more volatile than the *ex ante* variables, and that the LP and EW estimates based on the *ex post* data are substantially downward biased. Section 4 introduces the BEW estimator and provides further evidence that the LP and EW estimates are biased downward. This section also develops and implements a test of the equality of long memory in the Fisher components. Section 5 concludes.

2. MEMORY ESTIMATION AND THE APPARENT EMPIRICAL INCOMPATIBILITY OF THE FISHER COMPONENTS

We calculate three-month inflation rates using the US monthly CPI (all commodities, with no adjustment) and take the US three-month Treasury Bill rate as the nominal interest rate.[1] Instead of using monthly overlapping data as in Phillips (1998), we compute and employ quarterly non-overlapping data in order to make them conform to the data from the *Survey of Professional Forecasters*. In addition, the use of quarterly data avoids the possibly spurious serial correlation resulting from the horizon of the variables being longer than the observation interval. No variables are seasonally adjusted. Since low frequency (specifically, frequency zero) behaviour is the focus of interest, seasonality does not play an important role. The timing of the data is as follows: data are collected in January, April, July and October each year. A January observation of the three-month inflation rate is calculated from the January to April CPI data. A January interest rate observation uses the end of January three-month TB rate. The timing of the TB rate and the inflation rate is slightly different, as the monthly CPI is based on the prices that are taken throughout a month, instead of at the end of a month. Given data availability, it is not possible to match the timing of these two variables precisely.

Using quarterly data in the US over the period 1934 : 1–1999 : 4, we employ both the exact Whittle and the log-periodogram approaches to estimate the fractional differencing parameters. The advantages of the exact local Whittle estimator are its robustness to nonstationarity, its consistency and asymptotic normality for all values of d. However, its properties are unknown in the presence of additive short memory disturbances. The log-periodogram estimator is easy to implement and has been shown to be consistent and asymptotic normal (Sun and Phillips, 2003) for both fractional processes and perturbed fractional processes. However, the LP estimator is inconsistent when $d > 1$ (Kim and Phillips, 1999). In this case, two popular approaches are to difference or taper the data (Velasco, 1999a,b; Lobato and Velasco, 2000).

For a given time series $\{x_t, t = 1, 2, \ldots, n\}$, tapering produces a new time series of the form $\{h_t x_t\}$ where a taper $\{h_t\}$ is used to weight the original observations. A popular choice of taper is the Hanning taper defined by $h_t = 1/2[1 - \cos(2\pi t/n)]$. Obviously, any form of tapering distorts the trajectory of the original time series, and this in turn leads to an inflation of the asymptotic variance of estimates obtained from the tapered series. We therefore adopt the first option and difference the data using the filter $(1 - L)^{1/2}$. This half-difference filter has been used in earlier research, e.g. by Gil-Alana and Robinson (1997). To apply a fractional filter such as $(1 - L)^{1/2}$

[1] **CPI**: Bureau of Labor Statistics, *Monthly Labor Review*. Code: CUUROOOOSA0. http://www.bls.gov/data/home.htm. **Three-month Treasury Bill rate**: Board of Governors of the Federal Reserve System, Federal Reserve Bulletin. Code: TB3MS. http://research.stlouisfed.org/fred/.

to a given time series of fixed length, we assume that the prehistorical values of the time series are zero (see the footnote below). Since the choice of the fractional filter $(1 - L)^{1/2}$ is somewhat arbitrary, it is worthwhile using alternate filters to check robustness of the results. In the empirical work reported below, similar estimates to those reported were obtained after applying the filters $(1 - L)^{0.75}$ and $(1 - L)$.

2.1. EW and LP Estimation

The exact Whittle estimator was proposed in Phillips (1999) and its asymptotic theory was developed in Shimotsu and Phillips (2002a). For a fractional process defined as[2]

$$x_t = (1 - L)^{-d} w_t = \sum_{k=0}^{t-1} \frac{\Gamma(d + k)}{\Gamma(d)\Gamma(k + 1)} w_{t-k} \tag{1}$$

where $\{w_t\}$ is a weakly dependent process with continuous spectral density, exact Whittle estimation of the memory parameter d involves maximizing the following Whittle log-likelihood function:

$$Q_m(G, d) = \frac{1}{m} \sum_{j=1}^{m} \left(\log G\lambda_j^{-2d} + \frac{1}{G} I_{\Delta^d(x)}(\lambda_j) \right) \tag{2}$$

where G is a positive constant, $I_{\Delta^d(x)}(\lambda_j)$ is the periodogram of $(1 - L)^d x_t$ with the fractional filter $(1 - L)^d$ defined in the same way as in Robinson (1994) and Phillips (1999), and m is a bandwidth parameter satisfying $m/n + 1/m \to 0$ so that the band $\{\lambda_j = 2\pi j/n, j = 1, 2, \ldots, m\}$ concentrates on the zero frequency as the sample size $n \to \infty$.

Shimotsu and Phillips (2002a) show that the exact Whittle estimator $(\hat{G}_{EW}, \hat{d}_{EW})$ is consistent and that $\hat{d}_{EW}$ has the following limiting distribution as $n \to \infty$:

$$\sqrt{m}(\hat{d}_{EW} - d) \xrightarrow{d} N(0, 1/4) \tag{3}$$

for all values of d. The robustness of the asymptotic properties of $\hat{d}_{EW}$ is especially appealing for practical work when the domain of the true order of fractional integration is controversial. The EW estimate also provides guidance on the order of the fractional difference that can render the data stationary.

