Notes for the Extended Model Life Tables (version 1.3)

The version 1.3 of model life tables is an update and extension from the previous version 1.2 of the abridged model life tables made available by the Population Division (see notes for version 1.2 at the end of this file). Unlike the version 1.2, in which the complete model life tables were derived from the abridged model life tables, life tables in the version 1.3 were generated the other way around. That is, the abridged model life tables in the version 1.3 were computed using the complete model life tables. The complete model life tables in the version 1.3 were constructed using the following procedures.

1. **Complete model life tables**

1.1 **Age 0:** ${}_1m_0$, ${}_1q_0$ and ${}_1a_0$ were directly copied from the previous version 1.2.

1.2 **Ages from 1 to 4** The formula ${}_1q_x = A^{(x+B)^C}$ developed by Heligman and Pollard (1980) was applied to fit ${}_1q_1$, ${}_1q_2$, ${}_1q_3$, and ${}_1q_4$. A, B, and C are parameters to be estimated from the equation for each sex by each level of life expectancy at birth from 20 to 100. A constraint of probability of death was imposed to ensure that the fitted ${}_4\hat{q}_1$ (=1- $(1-{}_1\hat{q}_1)*(1-{}_1\hat{q}_2)*(1-{}_1\hat{q}_3)*(1-{}_1\hat{q}_4)$) equals the original ${}_4q_1$ or the difference between fitted and the original probabilities is smaller than the specified tolerance (10E-6). In implementation, two additional constraints were imposed so that ${}_1\hat{q}_0$ and ${}_5\hat{q}_5$ also equal ${}_1q_0$ and ${}_5q_5$ in the model life table. Note that this approach is also adopted in the ICM procedure in the MortPak software developed by the UN (United Nations, 2013: 43). We then estimated ${}_1\hat{m}_1$, ${}_1\hat{m}_2$, ${}_1\hat{m}_3$, and ${}_1\hat{m}_4$ using the Greville formula [${}_1a_x = \frac{1}{2} - \frac{1}{12}({}_1\hat{m}_x - \frac{\ln({}_1\hat{m}_{x+1}/{}_1\hat{m}_{x-1})}{2})$](Greville, 1943; Chiang, 1984) with iteration by assuming that initial values of ${}_1a_1$, ${}_1a_2$, ${}_1a_3$ and ${}_1a_4$ were respectively 0.43, 0.45, 0.47, and 0.49 (Chiang, 1984: 144), and while imposing a constraint of ${}_4\hat{m}_1$ equal to ${}_4m_1$.

1.3 **Ages from 5 to 49:** We mainly relied on the Piecewise Cubic Hermite Interpolating Polynomial (***pchip***) method to obtain ${}_1q_x$ for ages 5 to 49. The choice of the ***pchip*** method is fully determined by the goodness of fit based on the comparisons across different methods using HMD data. The other interpolation methods (all were applied to Lx) included the Elandt-Johnson and Johnson method (Elandt-Johnson and Johnson, 1999: 111-114), Karup-King four-term third-difference formula, Sprague six-term fifth-difference formula, Beers six-term ordinary formula, and Beers six-term modified formula (Siegel and Swanson 2004: 677-732). Another method based on the change rate between ${}_5p_x$ and ${}_5p_{x+5}$ for each ${}_1p_x$, within a given age group, was also compared across six stipulated methods. The results reveal that the ***pchip*** method is the most robust.

The procedure to apply the ***pchip*** method is as follows. (1) we first took (1- $\sqrt[5]{1-{}_5q_x}$) as the average probability of death for a given age group($x+2.5$) and the interpolated probabilities of death for each single year of age ${}_1\hat{q}_x$ using the ***pchip*** method. A constraint of probability of death was imposed during the interpolation to ensure that the differences between the estimated probability of death for a given five-year group and the original probability would be smaller than the specified tolerance (10E-6). (2) A simple seven-term moving average was performed after the application of the *pchip* method to smooth the estimated probabilities of death ${}_1\hat{q}_x$. The smooth was performed iteratively to ensure the constraint noted in (1). (3) We then estimated ${}_1\hat{m}_x$ from ${}_1\hat{q}_x$ by assigning an initial value of 0.5 to the ${}_1a_x$ for each single year of age (as per the Human Mortality Database method protocol, see Wilmoth et al., 2007:38). We then estimated ${}_5\hat{m}_x^1$ to verify whether the ${}_5\hat{m}_x^1$ was equal to the original ${}_5m_x$ from the model life table of the version 2. If not, iterations for ${}_1a_x$ were performed to ensure ${}_5\hat{m}_x^k$ to be as close as possible to ${}_5m_x$ after k^{th} iterations. In this step, ${}_1\hat{a}_x$ was also updated. (4) Once ${}_5\hat{m}_x^k$ was estimated (i.e., ${}_1\hat{m}_x^k$ was estimated), we re-calculated the average probability of death for each of five-year age groups and re-implemented the procedures (3) and (4) until the changes of estimates were smaller than the specified tolerance between adjacent iterations (10E-6).