LP regression involves linear least squares over the same frequency band $(\lambda_j : j = 1, 2, \ldots, m)$ leading to the regression equation

$$\log I_{\Delta^{\tilde{d}}(x)}(\lambda_j) = \hat{\alpha} - \hat{\beta} \ln |1 - \exp(i\lambda_j)|^2 + error \tag{4}$$

for some $\tilde{d}$ corresponding to preliminary fractional differencing of the data. The LP estimate $\hat{d}_{LP}$ of d is then obtained by adding $\tilde{d}$ back into the estimate $\hat{\beta}$, giving $\hat{\beta} + \tilde{d}$. In our empirical work

[2] Two main approaches have been used in the literature to define a fractional process x_t. The first, which is adopted in Hosking (1981), among others, defines a stationary fractional process as an infinite order moving average of innovations: $x_t = \sum_{k=0}^{\infty} \Gamma(d + k)/\Gamma(d)\Gamma(k + 1)w_{t-k}$ and defines a nonstationary $I(d)$ process as the partial sum of an $I(d - 1)$ process (Hurvich and Ray, 1995; Velasco, 1999a,b). The second, which is used in Robinson (1994) and Phillips (1999), truncates the fractional difference filter and defines $x_t = \sum_{k=0}^{t-1} \Gamma(d + k)/\Gamma(d)\Gamma(k + 1)w_{t-k}$ for all values of d. For a more detailed discussion of the definitions and their implications, see Shimotsu and Phillips (2002b).

reported below, we take $\tilde{d}$ to be 0.5, 0.75 and 1, and find that the estimates for different $\tilde{d}$ match fairly closely. We therefore only report the case for $\tilde{d} = 0.5$. Sun and Phillips (2003) show that the LP estimator is consistent and has the following limiting distribution even in the presence of perturbations:

$$\sqrt{m}(\hat{d}_{LP} - d) \xrightarrow{d} N(0, \pi^2/24) \quad (5)$$

2.2. Empirical Memory Estimates for the Fisher Components

Since the EW and LP estimates depend on the choice of bandwidth m, several different bandwidths were used. Figures 1 and 2 present the empirical estimates of (d_i, d_π, d_r), the long memory parameters for the three *ex post* Fisher components, using the LP estimator and the EW estimator, respectively. Both the EW estimates and the LP estimates appear fairly robust to the choice of m. A salient feature of both figures is that the 95% confidence bands for d_i and d_π do not overlap each other while the confidence bands for d_r and d_π are almost indistinguishable. The EW and LP estimates suggest that $d_i > d_\pi$ and $d_r = d_\pi$, a configuration that is incompatible with the *ex post* Fisher equation. For, in a model where the three fractionally integrated variables $y_t \equiv I(d_y), x_t \equiv I(d_x)$ and $z_t \equiv I(d_z)$ satisfy the linear relationship $y_t = x_t - z_t$, the long-run behaviour of y_t is characterized by the dominant component of x_t and z_t, e.g., if $d_x > d_z$, then $d_y = d_x$. In the present case, $r_{t+1} = i_t - \pi_{t+1}$. So, if $d_i > d_\pi$, then $d_r > d_\pi$. This conclusion is clearly at odds with the empirical estimates.

The estimates obtained here are similar to those reported in Phillips (1998) where the empirical incompatibility of the long-run behaviour of the Fisher components was discovered. Phillips used the local Whittle estimator (Robinson, 1995) that was originally proposed for stationary fractional processes, extending it to the nonstationary case $d \in (1/2, 1]$. So, local Whittle, exact local Whittle and LP estimators all reveal the same empirical incompatibility.

To check the robustness of the results, we re-estimated the fractional parameters using the data after World War II. In doing so, it is of particular interest to see whether the extensive controls

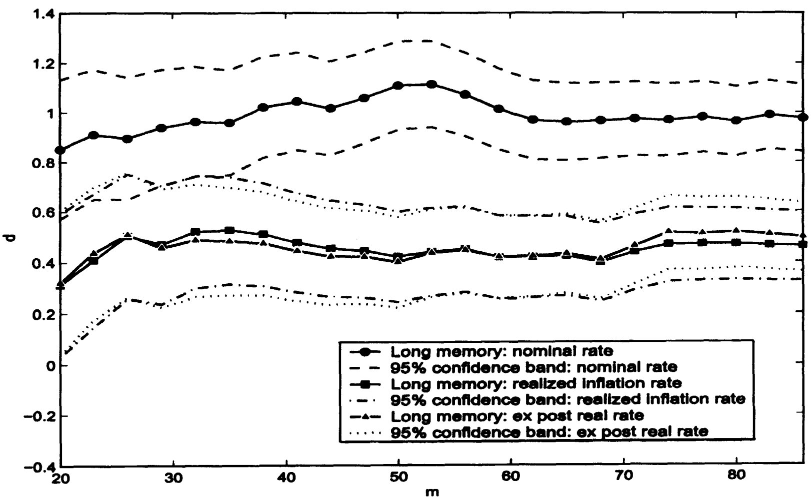

Figure 1. Log-periodogram estimates of d (sample size = 264)

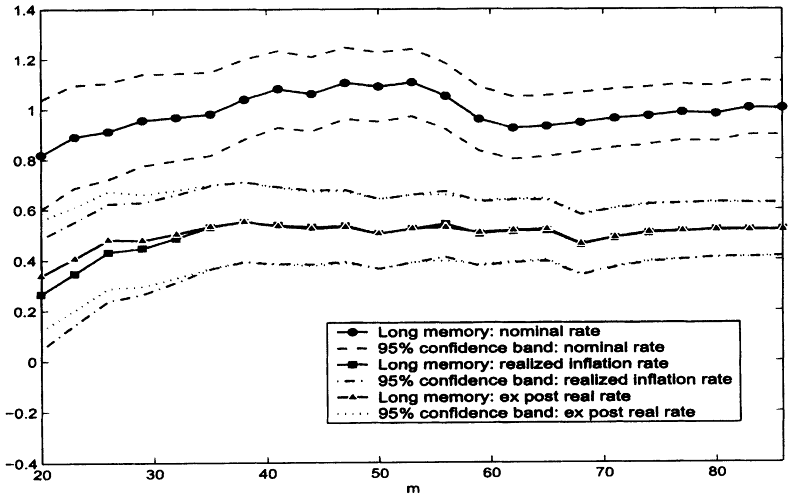

Figure 2. Exact-local Whittle estimates of d (sample size = 264)

on prices and nominal rates during the war period affect our results. Using the subsample ranging from 1948 : 1 to 1999 : 4, we find the subsample LP and EW estimates are close to those based on the full sample and that the empirical incompatibility remains.

3. UNDERSTANDING THE EMPIRICAL IMBALANCE IN THE FISHER EQUATION

The estimates reported in the previous section are all based on the *ex post* time series,[3] which are either directly observable or indirectly available from the *ex post* Fisher equation. We argue that the empirical estimates based on the *ex post* data underestimate the true degree of persistence of the underlying *ex ante* variables and hence that of the *ex post* variables themselves. Since the long-run properties of the underlying variables are the focus of interest, the *ex post* variables can be regarded as proper proxies for the *ex ante* variables. On theoretical grounds, this will be true as long as the forecasting errors are stationary and weakly dependent (i.e., have short memory). However, when the unexpected shocks are so large that the forecasting errors have greater variation than the innovations that drive the *ex ante* variables, the actual variables observed *ex post* may appear to be less persistent than they really are because the slowly moving nature of the persistent component is buried in the volatile short-run fluctuations. This interpretation gains support from the evidence presented below from inflation forecasts.

3.1. Results from Inflation Forecasts

Under the assumption of rational expectations, the realized inflation rate differs from the expected inflation rate by an unexpected shock, i.e.

$$\pi_{t+1} = \pi_t^e + e_{t+1} \tag{6}$$

[3] To avoid confusion, we should note that we sometimes refer to the nominal rate as an *ex post* variable because it has *ex ante* features and is observable *ex post*.

where the unexpected shock (forecasting error) e_{t+1} is a martingale difference process. If we further assume that $\{\pi_t^e\}$ is a fractional process that is uncorrelated with $\{e_t\}$, then the realized inflation rate is a fractional process with uncorrelated additive disturbances. Such a process is a special case of the perturbed fractional process studied in Sun and Phillips (2003). A general perturbed fractional process allows the additive disturbances to be any stationary and weakly dependent process. The uncorrelatedness between the forecasting errors and the innovations that drive the inflation forecast seems plausible. This is supported by a simple calculation of cross-correlation coefficients using the data on inflation and inflation forecasts.

The strong dependence in the inflation expectations data is consistent with Fisher's original study (Fisher, 1930). Fisher found that the duration of the expectation formation process was long and that realized inflation was quite volatile. He constructed inflation expectations series by taking moving averages of realized inflation over as many as 15 to 40 years. Here we employ modern techniques to model the same phenomenon, using a persistent (long memory) process to model expected inflation and additive disturbances (representing unexpected shocks) to allow for the greater volatility of realized inflation. When there is large variation in the unexpected shock component, realized inflation appears less persistent because the slow moving component is less evident in the time series. Therefore, estimates of strong dependence tend to be downward biased with the bias depending on the relative variation in the forecasts and the unexpected shocks, as shown by Sun and Phillips (2003).

To compare variation, we need to obtain the expected inflation rates. Prior studies of this issue can be grouped into two categories. One models expectation formation explicitly and then estimates expected inflation from the observed time series of realized (*ex post*) values (e.g., Hamilton, 1985). The other uses survey data on inflation forecasts or inflation expectations. Several surveys are available and among these the *Survey of Professional Forecasters* is the oldest quarterly survey of macroeconomic forecasts in the United States.[4] The survey respondents include a diverse group of forecasters who share one thing in common: they forecast as part of their current jobs. Hence it is reasonable to believe that their forecasts represent an overview of expectations about macroeconomic activity in general and expected inflation in particular. This position is supported by the study of Keane and Runkle (1990). In analysing the characteristics of these forecasts, Keane and Runkle found that they were unable to reject the hypothesis that the price level forecasts are unbiased and rational.

In this paper, we use forecasted inflation from the *Survey of Professional Forecasters* as expected inflation. Before turning to the survey data, we first describe the timing of the survey. The survey participants were asked to make macroeconomic forecasts for the next quarter.[5] The deadlines were usually close to the middle of February, May, August or November, depending on the quarter forecast. For example, to forecast the second quarter CPI in a year, the survey participants were asked to report their forecasts by mid-February. Therefore, the forecasts were made around the middle of a quarter instead of at the end of a quarter. The timing of inflation forecasts thus does not exactly match that of the CPI survey and the nominal interest rate. As a consequence, the implied *ex ante* real rate, defined as the difference between the nominal

[4] The survey began in 1968 and was conducted by the American Statistical Association and the National Bureau of Economic Research. The Federal Reserve Bank of Philadelphia took over the survey in 1990. The survey is publicly available at no cost and is often reported in major newspapers and financial news wires. For more information see http://www.phil.frb.org/econ/spf/.

[5] The survey participants actually made forecasts for the next five quarters. In this paper, we use only one-quarter ahead forecasts.

interest rate and the inflation forecast, is subject to an error arising from this misalignment. If this error is $I(0)$ stationary, the implied *ex ante* real rate differs from the true *ex ante* real rate by a stationary component. The estimated persistence in the implied *ex ante* real rate is expected to reflect the persistence in the *ex ante* real rate, with a possible downward bias in small samples. Given data availability, we make the assumption that the error arising from such misalignment is $I(0)$.

Using data from the *Survey of Professional Forecasters*, we extract a quarterly series of expected inflation rates from 1981:4 to 1999:3. Figure 3 graphs this expected inflation series against that of realized inflation. Expected inflation appears much smoother than realized inflation, revealing the volatility induced by the presence of unexpected shocks in realized inflation rates. Over the time period shown, there are spikes in realized inflation corresponding to both positive and negative shocks. The volatility of these shocks makes the realized inflation series appear less persistent. It is also obvious from Figure 3 that the shocks tend to change sign from quarter to quarter. This may be attributable to measurement errors in prices, arising from the survey sampling and reporting errors in the monthly price level. If the resulting measurement error in CPI inflation is an $I(0)$ process, then it contributes nothing at frequency zero for CPI inflation, but its local spectral shape runs counter to that of the underlying $I(d)$ process. This makes the realized inflation series appear less persistent in small samples. In short, both the volatility of the shocks and the negative autocorrelation in the inflation measurement error obscure the slow moving component of expected inflation. It can therefore be misleading to infer the persistence of expected inflation using realized inflation as a proxy.