1.4 **Ages from 50 to 79**: We applied the Gompertz model to estimate ${}_1\hat{m}_x$ for ages between 60 and 79 based on ${}_5m_x$ for ages 50-54, 55-59, 60-64, 65-69, 70-74, and 75-79 from the version 1.2. The Gompertz function $\mu_x = ae^{bx}$ is from the formula presented in Chapter Two of the book "Force of Mortality at Ages 80 to 120" by Thatcher, Kannisto, and Vaupel (1998) (available at http://www.demogr.mpg.de/Papers/Books/Monograph5/start.htm), where a and b are parameters, and μ_x is the force of mortality at exact age x. The model was fitted by the maximum likelihood method using the formula of $L(x) = -D(x)\log(\hat{q}_x) - (N - D)\log(1 - \hat{q}_x)$; where *L(x)* is the likelihood function, *D* is number of persons who survive to age x but die before they reach age $x+n$; *N* is number of persons who reach age x; $\hat{q}_x$ is estimated probability of dying from age x to $x+n$. Once μ_x is fitted, we could get its survival function [$S(x) = \exp((\frac{a}{b}(1-e^{bx}))$], and then ${}_1\hat{m}_x$ and ${}_1\hat{q}_x$. Subsequently, ${}_1\hat{a}_x$ could be easily obtained.

1.5 **Ages 80 and older:** We applied the Kannisto model to estimate ${}_1\hat{m}_x$ for ages 80 and beyond based on ${}_1m_x$ for ages 80-99 derived from the extrapolations from the Gompertz model in Step 1.4. This approach is based on evidence of inadequate fitting of mortality for ages beyond 100 by the Gompertz model, yet relative accuracy of fitting for ages before 100 (Thatcher, Kannisto, and Vaupel, 1998). The Kannisto function $\mu_x = c + \frac{ae^{bx}}{1+ae^{bx}}$ is from the formula presented in Chapter Two of the book "Force of Mortality at Ages 80 to 120" by Thatcher, Kannisto, and Vaupel (1998), where *a, b*, and *c* are parameters, and μ_x is the force of mortality at exact age x. The model was also fitted by the maximum likelihood method using the same likelihood formula for mortality at ages 50 to 79. Once μ_x is fitted, we could get its survival function [$S(x) = \exp(-((cx + \ln(ae^{bx}+1)/b) - \ln(a+1)/b))$], and then ${}_1\hat{m}_x$ and ${}_1\hat{q}_x$. Subsequently, ${}_1\hat{a}_x$ could be easily obtained.

1.6 In a final round, minor adjustments were made for ${}_1\hat{m}_x$ (and thus also for ${}_1\hat{a}_x$) proportionally according to the distribution of dx to ensure that T_0 equals to E0*100,000, where E0 is a given level of life expectancy at birth. We used Matlab to implement this process.

2. Abridged model life tables

Abridged life table were constructed from the completed life table. The main reason we used the approach described above to reconstruct the model life tables (i.e., generating abridged model life tables using completed model life tables) is that we have detected noticeable biases for ${}_5a_x$ in the abridged life table using Greville's formula [${}_na_x = \frac{n}{2} - \frac{n^2}{12}({}_nm_x - \frac{\ln({}_nm_{x+n}/{}_nm_{x-n})}{2n})$] when ${}_5m_x$ is greater than 0.5. In other words, we are not able to construct an accurate abridged life table when only ${}_5m_x$ or ${}_5q_x$ is available and when mortality at some age groups is larger than 0.5 as it normally happens at older ages (e.g., ages 90 or over under a level of life expectancy at birth of 82.5 years).

The version 1.3 was primarily prepared by Danan Gu.

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Notes for the Extended Model Life Tables (version 1.2)

Two sets of standard model life table families (Coale-Demeny 1966 and 1989, and United Nations, 1982) are commonly used to derive a variety of mortality indicators, and, as underlying mortality patterns for estimation and projection by the United Nations and the demographic research community at large. But these two sets of model life tables - designed primarily to be used in developing countries or for historical populations cover mortality patterns only for a life span from ages 20 to 75. A first extension of these model life tables was produced by Thomas Buettner in 1998, which extended the initials sets of model life tables from e(0)=75.0 up to 92.5 using both a limit life table as an asymptotic pattern and the classic Lee-Carter approach to derive intermediate age patterns (Buettner, 2002).

With the extension of the projection horizon for all countries up to 2100, and as part of the 2012 Revision of the UN World Population Prospects, it was necessary to allow life expectancy at birth to go beyond 92.5 years. In addition, in-depth analysis of the initial 1998 extension revealed substantial deviation for out-of-sample predictions compared to the Human Mortality Database experience at very low mortality levels (especially for Coale-Demeny models, see Figure 1 in Wilmoth et al., 2009), and the need to improve a smoother transition between the existing set of model life tables up to age 75 and their extension. A new set of extended model life tables was computed in Spring 2010 by the staff of the Population Division (Gerland and Li) based on the modified Lee-Carter approach. After extensive cross-validation against the Human Mortality Database (HMD) performed by Kirill Andreev, some constraints have been imposed to ensure some convergence toward the HMD mortality experience at high levels of e(0). The nine families of model life tables extended up to e(0)=100 were smoothly blended to the existing ones to insure smooth mortality surfaces by age and sex and e(0) levels.

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