We note, in passing, that there are other sources of measurement error that may have long-lasting effects, particularly those arising from changes in the quality of goods and changes in consumer spending patterns over time. Such measurement errors do not bear directly on the difference between expected and realized inflation, as forecasters presumably seek to predict what the BLS will later announce. However, these measurement errors may affect the estimated persistence in expected and realized inflation. This issue might be addressed by comparing different consumer

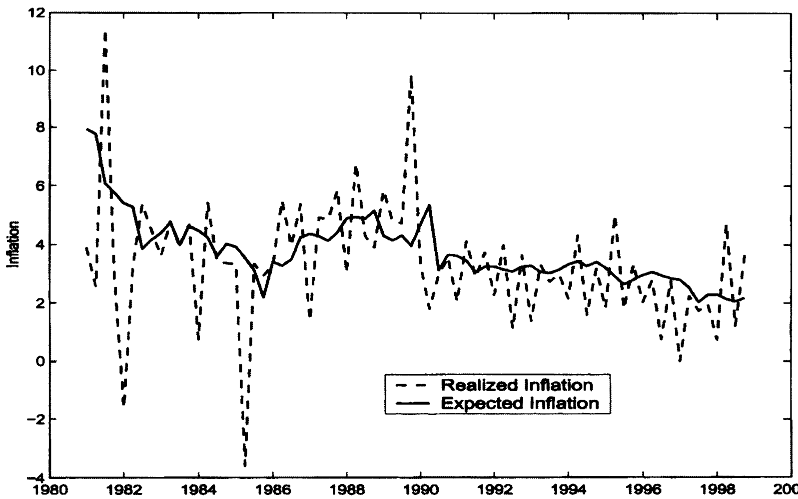

Figure 3. Expected and realized inflation (sample size = 72)

price indices, such as CPI data based on arithmetic averages with that based on geometric means. Such analysis is obviously beyond the scope of the present paper.

Figure 4 graphs the EW estimates of the long memory parameter using quarterly inflation forecasts over 1981:4–1999:3. Although the sample size is small, the results are indicative. Comparing the empirical estimates based on inflation forecasts with those based on realized inflation, we observe a substantial difference. In particular, the EW estimates using realized inflation are less than the lower limits of the 95% confidence intervals based on expected inflation. The difference increases as m increases. This is consistent with the fact that when m is larger, the estimator is less able to avoid contamination from short memory (higher frequency) effects arising from sources such as a stationary disturbance.

In much the same way as for inflation, the *ex post* real rate is more volatile than the *ex ante* real rate because of the presence of the additive short memory component. Figure 4 also shows the differences in the two estimates obtained from the *ex ante* and *ex post* real rate series. Again, the long memory parameter estimates for the *ex ante* real rate are generally larger than those for the *ex post* real rate. The qualitative observations made for the inflation rate series remain valid for the real rate series. These differences are reflected in the LP estimates as well. To save space, we do not present the graph for the LP estimates.

In sum, the empirical estimates obtained in Figure 4 suggest that expected inflation and the *ex ante* real rate may be just as persistent as the nominal rate. Under rational expectations, *ex post* and *ex ante* variables are characterized by the same degree of persistence because they differ by unanticipated shocks. Thus, under this assumption and according to these estimates, actual inflation and the real rate observed *ex post* may be as persistent as the nominal rate.

3.2. Evaluating the Small Sample Bias

The last subsection used inflation forecasts to estimate the long memory parameter directly. This subsection uses the forecasting data to evaluate the small sample biases of the LP and EW estimates when additive perturbations are present.

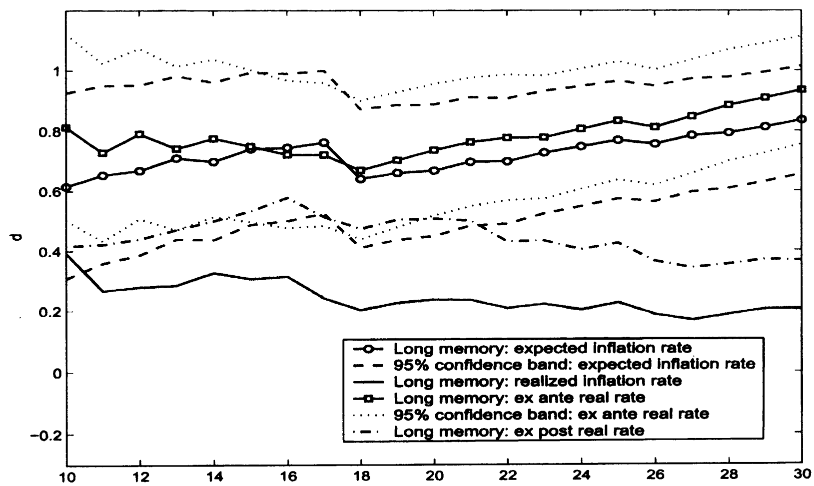

Figure 4. Exact local Whittle estimates of d based on inflation forecasts (sample size = 72)

We start by assuming that expected inflation follows a fractional process:

$$\pi_t^e = \mu + (1-L)^{-d} w_t = \mu + \sum_{k=0}^{\infty} \frac{\Gamma(d+k)}{\Gamma(d)\Gamma(k+1)} w_{t-k} \tag{7}$$

where $\{w_t\}$ is a Gaussian process with zero mean and continuous spectral densities $f_w(\lambda)$. We assume that the forecasting errors e_τ are uncorrelated with π_s^e for all τ and s. Sun and Phillips (2003) show that, under certain regularity conditions, the LP estimator based on actual inflation, $\pi_{t+1} = \pi_t^e + e_{t+1}$, is consistent and asymptotically normal. The limiting distribution is

$$\sqrt{m}(\hat{d}_{LP} - d) \Rightarrow N\left(b_{LP}, \frac{\pi^2}{24}\right) \tag{8}$$

where the asymptotic bias effect

$$b_{LP} = -(2\pi)^{2d} \left(\frac{f_w(0)}{f_e(0)}\right)^{-1} \frac{d}{(2d+1)^2} \frac{m^{2d}}{n^{2d}} \sqrt{m} \tag{9}$$

The asymptotic bias b_{LP} is always negative, just as one would expect when there is short memory contamination. The magnitude of the bias obviously depends on the signal–noise (SN) ratio $f_w(0)/f_e(0)$, which is the ratio of the long-run variance of the innovations that drive expected inflation to that of the forecasting errors. Again, this is not surprising, since the ratio measures the underlying force of expected inflation shocks relative to that of the forecasting errors. Because of the presence of the bias in (8), the larger is the force of the forecasting errors, the more difficult it is to recover good estimates of the long memory parameter from *ex post* observations.

Asymptotic results analogous to (8) for the EW estimator are not available in the literature and to derive such results is beyond the scope of the present paper. However, in related work without the effect of perturbations, Andrews and Sun (2004) show that the local Whittle estimator has the same asymptotic bias, but smaller asymptotic variance than the LP estimator for stationary long memory processes. We conjecture these results continue to hold for nonstationary perturbed long memory processes. This conjecture is supported by the simulation study reported in Table II below.

To evaluate the asymptotic bias $b_{LP}/\sqrt{m}$, we estimate the forecasting errors e_{t+1} by $\pi_{t+1} - \pi_t^e$ and the innovations w_t by $(1-L)^{\hat{d}_\pi}\tilde{\pi}_t^e$, where $\tilde{\pi}_t^e$ is the demeaned inflation forecast. To estimate the long-run variances $f_e(0)$ and $f_w(0)$, we employ the following formula:

$$lrvar = \gamma(0) + 2\sum_{j=1}^{p}\left(1 - \frac{j}{p+1}\right)\gamma(j) \tag{10}$$

where $\gamma(k)$ is the kth autocovariance function. The SN ratio $f_w(0)/f_e(0)$ can then be calculated as the ratio of the estimates of the long-run variances. The estimated ratio evidently depends on the choices of p and $\hat{d}_\pi$. Table I presents estimates of the inverted SN ratio obtained in this way for various selections of p and d_π. It shows that the variation in unexpected shocks is indeed relatively very large. Large variation in unexpected shocks leads to large small-sample bias. For

Table I. Estimates of the inverted signal–noise ratio ($n = 72$)

	$d = 0.6$	$d = 0.7$	$d = 0.8$	$d = 0.9$	$d = 1$
$p = 1$	6.69	7.35	7.84	8.18	8.37
$p = 3$	6.24	7.61	8.99	10.34	11.60
$p = 5$	5.74	7.36	9.12	10.91	12.67
$p = 7$	5.93	8.05	10.54	13.33	16.30
$p = 9$	6.14	8.55	11.41	14.62	17.99
$p = 11$	6.22	8.94	12.33	16.31	20.68
$p = 13$	6.60	9.63	13.44	17.92	22.80

example, when $n = 264, m = n^{1/2}$, $f_e(0)/f_w(0) = 12$ and $d = 0.8$, the asymptotic bias $b_{LP}/\sqrt{m}$ is -0.3105, or 39%.

To examine the effectiveness of the asymptotic results for finite samples, we conduct a Monte Carlo simulation. Let $x_t = (1 - L)^{-d} w_t$ and $y_t = x_t + e_t$ where $w_t \sim iid\ N(0, 1)$ and $e_t \sim iid\ N(0, 12)$. The *iid* assumptions are innocuous, as the bias depends only on the signal–noise ratio $f_w(0)/f_e(0)$. The two variances are chosen to calibrate to the signal–noise ratio in the data (see Table I). For each replication with sample size $n = 264$ and $m = n^{1/2}$, we estimate the long memory parameter using the original process $\{x_t\}$ and using the perturbed process $\{y_t\}$. Table II gives the averages and standard errors of the LP and EW estimates obtained from 1000 replications. As expected, the EW estimator has more or less the same finite sample bias but smaller variance than the LP estimator. Both the EW estimator and the LP estimator have a large finite sample bias. Thus, on bias grounds alone, estimates around 0.55 obtained from *ex post* data as shown in Figures 1 and 2 could come from a model where the true memory parameter is as large as 0.8. When the bias is so large, memory parameter estimates obtained from *ex post* inflation and the real interest rate series can therefore be seriously misleading.

4. FURTHER EVIDENCE USING A NEW BIVARIATE EXACT LOCAL WHITTLE ESTIMATOR

The small-sample biases discussed above arise because expected inflation and the *ex ante* real rate are not directly observable. Of course, we can use survey data on expectations such as that from the *Survey of Professional Forecasters* as proxies. However, time series of expectations data

Table II. Average estimates using original series and perturbed series ($n = 264$)

		$d = 0.5$	$d = 0.6$	$d = 0.7$	$d = 0.8$	$d = 0.9$	$d = 1$
LP	$\hat{d}_x$	0.5028	0.6133	0.6971	0.8053	0.9003	1.0095
	Std($\hat{d}_x$)	(0.2013)	(0.2018)	(0.2059)	(0.2011)	(0.2057)	(0.2132)
	$\hat{d}_y$	0.2249	0.3326	0.4432	0.5778	0.6944	0.8227
	Std($\hat{d}_y$)	(0.2170)	(0.2033)	(0.2168)	(0.2069)	(0.2059)	(0.2146)
EW	$\hat{d}_x$	0.4805	0.5823	0.6752	0.7826	0.8752	0.9768
	Std($\hat{d}_x$)	(0.1821)	(0.1780)	(0.1699)	(0.1750)	(0.1712)	(0.1789)
	$\hat{d}_y$	0.2084	0.3072	0.4212	0.5512	0.6611	0.7826
	Std($\hat{d}_y$)	(0.1676)	(0.1691)	(0.1737)	(0.1664)	(0.1632)	(0.1688)

like the inflation forecasts series we have used earlier are not long series, so empirical estimates based on them may not be very accurate, particularly for a parameter that characterizes long-range dependence in the data. In this section, therefore, we explore the structure of the Fisher equation further and propose a new estimator that is based on the *ex post* data to achieve bias reduction.

4.1. The Bivariate Exact Whittle Estimator

Observe that the *ex post* real rate and the realized inflation rate can be represented in system format as

$$r_{t+1} = r_t^e - e_{t+1}$$
$$\pi_{t+1} = \pi_t^e + e_{t+1} \tag{11}$$

Under the assumption that r_t^e and π_t^e are fractional processes, both r_{t+1} and π_{t+1} are perturbed fractional processes. Furthermore, the perturbations are from the same source, i.e. the unexpected inflation shocks. Therefore, we can expect that it is more efficient to estimate the fractional parameters jointly.

Assuming that the *ex ante* variables and forecasting errors are uncorrelated, the generalized spectral density matrix $f(\lambda)$ of $x_t := (r_t, \pi_t)'$ satisfies

$$f(\lambda) \sim \Lambda G \Lambda^* + \kappa H \text{ as } \lambda \to 0+ \tag{12}$$

where $\Lambda = diag(e^{\frac{\pi}{2}d_1 i}\lambda^{-d_1}, e^{\frac{\pi}{2}d_2 i}\lambda^{-d_2})$, $d = (d_1, d_2)'$, $\kappa = \sigma_e^2/(2\pi)$, G is a symmetric positive definite real matrix,

$$H = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \tag{13}$$

and the affix * denotes complex conjugate transpose. For more details on the generalized spectral density, see Solo (1992), which gave a formal justification of $f(\lambda)$ as a spectral density in terms of the limit of the expectation of the periodogram. Define $\Delta^d(x_t) = (\Delta^{d_1} r_t, \Delta^{d_2}\pi_t)'$ and

$$I_{\Delta^d(x)}(\lambda_j) = \frac{1}{2\pi n}\left|\sum_{t=1}^{n} \Delta^d(x_t)\exp(it\lambda_j)\right|^2, \quad \Lambda_j = diag(e^{\frac{\pi}{2}d_1 i}\lambda_j^{-d_1}, e^{\frac{\pi}{2}d_2 i}\lambda_j^{-d_2}) \tag{14}$$

Then the (negative) exact local Whittle likelihood is

$$Q_m(G, \kappa, d) = \frac{1}{m}\sum_{j=1}^{m}\left(\log|\Lambda_j G \Lambda_j^* + \kappa H| + tr[(G + \kappa \Lambda_j^{-1} H \Lambda_j^{*-1})^{-1} I_{\Delta^d(x)}(\lambda_j)]\right)$$

Minimizing $Q_m(G, \kappa, d)$ yields the bivariate exact Whittle (BEW) estimator

$$(\hat{G}_{BEW}, \hat{\kappa}_{BEW}, \hat{d}_{BEW}) = \arg\min Q_m(G, \kappa, d) \tag{15}$$

When x_t is a scalar time series, both G and $I_{\Delta^d(x)}$ reduce to positive scalars. In this case, we get the univariate exact Whittle (UEW) estimator. Observe that the UEW estimator is different from the EW estimator of Shimotsu and Phillips (2002a) because, unlike the EW estimator, the UEW estimator takes account of the additive perturbations in the observed series. To avoid confusion, we note that the acronym 'UEW' in what follows always refers to the univariate exact Whittle estimator that accounts for additive perturbations, while 'EW' refers to the original exact Whittle estimator of Shimotsu and Phillips (2002a).

The BEW estimator is motivated by the bivariate Whittle (BW) estimator of Lobato (1999), the exact Whittle (EW) estimator of Shimotsu and Phillips (2002a), and the nonlinear log-periodogram (NLP) estimator of Sun and Phillips (2003). In view of the established properties of the latter three estimators, we expect, under certain regularity conditions, the BEW estimator to be more efficient than the corresponding univariate exact Whittle estimator, to be consistent and asymptotically normal for all values of d, and to be less biased than Lobato's BW estimator in the presence of stationary perturbations.

A theoretical development of the asymptotic properties of the BEW is beyond the scope of the present paper and is left for future research. Instead, we provide some simulation evidence here to justify the new estimator and reveal its finite sample performance in relation to existing procedures. To save space, we only consider the following data generating process:

$$z_{1t} = (1-L)^{-d_1} v_{1t} - \varepsilon_t \tag{16}$$

$$z_{2t} = (1-L)^{-d_2} v_{2t} + \varepsilon_t \tag{17}$$

where $d_1 = d_2$ are long memory parameters, $\{v_{1t}\}$, $\{v_{2t}\}$ and $\{\varepsilon_t\}$ are independent and each is *iid* $N(0, 12)$. For each simulated sample of size $n = 264$, we estimate d_1 and d_2 using the EW, LP and BEW estimators with bandwidth $m = \sqrt{n}$. The EW and LP estimators are based on the individual time series $\{z_{1t}\}$ and $\{z_{2t}\}$, whereas the BEW estimator is based on the bivariate series $\{(z_{1t}, z_{2t})'\}$.

Table III presents the average estimates and the standard deviations (in parentheses) using 500 simulation repetitions. We note the following two main features of the simulation results. First, the BEW estimator achieves substantial bias reduction, producing results that are only slightly downward biased. By contrast the EW and LP estimates both show very significant downward bias, amounting to as much as 50% in some cases. Second, the variance of the BEW estimator appears to lie between that of the EW and LP estimators. It is not so surprising that the BEW estimates have greater variance than the EW estimates. Because it utilizes the system structure, the BEW estimate is expected to be more efficient than the corresponding UEW estimator, which does not make use of the system formulation. On the other hand, the UEW estimator can be expected to have larger variance than the EW estimator because UEW estimation involves the extra parameter arising from the perturbation effects. Apparently, the latter factor outweighs the former, which leads to the BEW estimator having larger variance than the EW estimator.

4.2. Empirical Estimation and Inference using BEW

We now use the BEW estimator to estimate the fractional difference parameters using actual inflation and the *ex post* real rate. Figure 5 presents the results, using the data from 1934 : 1 to 1999 : 4. Compared with Figures 1 and 2, we find that the BEW estimates are significantly higher than both the EW and LP estimates. For the bandwidths considered, the estimated integration orders are larger than 0.75 and centre around 0.9. So, the BEW estimates appear to fall in the same general range as the estimated integration order of the nominal interest rate (based on either EW or LP estimation). These empirical estimates of the long range dependence of the Fisher components are therefore generally consistent with the Fisher equation.

Table III. Finite sample performance of the EW, LP and BEW estimators ($n = 264$)

	EW estimator		LP estimator		BEW estimator	
(d_1, d_2)	$\hat{d}_1$	$\hat{d}_2$	$\hat{d}_1$	$\hat{d}_2$	$\hat{d}_1$	$\hat{d}_2$
(0.4, 0.4)	0.1383	0.1440	0.1306	0.1490	0.3800	0.3779
	(0.1330)	(0.1291)	(0.2068)	(0.2060)	(0.1730)	(0.1749)
(0.6, 0.6)	0.3146	0.3109	0.3273	0.3194	0.5678	0.5761
	(0.1579)	(0.1615)	(0.2025)	(0.2065)	(0.1864)	(0.1903)
(0.8, 0.8)	0.5418	0.5392	0.5709	0.5670	0.7422	0.7449
	(0.1645)	(0.1776)	(0.2056)	(0.2140)	(0.1921)	(0.1813)
(1.0, 1.0)	0.7892	0.7843	0.8417	0.8302	0.9369	0.9319
	(0.1570)	(0.1630)	(0.1983)	(0.2093)	(0.2040)	(0.2000)

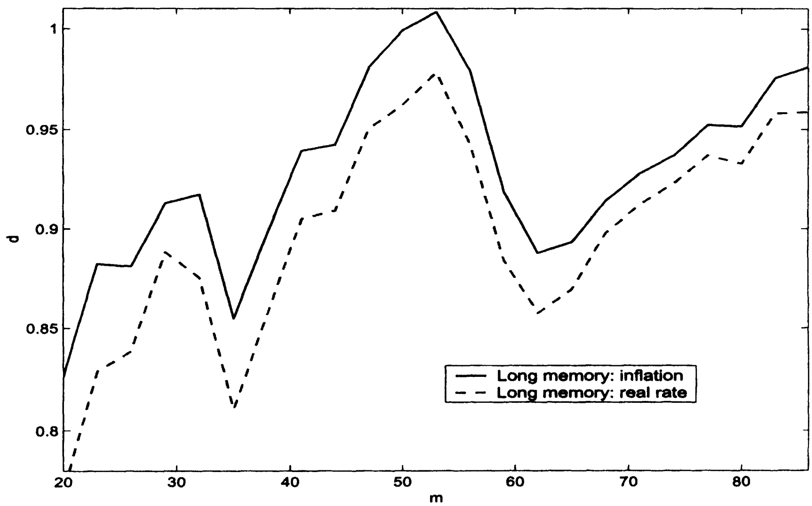

Figure 5. Bivariate exact Whittle estimates of d (sample size = 264)

The empirical estimates in Figures 1, 2 and 5 show that inflation and the real rate may well be integrated of the same order. To investigate this further, we formally test the null $H_0: d_\pi = d_r$ against the alternative $H_1: d_\pi > d_r$. To do so, we construct the test statistic $T = \sqrt{m}(\hat{d}_\pi - \hat{d}_r)$, where $(\hat{d}_\pi, \hat{d}_r)$ is the BEW estimate of (d_π, d_r), rejecting the null if T is larger than some critical value.

Since an asymptotic theory of the BEW estimator has not yet been established, we use simulations to compute the critical value for a given size. The experiment is designed as follows. The data generating processes for r_t and π_t are

$$r_{t+1} = r_t^e - e_{t+1} = (1 - L)^{-d_r} u_t - e_{t+1} \tag{18}$$

$$\pi_{t+1} = \pi_t^e + e_{t+1} = (1 - L)^{-d_\pi} v_t + e_{t+1} \tag{19}$$

where $d_r = d_\pi = d_0$, $\{e_t\}$ is *iid* $N(0, \sigma_e^2)$ and (u_t, v_t) follows a fourth-order vector autoregressive model. Since the long memory cannot be precisely estimated, we use a range of long memory parameters. For each value of $d_0 = 0.5, 0.6, 0.7, 0.8, 0.9, 1$, we first apply the fractional difference operators $(1-L)^{d_0}$ to the forecasted inflation rates (extracted from the *Survey of Professional Forecasters*) and the forecasted real rates (calculated by subtracting forecasted inflation from nominal rates) to get $\{u_t, v_t\}$ and then use them to estimate the VAR in the DGP. The choice of bandwidth in the bivariate Gaussian semiparametric estimator necessarily involves judgement. Although too large a value of m causes contamination from high frequencies, too small a value of m leads to imprecision in estimation. Hence, several values of m are employed. For each value of $m = n^l$, $l = 1/2, 2/3$ and 3/4, we perform 500 replications with sample size $n = 264$. Table IV contains the critical values so obtained for $l = 1/2, 2/3$ and size of 5% and 10%.

The power properties of this test are also examined using simulations. The data generating processes are the same as before except $d_\pi > d_r$. For each combination of d_π and d_r such that $d_\pi = d_r + (k-1)/10, k = 1, \ldots, 6$, we first apply the fractional difference operator $(1-L)^{d_0}$ with $d_0 = d_r$ to the forecasted inflation rates and the forecasted real rates to get $\{u_t, v_t\}$ and then use them to estimate the VAR in the DGP. We report the powers based on 500 replications when the size is 10%, $n = 264$ and $m = n^{1/2}$ in Table V. Apparently, the power increases with the difference between d_π and d_r and is reasonably high when the difference is greater than 0.4.

We now perform this test in the empirical application using the *ex post* data. The results for m at equispaced points in the interval [10, 90] are tabulated in Table VI. For all values of $m \geq 10$,

Table IV. Empirical critical values for the T-test ($n = 264$)

		$d_0 = 0.5$	$d_0 = 0.6$	$d_0 = 0.7$	$d_0 = 0.8$	$d_0 = 0.9$	$d_0 = 1.0$
5%	$l = 1/2$	1.4829	1.7076	1.7600	1.8710	1.8994	1.9060
	$l = 2/3$	0.9174	0.9994	1.0037	0.9775	0.9125	0.9156
10%	$l = 1/2$	1.0538	1.2628	1.3379	1.2526	1.2807	1.2625
	$l = 2/3$	0.7015	0.7467	0.7096	0.7345	0.6365	0.6530

Table V. The power of the T-test ($n = 264$)

	k					
	0	1	2	3	4	5
$d_1 = 0.5$	10.00	19.80	32.20	59.00	81.40	93.50
$d_1 = 0.6$	10.20	15.20	25.60	47.40	71.20	89.00
$d_1 = 0.7$	10.00	16.80	27.00	45.00	68.00	82.80
$d_1 = 0.8$	10.20	17.40	29.40	47.20	65.80	83.60
$d_1 = 0.9$	9.80	15.60	29.40	46.80	67.20	81.20

Table VI. Bivariate estimates and tests: *ex post* data ($n = 264$)

m	10	20	30	40	50	60	70	80	90
$\hat{d}_\pi$	0.8645	0.8266	0.8961	0.9298	0.9993	0.8974	0.9268	0.9516	0.9780
$\hat{d}_r$	0.6610	0.7700	0.8537	0.8907	0.9624	0.8694	0.9105	0.9327	0.9516
T	0.6504	0.2530	0.2320	0.2468	0.2608	0.2175	0.1358	0.1690	0.2505
$\hat{d}$	0.7572	0.8181	0.8958	0.9244	0.9876	0.8929	0.9271	0.9501	0.9717

including the values not presented here, $\hat{d}_\pi$ is only slightly larger than $\hat{d}_r$. Using the critical values given in Table IV (and those not reported here for intermediate values), we cannot reject the null at the level of 10%. According to this evidence, therefore, inflation and the real rate are integrated of the same order. From the *ex ante* Fisher equation, the nominal rate also has the same order of integration. As a consequence, all three *ex ante* and *ex post* Fisher components share the same degree of persistence. The last row of Table VI reports the restricted estimates when we impose the restriction that $d_\pi = d_r$. As might be expected, the restricted estimates fall between the unrestricted ones, which are presented in the first two rows in Table VI. Combining this with the estimates based on the univariate nominal rate series, these empirical findings suggest that the integration order of the Fisher components lies between 0.75 and 1.

Equality of the integration orders of the three Fisher components has important implications for the long-run Fisher hypothesis, which states that the nominal rate moves with the expected inflation rate in the long run. Since both variables appear to be fractionally nonstationary, validity of the long-run Fisher hypothesis requires the existence of a fractional cointegrating relationship between these variables. In addition, the full Fisher effect implies the cointegrating vector must be $(1, -1)$. In effect, therefore, the long-run Fisher hypothesis requires that the residual in this relationship, i.e., $i_t - \pi_t^e$, be less persistent than i_t and π_t^e. However, this residual is the *ex ante* real rate, which according to the evidence above is as persistent as π_t^e. It follows from these findings that the relationship is not cointegrating and the long-run Fisher hypothesis does not hold.

5. CONCLUSIONS

Many empirical studies in the past have investigated the orders of integration of the Fisher equation variables. Given the recent development of robust semiparametric estimation methods for stationary and nonstationary long memory, this practice is now being extended to include analyses of the degree of persistence using fractional models and estimates of the degree of long-run dependence in each of the series. For time series that may be perturbed by weakly dependent noise, we show here that estimating the degree of persistence is more difficult because of the presence of a strong downward bias in conventional estimates of long memory.

In the present context, *ex post* data can be viewed as noisy observations of the *ex ante* variables. Our findings reveal that conventional semiparametric estimation using *ex post* data substantially underestimates the true degree of persistence in the *ex ante* variables and, hence, that of the *ex post* variables themselves. The bivariate exact Whittle estimator introduced here explicitly allows for the presence of additive perturbations or short memory noise in the data. This new estimator enhances our capacity to separate low-frequency behaviour from high-frequency fluctuations and gives us estimates of long range dependence that are much less biased when there is noise contaminated data. Evidence based on this new estimator supports the hypothesis that the three Fisher components are integrated of the same order. Accordingly, we find little support for the presence of a cointegrating relation among the Fisher variables and, therefore, little support for the long-run Fisher hypothesis.

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REFERENCES

Andrews DWK, Sun Y. 2004. Adaptive local polynomial whittle estimation of long-range dependence. *Econometrica* (forthcoming).

Croushore D. 1993. Introducing: the Survey of Professional Forecasters. *Business Review*, the Federal Reserve Bank of Philadelphia, Nov. 3–15.

Fisher I. 1930. *The Theory of Interest*. A.M. Kelly: New York.

Geweke J, Porter-Hudak S. 1983. The estimation and application of long-memory time series models. *Journal of Time Series Analysis* **4**: 221–237.

Gil-Alaña LA, Robinson PM. 1997. Testing of unit root and other nonstationary hypotheses in macroeconomic time series. *Journal of Econometrics* **80**: 241–268.

Hamilton JD. 1985. Uncovering financial market expectations of inflation. *Journal of Political Economy* **93**: 1224–1241.

Hurvich CM, Ray BK. 1995. Estimation of the memory parameter for nonstationary or noninvertible fractionally integrated processes. *Journal of Time Series Analysis* **16**: 17–42.

Keane M, Runkle DE. 1990. Testing the rationality of price forecasts: new evidence from panel data. *American Economic Review* **80**: 714–735.

Kim CS, Phillips PCB. 1999. Log periodogram regression: the nonstationary case. Mimeo, Cowles Foundation, Yale University.

Lobato IN. 1999. A semiparametric two step estimator in a multivariate long memory model. *Journal of Econometrics* **90**: 129–153.

Lobato IN, Velasco C. 2000. Long memory in stock market trading volume. *Journal of Business and Economics Statistics* **18**: 410–427.

Maynard A, Phillips PCB. 2001. Rethinking an old empirical puzzle: econometric evidence on the forward discount anomaly. *Journal of Applied Econometrics* **16**: 671–708.

Mishkin FS. 1992. Is the Fisher effect for real? *Journal of Monetary Economics* **30**: 195–215.

Mishkin FS. 1995. Nonstationarity of regressors and test on the real rate behavior. *Journal of Business and Economics Statistics* **13**: 47–51.

Phillips PCB. 1998. Econometric analysis of Fisher's equation. Cowles Foundation Discussion Paper #1180, Yale University.

Phillips PCB. 1999. Discrete Fourier transforms of fractional processes. Cowles Foundation Discussion Paper #1243, Yale University.

Robinson PM. 1994. Time series with strong dependence. In *Advances in Econometrics, Sixth World Congress*, vol. 1, Sims CA (ed.). Cambridge University Press: Cambridge; 47–96.

Robinson PM. 1995. Gaussian semiparametric estimation of long range dependence. *Annals of Statistics* **23**: 1630–1661.

Rose AK. 1988. Is the real interest rate stable? *Journal of Finance* **43**: 1095–1112.

Shimotsu K, Phillips PCB. 2002a. Exact local Whittle estimation of fractional integration. Cowles Foundation Discussion Paper #1367, Yale University.

Shimotsu K, Phillips PCB. 2002b. Local Whittle estimation of fractional integration and some of its variants. Mimeo, Yale University.

Solo V. 1992. Intrinsic random functions and the paradox of 1/f noise. *SIAM Journal of Applied Mathematics* **52**: 270–291.

Sun Y, Phillips PCB. 2003. Nonlinear log-periodogram regression estimator for perturbed fractional processes. *Journal of Econometrics* **115**: 355–389.

Velasco C. 1999a. Gaussian semiparametric estimation of non-stationary time series. *Journal of Time Series Analysis* **20**: 87–127.

Velasco C. 1999b. Non-stationary log-periodogram regression. *Journal of Econometrics* **91**: 325–371.

